Inverted Yield Curves and the Conundrum:
A Dynamic Model of the Term Structure of Interest Rates

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Abstract

This paper estimates a dynamic term structure with the inflation rate and the output gap as the explanatory variables. The inflation variable is associated with a level change in the yield curve and the output gap is associated with a slope effect. The use of survey data improves the goodness-of-fit of the term structure models. Survey data contains information that is not captured by rational expectations models using historical data. The estimates and results of the term structure model can be used to analyze periods when the yield curve is not “normal”. Inverted yield curves are caused by the combination of high inflation and a rising output gap that both have a slope component. Short term yields rise more than long term yields in response to these variables, and when they go beyond a threshold, the yield curve inverts. The “conundrum” in 2004 and 2005 where the Fed raised short term rates and caused the yield curve to flatten as long term rates fell can be seen as an intermediate stage in the eventual inversion of the yield curve. This model predicts that the combination of high inflation along with the increasing output gap were responsible for the conundrum and for the subsequent inverted yield curve in the early part of 2006.

I would like to thank John Taylor for his mentorship and his exceptional help in guiding my research. I am also grateful to Albert Chun and Geoffrey Rothwell, without whom this project would not have been possible. Finally, I would like to thank my family and friends for their patience and constant encouragement.
1. Introduction

How do inflation and the output gap affect interest rates? There is a broad body of economic literature supporting the idea that a simple rule using inflation and the output gap accurately describe the monetary policy decisions of the Federal Reserve. This paper explores whether the same variables can be used to describe the dynamics of the full term structure of interest rates. Understanding the nature of this relationship has important implications for policy-makers.

Monetary policy relies on adjustments of the short rate as the primary policy tool to stimulate or slow down economic activity. The short rate is a monetary transmission mechanism by which the central bank affects real economic variables like output and employment. The effect of the short rate on interest rates, exchange rates, asset prices, and firm balance sheets are the channels through which monetary policy affects these real economic variables. For many countries, simple rules based on a few macroeconomic variables are very effective at describing monetary policy and the short rate. Since consumers and businesses borrow money for longer terms, policy makers need to understand how changes in the short rate affect long term yields. Additionally, since they are trying to predict future rates based on current data, they must also understand how macroeconomic variables change over time and how long term yields respond to them.

The US Treasury Bond market is one of the deepest and most liquid markets in the world. Any model that tries to capture the dynamics of the term structure of interest rates must do so in an arbitrage free setting for the predictions of the model to be useful.

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Recent research has shown that macroeconomic variables play a significant role in explaining the term structure of interest rates. Most researchers have studied the dynamics of the term structure using historical data on macroeconomic variables. In this paper, I take a similar approach using data on inflation and the output gap to generate rational expectations of these variables in the future and describe the dynamics in yields using those expectations. I also expand on the existing work by estimating a different model using data from surveys of economic forecasters. Future yields should depend on market expectations of the future state of the economy. While people’s expectations about the future state may depend on the current state, their expectations should contain views that are based on more than just historical data. I first explore whether historical data provide a good fit for yields. I then explore whether the use of survey data fits yields better than the forecasts generated using rational expectations based on historical values. Finally, I apply the results of the model to analyze inverted yield curves and the interest rate “conundrum” during 2004 and 2005.

2. Previous Research

Models of the term structure of interest rates have been explored by many researchers. The work begins with Vasicek (1977) and the Cox-Ingersoll-Ross (1985). These model the term structure of interest rates in a general equilibrium setting and use short rates as the factor that predicts the full term structure of interest rates. Since then, researchers have focused on developing models which incorporate factors other than

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observed yields to explain interest rates. Work by Litterman and Scheinkman (1991), Chen and Scott (1993), and others focused on using latent factors to estimate the dynamics of the yield curve. Duffie and Kan (1996) generalized this approach, and showed that yields could be described by an affine function of a set of general state variables. They also showed that many of the other models including the CIR model can be interpreted as special cases of their model. The generality of this model has made it the workhorse model in the term structure literature.

