Microeconomics Comprehensive Exam
Stanford Economics Department
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Instructions: You have six hours to complete the exam. Please use a *separate* blue book for each question. Be clear and concise in your answers. Points will be subtracted for wrong and/or rambling answers. Good luck!

Question 1. Neoclassical and Labor-Managed Firms *(50 points)*

Consider a world where capital and labor are inputs to production. Given an output price p and input prices w, r, the standard *neoclassical firm* chooses inputs to maximize profits:

$$\max_{k,l\geq 0} pf(k,l) - wl - rk.$$

The firm will choose to operate if and only if it can achieve non-negative profits.

In contrast, the objective of a *labor-managed firm* is to maximize profits *per unit of labor*:

$$\max_{k,l\geq 0} \frac{1}{l}\left[pf(k,l) - wl - rk\right]$$

If a firm is labor-managed, it will operate if and only if it can give a revenue share to each worker that exceeds foregone wages (i.e. if the solution to its maximization problem is non-negative).

Assume that $f(\cdot)$ is continuous, increasing and differentiable, that $f(0,0) = 0$, and that solutions to the problems you encounter are unique.

For parts (a)–(c) assume that capital is fixed at some level $\overline{k} > 0$.

(a) Show that in response to a higher product market price p, the neo-classical firm will raise its labor input.

(b) How will a labor-managed firm respond to this same increase, assuming that $f(\cdot)$ is concave?

(c) Show that if prices (p, w, r) are such that a labor-managed firm would operate at positive levels, then the labor managed firm will choose a lower level of labor input than the neo-classical firm.

(d) Are your conclusions to (a)–(c) still valid if capital is a choice variable?

(e) Show that the set of prices (p, w, r) at which the neo-classical and labor-managed firms would operate is identical.

(f) Paul Samuelson wrote that "in a perfectly competitive economy, it does not matter whether capital hires labor, or labor hires capital." Show that if prices (p, w, r) are such that the neo-classical firm earns zero profits, the neo-classical and labor-managed firms would make identical choices.

Question 2. General equilibrium (25 points)

Consider a standard Arrow Debreu exchange economy: There are H housholds and L commodities. Each household $h = 1, ..., H$ has a utility function which is continuous and increasing. Each household has strictly positive individual endowments $e^h \in I\!R^L_{++}$.

(a) Define a Walrasian equilibrium and Pareto efficiency.

(b) Give an example (utility functions and individual endowments) to show that without further assumptions on utility Walrasian equilibrium does not always exist.

(c) Suppose that $\sum_{h \in \mathcal{H}} e^h_l = 1$ for all commodities $l = 1, ..., L$. Suppose that for each h $u^h(c) = \sum_{l=1}^{L} v^h(c_l)$, where $v^h : I\!R_+ \rightarrow I\!R$ is increasing, strictly concave, differentiable and satisfies $\lim_{x \rightarrow 0} v^{h\prime}(x) = \infty$. Prove that the economy has a at most one competitive equilibrium. (Hint: One way to do this is to consider first order conditions across households and market clearing.).

Question 3. Choice under Uncertainty (25 points)

Consider the standard von Neumann-Morgenstern model of choice under uncertainty. Suppose the prize space (in dollars) is

$$\mathcal{Z} = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

and consider choices by an agent whose preferences (over lotteries) satisfy the von Neumann-Morgenstern axioms. Each lottery can be written as an 8-vector of probabilities, the first element is the probability of getting \$1, the 8th element the probability of getting \$8.

(a) A risk averse von Neumann-Morgenstern agent (with increasing utility) has to compare the following two gambles

$$p = (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}) \quad \text{and} \quad q = (\frac{2}{8}, 0, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, 0, \frac{2}{8})$$

Can you say unambiguously which one she would prefer ? Justify your answer.

For the purposes of parts b) and c), you can assume that the agent's Bernoulli utility is actually defined over the entire interval $[0, 10]$ and that it is differentiable (i.e. the agent has preferences over probability distributions over $[0, 10]$ but we restrict ourselves to lotteries which only put positive probability on elements in $\mathcal{Z}$).

(b) A risk averse von-Neumann-Morgenstern agent whose preferences exhibit constant relative risk aversion has to compare the following two gambles

$$p = (\frac{1}{4}, \frac{1}{4}, 0, 0, \frac{1}{8}, \frac{1}{8}, 0, \frac{1}{4}) \quad \text{and} \quad q = (0, 0, 0, 1, 0, 0, 0, 0)$$

Can you say unambiguously which gamble she would prefer ?

Assume that the coefficient of relative risk aversion is above one. Can you now say which gamble the agent would prefer?

Question 3. continued.