2.1 Latent Factor Models

Some of the early work on term structure models focused on traditional factor analysis. Litterman and Scheinkman (1991) computed the principal components of yield changes and found that the first three principal components explained about 96% of the variation in yields. They called the three factors “level,” “slope,” and “curvature.” The level factor refers to a parallel shift in the yield curve, the slope factors refers to a steepening or flattening, and the curvature factors refers to the twisting between intermediate term and short and long term yields. Other researchers have found similar results.4 The level-slope-curvature factors are closely related to the latent factors that have been used for affine term structure models. Rather than using observed state variables, the state variables are backed out from the observable yields. This approach has been used in continuous time by Dai and Philippon (2004), and Dai and Singleton (2002) among others. The latent factors used in affine term structure models behave essentially like the level, slope, and curvature factors. In a survey, Piazzesi (2003)

4 See Knez, Litterman, and Scheinkman (1994) and Bliss (1997)
reports that this relationship is robust across time periods, data sets, and model specifications. She claims, however, that there is no robust mapping between the Litterman and Scheinkman labels and the stochastic mean and stochastic volatility terms that characterize the affine term structure models. The latent factor approach is how researchers in finance have dealt with questions about the term structure of interest rates. The major drawback of this approach is that the factors are not observable, and so they do not lend themselves to good forecasting methods. They also do not provide any explanation of how macroeconomic variables affect the term structure.

2.2 Macroeconomic Factor Models

Macroeconomists have taken an approach that is different from the models used in the finance literature. Work by Taylor (1993) and McCallum (1994) have focused on using monetary policy rules to describe the dynamics of the short rate. These approaches have been very successful at describing monetary policy. However, these models assume a simple relationship between the short rate and the longer term yields. As a result, although the models describe short rates very well, they do not fit longer term yields is very well. Some more recent work in the macro literature has focused on incorporating macroeconomic variables in the term structure model. Evans and Marshall (2000) use a vector auto regression (VAR) model of this form that includes factors for GDP and inflation. Their model, however, does not impose the restrictions of no-arbitrage. Even though the model does a better job of explaining the effects of macroeconomic variables
on the full term structure, the lack of no-arbitrage restrictions means that the model is fundamentally missing out on important aspects of term structure dynamics.

### 2.3 Macro-Finance Models

Researchers in the macro-finance field have focused on integrating the approaches of the finance and macroeconomic literature. Ang and Piazzesi (2003) estimate a multifactor model that uses both macroeconomic and latent variables with no-arbitrage restrictions. They use a discrete time version of the Duffie and Kan (1996) model which generates bond prices as exponential affine functions of state variables. Ang and Piazzesi (2003) show that incorporating macroeconomic factors and latent factors in a term structure model fits yields better. Additionally, they show that there is some tentative mapping between macroeconomic factors and the latent factors. They find that inflation helps explain the variation in the yields that was predicted by the “level” factor, and monetary policy changes explain some of the variation in yields that was predicted by the “slope” factor. Other papers in the field like Ang, Dong, and Piazzesi (2005), Ang, Piazzesi, and Wei (2003), Duffee (2002), and Rudebusch and Wu (2003) have all adopted the macro-finance approach and found that macroeconomic factors do explain dynamics of the yield curve.

Research in this field is exploding and many papers are being written about the macro-finance models. However, very little work has been done with survey data. Ang, Dong, and Piazzesi (2005) estimate a number of no-arbitrage Taylor rules, but their data relies on using historical macroeconomic data to predict the future states rather than using...
survey data. Chun (2006) uses expectations of macroeconomic variables to estimate a term structure model. He uses data from the Blue Chip Financial Forecasts to estimate how forecasts of macroeconomic variables affect the term structure of interest rates.

This paper takes an approach that is similar to Chun (2006), with a few key differences. First, while Chun’s model is in continuous time, this paper will estimate a discrete time model. Secondly, this paper will only look at monetary policy rules based on the specification in Taylor (1993) where the central bank reacts to inflation and the output gap. Chun estimates the model with other rules based on McCallum (1994). Finally, Chun’s specification of the Taylor rule with inflation and output uses forecasts of output growth rather than the output gap. This specification is because output growth should be higher when output is far below potential output, and should be lower when output above potential output. Therefore, his specification of the policy rule would give the opposite sign on output from what would be expected. Instead, this paper will use the output gap in the Taylor rule.

3. Model

3.1 The Affine Term Structure Model

Affine Term Structure Models (ATSMs) describe yields as affine (constant plus linear) functions of some underlying state variables. Yields are specified as

\[ y(\tau) = A(\tau) + B(\tau)X, \]
where $A(\tau)$ is a scalar and $B(\tau)$ is a vector of coefficients on the state variables. Both $A(\tau)$ and $B(\tau)$ are functions of maturity $\tau$. The coefficients make the equations consistent with the conditions of no-arbitrage. Functionally, bond yields for long maturities are risk-adjusted expectations of future short rates. Assumptions about the risk-adjusted dynamics of the state variables govern the expectations. Zero-coupon bond prices are then exponential affine functions of the state vector $X_\tau$ of the form

$$P_{\tau} = \exp\{-A(\tau) - B(\tau)X_\tau\}$$

The no-arbitrage condition is imposed through cross equation restrictions across time. These restrictions are derived from the equations that specify the dynamics of the state vector and risk premium. The functions $A(\tau)$ and $B(\tau)$ are non-linear functions of the parameters of the state vector and the risk premium dynamics.

Research in this area of term structure models has been very active mainly because affine models are highly tractable. The models can be specified in both discrete and continuous time. Continuous time models are useful because Ito’s lemma can be used to transform the problem into a partial differential equation with a closed form solution. However, it is difficult to test a continuous time model because of data restrictions. On the other hand, Ang and Piazzesi (2003) derived equations for $A(\tau)$ and $B(\tau)$ in a discrete time setting and were able to estimate a model based on monthly data.

### 3.2 Affine Term Structure Model in Discrete Time
In this paper, I estimate a discrete time VAR and an affine term structure model with a specified short rate process. Analytically, the model is very similar to the one proposed by Ang and Piazzesi (2003). While it is possible to explain changes in the overnight rate with a few macroeconomic variables, estimating the entire term structure of interest rates poses additional modeling challenges. Long term bonds held for a short period have a risk premium over short term bonds held for that same period. Therefore, assumptions about the parameterization of the risk premium must be made. Furthermore, movements in the cross section of yields are highly correlated, so restrictions need to be imposed on the term structure model to satisfy the no-arbitrage condition. An affine term structure model captures this by specifying yields as an affine (constant plus linear term) function of maturity. This structure makes yield equations for differing maturities consistent. It also makes the model computationally simpler than using other simulation methods.

The short rate process in my model follows a simple Taylor Rule\(^5\)

\[
\begin{align*}
r_t &= \delta_0 + \delta'_r \bar{X}_t,
\end{align*}
\]

The short rate \(r_t\) is the 1-month Treasury rate. \(\delta_r\) is a 2 by 1 vector which has the coefficients for the state variables and \(\bar{X}_t\) are the variables for inflation and the output gap in the model using historical data and expected inflation and expected output gap in the model using survey data. The evolution of the state variables (inflation and the output gap) is specified as a first order Gaussian VAR of the form

\[
\begin{align*}
\bar{X}_t &= \bar{\mu} + \Phi \bar{X}_{t-1} + \Sigma \bar{\varepsilon}_t,
\end{align*}
\]

\(^5\) See Taylor (1993)
where $X_t$ is the same 2 by 1 vector of the variables inflation and the output gap, $\mu$ is a 2 by 1 vector that contains the constant, $\Phi$ is a 2 by 2 matrix with the coefficients on the lag of the state variables, and $\Sigma$ is a covariance matrix that is assumed to be lower triangular.

The next step is to specify a pricing kernel to price bonds under a risk-neutral measure. I will use the assumption of no-arbitrage (Harrison and Kreps, 1979) to guarantee the existence of an equivalent martingale measure $Q$ to generate risk neutral prices where the expectation is taken under the measure $Q$. Ang and Piazzesi (2003) show that given these assumptions, we can derive a pricing kernel $m_{t+1}$ as

$$m_{t+1} = \exp\left(-\frac{1}{2}\lambda_t\bar{\lambda}_t - \delta_0 - \delta_1X_t - \bar{\lambda}_t\bar{\sigma}_{t+1}\right)$$

where $\lambda$ is parameterized as the affine process

$$\bar{\lambda}_t = \bar{\lambda}_0 + \bar{\lambda}_1\bar{X}_t$$

In this parameterization, $\lambda_0$ is a 2 by 1 vector and $\lambda_1$ is a 2 by 2 matrix. The price of a bond can be solved recursively by the first equation below, which simplifies to the second equation

$$p_{t+1} = E_t\left[m_{t+1}p_{t+1}^n\right]$$

$$p_t^n = \exp\{A(t) + B(t)\bar{X}_t\}$$

where the coefficients follow the difference equations (Ricatti equations)

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6 This affine specification has been used frequently in the literature. For discrete time examples, see Ang and Piazzesi (2003), Ang, Piazzesi and Wei (2004), And, Dong and Piazzesi (2005), and Dai and Philippon (2004).