(c) Suppose a risk averse (von Neumann-Morgenstern) agent has preferences which exhibit decreasing absolute risk aversion. Suppose we observe

$$(\frac{1}{2}, 0, 0, \frac{1}{2}, 0, 0, 0, 0) \succ (0, 1, 0, 0, 0, 0, 0, 0)$$

Can you say unambiguously how he will choose between

$$p = (0, 0, 0, 0, \frac{1}{2}, 0, 0, \frac{1}{2}) \qquad \text{and} \qquad q = (0, 0, 0, 0, 0, 1, 0, 0)$$

Justify your answer.

Question 4. Public Good Provision (35 points)

Two agents 1 and 2 can contribute to the provision of a public good. The good will be provided if and only if the sum of their contributions exceeds some critical level X. In this case, each agent obtains a benefit B. However, contributing is costly and contributions are non-refundable. The cost of contributing x is $c(x) = x$. Assume that $2B > X$ — providing the good is efficient.

Suppose the agents simultaneously choose contribution levels x_1 and x_2, after which the good is provided if and only if $x_1 + x_2 \geq X$.

(a) Identify the pure strategy Nash equilibria for the case of $B \geq X$.

(b) Identify the pure strategy Nash equilibria for the case of $B < X$.

Suppose instead that agents contribute sequentially, with agent 1 contributing in period 1 and agent 2 in period 2. Assume contributions are observable and that the good is provided as soon as the total contribution level exceeds X (that is, either in the first period, the second period, or never). Suppose both agents discount at a rate δ. If agent i contributes x_{it} in period t, his payoff is $\delta^{t-1}B - \sum_t \delta^{t-1}x_{it}$ if the public good is provided in period t and $-\sum_t \delta^{t-1}x_{it}$ if it isn't provided.

(c) Derive the unique subgame perfect equilibrium. Under what conditions is the public good provided in the first period, the second period, or never.

(d) Supposing that $B = X$, identify a Nash equilibrium that is not subgame perfect.

Question 5. Arms Race (30 points)

Consider the following stylized model of an arms race. Two countries A and B must decide whether or not to initiate weapons programs. Building weapons yields a direct return x_i ($i = A, B$), typically but not necessarily negative, and also offers some protection if the other country arms. Payoffs can be represented in the following matrix, where $0 < a < 1$.

	Don't Build	Build
Don't Build	$0, 0$	$-1, x_B$
Build	$x_A, -1$	$x_A - a, x_B - a$

We assume that the countries will choose simultaneously and non-cooperatively whether or not to arm.

(a) What is a dominant strategy? For what values of x_A does country A have a dominant strategy.

(b) What are the pure strategy Nash equilibria of the arms race game?

Now assume that the countries' returns to arming, x_A and x_B, are private information (so only A knows x_A). Assume also that x_A and x_B are independent draws from a uniform distribution on $[b-1, b]$, where $0 < b < a$.

(c) Prove that country i's equilibrium strategy must have a cut-off form (i.e. build if x_i is above some threshold).

(d) Identify the Bayesian Nash equilibrium or equilibria.

Question 6. Price as a Signal of Quality *(35 points)*

A monopolist with a new product faces a unit mass of identical consumers. The monopolist's product may be of high or low quality. Consumers value high quality at v_H and low quality at v_L. However, while the monopolist knows the quality of his product, consumers do not. Instead, they assign a commonly known probability γ to the product being high quality. The monopolist's marginal costs of production are c. Assume that $v_H > c > v_L$ so that only a high quality monopolist can succeed under complete information. Assume also that $\gamma v_H + (1 - \gamma) v_L < c$.

Consider a game where the monopolist sets a price and consumers then decide whether or not to purchase.

(a) Prove that in Perfect Bayesian Equilibrium (PBE) no trade will occur at a price above marginal cost.

Now suppose that the monopolist can produce and each consumer is interested in purchasing in periods 1 and 2. In each period the monopolist sets a price and consumers decide whether to purchase. If a consumer purchases in the first period, she learns product quality and believes accurately that product quality will be the same in the second period. There is no discounting.

(b) Can you construct a separating PBE in which the monopolist produces in the first period only if his quality is high?

(c) Assuming a separating equilibrium exists, what pattern do prices follow? Give an economic interpretation.

(d) Is there also a pooling equilibrium in which no trade occurs? Either construct such an equilibrium or explain why it does not exist.

Question 7: *Arbitrage Free Pricing* (30 points)

Consider a standard two period economy dated $t = 0$ and $t = 1$ in which consumption takes place at both date 0 and date 1. Agents have utility of consuming in both periods. Uncertainty at date 1 is represented by three states of nature. There is only one consumption good which is used as a numenraire hence the spot price of a unit of consumption good at dates 0 and 1 is 1.