7 For the derivation, see Appendix A or Ang and Piazzesi (2003). Note: the Ang and Piazzesi (2003) derivation contains a sign error but the final recursive equations are correct. This sign error has been corrected in the derivation provided in Appendix A.
\[ \bar{A}(t+1) = \bar{A}(t) + \bar{B}(t)(\bar{\mu} - \Sigma \bar{x}_0) + \frac{1}{2} \bar{B}(t)\Sigma' \bar{B}(t) - \delta \]

\[ \bar{B}(t+1) = \bar{B}(t)(\Phi - \Sigma \bar{x}_t) - \delta_i \]

The continuously compounded yield is given by

\[ y_t = -\frac{\log p_t^n}{n} = A(t) + B'(t)\bar{X}_t \]

where the coefficients are \( A(t) = -\bar{A}(t)/n \) and \( B(t) = -\bar{B}(t)/n \).

### 3.3 The Data-Based Model

The first model I estimate is a Data-Based Model using historical inflation and the output gap as the state variables. In this specification, the Central bank adjusts the short rate based on the deviation of inflation from the target rate of inflation and the gap between actual and potential output. Therefore, the short rate equation is expressed as a Taylor Rule of the form

\[ r_i = \delta^* + \delta_{11}(\pi_i - \pi^*) + \delta_{12}y_i \]

The equation I estimate is

\[ r_i = \delta_0 + \delta_{11}\pi_i + \delta_{12}y_i \]

where \( \delta_0 = \delta^* - \delta_{11}\pi^* \).

I first estimate a restricted version of this model where inflation is the only factor and \( \delta_{12} \) is restricted to be zero. This one-factor model will allow me to test whether inflation is an effective variable to explain changes in yields. The important question in
this model is whether inflation is more important in determining yields in the short or the long run. Since most central banks now follow an inflation targeting regime, this would explain the effects of inflation targeting on the entire yield curve. Furthermore, when the economy is not in recession, it is likely that the central bank is more concerned about inflation than output, so this model would help us see how inflation predicts yields during normal economic times.

Then I estimate the complete model in order to determine how the Taylor rule using historic data describes changes in yields. Taylor (1993) argues that a monetary policy rule with inflation and the output gap as the explanatory variables is very effective at describing the policy of the central bank. This model tests how well these variables explain the full term structure of interest rates. It also allows us to examine the effect of monetary policy, which adjusts the short rate, can have on longer term yields.

3.4 The Survey-Based Model

The second model I estimate is based on a Taylor Rule that uses survey data. In this rule, the central bank responds to expected inflation and the expected output gap as predicted in surveys of forecasters. The monetary policy rule is expressed as

\[
    r_i = \delta^* + \delta_{11}E[\pi_{t+1} - \pi^*] + \delta_{12}E[y_{t+1}]
\]

The corresponding function for the short rate process in the model is

\[
    r_t = \delta_0 + \delta_{11}E[\pi_{t+1}] + \delta_{12}E[y_{t+1}]
\]

where \( \delta_0 = \delta^* - \delta_{11}\pi^* \).
Once again, I estimate the restricted one-factor models to test if inflation is better at fitting yields in certain periods, and if it is better at predicting different maturities of the yield curve. Then I estimate the full model to test whether the survey data explains changes in yields better than the historical data.

4. Data

I use data on yields for zero coupon bonds, historical inflation and output, and expected inflation and output to estimate this model. The data is from the period between Q2 1982 to Q4 2004. This consists of 91 quarters of observation and includes periods...
Table 1: Descriptive Statistics of Yields

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1-month</th>
<th>1-year</th>
<th>2-year</th>
<th>3-year</th>
<th>4-year</th>
<th>5-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.084%</td>
<td>6.129%</td>
<td>6.507%</td>
<td>6.781%</td>
<td>6.993%</td>
<td>7.122%</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.250%</td>
<td>0.309%</td>
<td>0.308%</td>
<td>0.301%</td>
<td>0.295%</td>
<td>0.287%</td>
</tr>
<tr>
<td>Median</td>
<td>5.108%</td>
<td>5.852%</td>
<td>6.113%</td>
<td>6.281%</td>
<td>6.485%</td>
<td>6.529%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.384%</td>
<td>2.997%</td>
<td>2.983%</td>
<td>2.919%</td>
<td>2.856%</td>
<td>2.786%</td>
</tr>
<tr>
<td>Sample Variance</td>
<td>0.057%</td>
<td>0.090%</td>
<td>0.089%</td>
<td>0.085%</td>
<td>0.082%</td>
<td>0.078%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.143</td>
<td>0.640</td>
<td>0.550</td>
<td>0.519</td>
<td>0.535</td>
<td>0.250</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.186</td>
<td>0.585</td>
<td>0.632</td>
<td>0.684</td>
<td>0.736</td>
<td>0.725</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.794%</td>
<td>1.039%</td>
<td>1.299%</td>
<td>1.700%</td>
<td>2.114%</td>
<td>2.500%</td>
</tr>
<tr>
<td>Maximum</td>
<td>11.779%</td>
<td>15.433%</td>
<td>15.592%</td>
<td>15.529%</td>
<td>15.824%</td>
<td>15.001%</td>
</tr>
</tbody>
</table>

Source: Fama Risk Free Rate Files and Fama-Bliss Discount Bond Files

of high and low interest rates, economic growth, and inflation, and encompasses multiple business cycles. Therefore, it should provide a rich set of data with which to fit the model. One drawback of using this period is that it could be subject to biases related to regime switches in the Federal Reserve since the period spans more than just Alan Greenspan’s tenure as Chairman.

Yield curve data is taken from the Center for Research in Security Prices (CRSP) database. I use data for 1-month, and 1, 2, 3, 4, and 5 year yields on zero-coupon bonds. The 1 month yields are taken from the Fama Risk Free Rate files. The 1 through 5 year yields are taken from the Fama-Bliss Discount Bond files. The descriptive statistics for the yield data are shown in Table 1. It is evident from Table 1 that the sample period includes the high rate environment of the early 1980s as well as the very low yield environment in 2004. Table 2 has the correlations of the yields. One of the characteristic features of the term structure of interest rates is the high degree of correlation between the various points on the yield curve. This reinforces the need for a model that imposes no-arbitrage restrictions, so that the shocks to yields are consistent across different maturities. Figure 1 plots yields for the different maturities over the sample
<table>
<thead>
<tr>
<th></th>
<th>1-month</th>
<th>1-year</th>
<th>2-year</th>
<th>3-year</th>
<th>4-year</th>
<th>5-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-month</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td>0.9695</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-year</td>
<td>0.9540</td>
<td>0.9936</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-year</td>
<td>0.9404</td>
<td>0.9838</td>
<td>0.9973</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-year</td>
<td>0.9252</td>
<td>0.9712</td>
<td>0.9908</td>
<td>0.9976</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>5-year</td>
<td>0.9121</td>
<td>0.9612</td>
<td>0.9846</td>
<td>0.9940</td>
<td>0.9985</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

The large range of yields over the period and the high correlation between yields are both evident in the chart.

Data for historical inflation is taken from the GDP Price Index published in the Philadelphia Fed’s Real-Time Data Set for Macroeconomists. The price index uses a fixed basket of goods, and tends to be less noisy than the Consumer Price Index, so I chose it as the measure of inflation. The data has quarterly annualized rates. The quarterly inflation rate is calculated taking the percent change in the index over three months. The quarterly data by itself is too noisy. Instead, in the estimation of the model, a four-quarter moving average of inflation rate is used. Data on Real GDP is also taken from the Real-Time Data Set for Macroeconomists. One major drawback of the data used for this study is that it uses data that has eventually been revised. To truly capture the effects on yields of macroeconomic variables, it would be better to have the data as market participants would have seen it when it came out at first rather than the revised versions. Also, it would be useful to examine the effects of revisions on the yields. While the Real-Time Data Set for Macroeconomists does have the data in order to carry out such a study, estimating a term structure based on that data is left to a future study.
### Table 3: Descriptive Statistics of State Variables

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Inflation</th>
<th>Output Gap</th>
<th>Inflation Forecast</th>
<th>Output Gap Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.592%</td>
<td>-0.092%</td>
<td>2.968%</td>
<td>-1.014%</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.098%</td>
<td>0.104%</td>
<td>0.120%</td>
<td>1.088%</td>
</tr>
<tr>
<td>Median</td>
<td>2.291%</td>
<td>-0.096%</td>
<td>2.735%</td>
<td>-0.230%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.930%</td>
<td>0.996%</td>
<td>1.142%</td>
<td>10.375%</td>
</tr>
<tr>
<td>Sample Variance</td>
<td>0.009%</td>
<td>0.010%</td>
<td>0.013%</td>
<td>1.076%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.654</td>
<td>-0.033</td>
<td>-0.048</td>
<td>3.219</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.830</td>
<td>-0.176</td>
<td>0.675</td>
<td>-0.631</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.038%</td>
<td>-2.759%</td>
<td>1.378%</td>
<td>-35.977%</td>
</tr>
<tr>
<td>Maximum</td>
<td>5.794%</td>
<td>1.906%</td>
<td>6.446%</td>
<td>30.649%</td>
</tr>
</tbody>
</table>