At date 0 the agents trade five securities with payoff vectors and market prices

for security 1	$r_1 = (2, 1, 0)$,	$q_1 = 1.0$
for security 2	$r_2 = (0, 1, 1)$,	$q_2 = 0.4$
for security 3	$r_3 = (0, 1, 2)$,	$q_3 = 0.6$
for security 4	$r_4 = (1, 2, 3)$,	$q_4 = 1.4$
for security 5	$r_5 = (3, 2, 1)$,	$q_5 = 1.6.$

All quantities are defined in units of the consumption good.

(1) Is the price system provided arbitrage free? If your answer is YES, prove it. If No, explain why not.
(2) If your reply to question (1) is NO, deduce an arbitrage free pricing by altering at most one price.
(3) Given that you now developed arbitrage free pricing, *assume that these prices arise in an REE with the specified securities.*

(i) What would be the market price of a call option on security 4 with a strike price of 2?
(ii) What is the risk free consumption interest rate on loans taken at date $t = 0$?

Question 8: *Risk Free Rate* (40 points)

Consider an infinite horizon economy with $N + 1$ securities. There are N long lived risky securities enumerated $j = 1,2, \ldots, N$ and a one period riskless security enumerated 0. Denote security prices and dividends by p_t^j, d_t^j respectively. The discounted price of the riskless debt instrument is denoted by $1/(1 + r_{0t})$.

We now consider the optimization of agent k. *All quantities below are associated with agent k and carry his index.* However, for simplicity of notation we avoid writing down the index k on all terms. For example, agent k's date t porfolio is denoted by $(\theta_t^0, \theta_t^1, \ldots, \theta_t^N)$. This agent solves the following dynamic programming problem:

$$\underset{c,\theta}{\text{Max}}\, E_Q \sum_{t=0}^{\infty} \beta^t \log(c_t) \qquad \text{with } \beta = .90$$

such that for all t

$$\theta_{t-1}^0 + \sum_{j=1}^{N} \theta_{t-1}^j (p_t^j + d_t^j) = c_t + \theta_t^0 \frac{1}{1 + r_{0t}} + \sum_{j=1}^{N} \theta_t^j p_t^j$$

$$(\theta_1^0, \theta_1^1, \ldots, \theta_1^N) \text{ is given.}$$

Denote the risky return on security j by

$$1 + R_{t+1}^j = \frac{p_{t+1}^j + d_{t+1}^j}{p_t^j}, \qquad j = 1, 2, \ldots, N.$$

In an extensive consumer survey agent k's past portfolios and consumptions were recorded in detail. Investigators found out that the empirical regularity of the data is described by the process:

$$c_{t+1} = \frac{c_t}{\mu_c + \rho_{t+1}^c}$$

$$R_{t+1}^j = .14 + \rho_{t+1}^j$$

where $\rho_{t+1}^c > \mu_c$ with probability 1 (hence consumption is always positive) and the vector

$$\begin{pmatrix} \rho_{t+1}^c \\ \rho_{t+1}^j \end{pmatrix} \text{ is distributed i.i.d with mean (0,0) and convariance matrix } \begin{bmatrix} \frac{2}{25}, & -\frac{1}{9} \\ -\frac{1}{9}, & \frac{1}{10} \end{bmatrix}.$$

1. Set up the Euler equations for the optimizing agent;
2. Can you deduce from the information provided what is the risk-free rate in this economy? If you can, please explain the steps you have taken to arrive at that deduction.
3. From the data provided what is the value of μ_c. Why?

Question 9: *Propositions Regarding Asset Pricing* (30 points)

Do you agree or disagree with the statements below? Please explain and support your answer. Be *very brief*; you will be penalized for wrong or irrelevant reasoning.

1. The Expectations Hypothesis implies that the forward rate is an unbiased forecast of the future spot rate.
2. In a *multi-period* economy a complete market structure requires that the number of linearly independent long lived securities be equal to the number of Arrow-Debreu states.
3. If the market structure is incomplete then there are no circumstances in which an agent can be guaranteed a riskless consumption flow in an REE with securities.
4. State prices are unique if the market structure is complete.
5. The common prior assumption can be deduced from Savage's utility theory axioms.
6. An allocation of a sunspot equilibrium is never Pareto Optimal if sunspots have a material effect on the allocation.
7. A standard *Negishi Theorem* can be used to show that a representative agent with time additive utility function can always be constructed for a competitive economy.
8. If the true parameter is in the support of the prior then, *for any parameter space*, a Bayesian agent learns the true parameter with true probability 1 as the amount of data increases without bound.