Source: Philadelphia Fed's Real-Time Data Set for Macroeconomists

Data on the forecasts of inflation and real GDP is taken from the Survey of Professional Forecasts, which is also maintained by the Philadelphia Federal Reserve. I use one quarter forecasts of inflation and real GDP. This data set exists from the Q3 1980 to the present. This study uses forecast data from Q2 1982 up to Q4 2004. The survey, which began in 1968, is the oldest survey of quarterly macroeconomic forecasts in the US. It was initially taken by the American Statistical Association and the National Bureau for Economic Research until 1990 when it was taken over by the Philadelphia Fed. 8 Professional forecasters from a variety of places, including Wall Street firms, economic consulting firms, academia, and private economists participate in the survey. 9 Keane and Runkle (1990) have shown that the forecasts in the survey are consistent with rational expectations.

In order to extract the output gap from the data on Real GDP, I use the Hodrick-Prescott filter to detrend the Real GDP series. 10 The filter is a statistical tool with which

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8 See www.frb.phil.org.
9 For more information on the Survey of Professional Forecasters, see Croushore (1993).
Table 4: Correlation of State Variables

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>GDP Growth Forecast</th>
<th>Inflation Forecast</th>
<th>GDP Growth Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.1237</td>
<td></td>
<td></td>
<td>1.0000</td>
</tr>
<tr>
<td>Inflation Forecast</td>
<td>0.8650</td>
<td>0.0183</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>Output Gap Forecast</td>
<td>-0.3356</td>
<td>0.2068</td>
<td>-0.3023</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

to extract a smooth series from a noisy one. The smooth series that is extracted from the data is taken to be potential GDP, and the residual between potential and actual GDP is the output gap. I divide the residual, which is in dollars, by the potential GDP to get the output gap in percentage terms. The same technique is applied to the data in both the data-based and the survey-based models to extract the output gap in both datasets.

Table 3 shows the descriptive statistics for the historical values and forecasts of the macroeconomic variables during the sample period. Table 4 shows the correlations between the variables. As one would expect, there is a strong correlation between the historical and forecast values for inflation. This is probably because the prior quarter’s inflation value is probably an important component in the forecast for the value in the next period. Figures 2 and 3 contain the plots of inflation and the output gap along with their forecasts. The forecasts seem to track the historical levels fairly well, although there are clear deviations. There is much more variability in the output gap series. The output gap extracted from historical data looks reasonable and corresponds to business cycles. However, the output gap extracted from the survey data has a pattern which does not track the actual data or business cycles. This will make it difficult to estimate a two variable term structure model for the survey-data model.
5. Estimation Procedure

The model requires the estimation of the parameters \( \Theta = (\delta_0, \delta_1, \mu, \Phi, \Sigma, \lambda_0, \lambda_1) \). The parameters of the short rate equation, \( (\delta_0, \delta_1) \) and the parameters of the VAR, \( \Theta_1 = (\mu, \Phi, \Sigma) \), are estimated using Ordinary Least Squares. Then I use a maximum likelihood (MLE) procedure to estimate the remaining parameters. In this second step, the estimated parameters from the first step are fixed and the remaining parameters for the risk premium, \( \Theta_2 = (\lambda_0, \lambda_1) \), are estimated.
Estimating $\Theta_2 = (\lambda_0, \lambda_1)$ requires non-linear maximum likelihood estimation. I assume that all the yields are measured with error so that

$$e_t = \hat{Y}_t - A - B^T X_t$$

where $\hat{Y}$ contains the observed yields. The error term $e$ is distributed $N(0, \Omega_y)$ and $\Omega_y$ is the covariance matrix of the residuals. Then the conditional density function for this step of the estimation is Gaussian and has the form
The log likelihood function is

\[ L = \frac{1}{T} \sum_{t=1}^{T} \log f(Y_t | X_t) \]

and the parameters for the risk premium are estimated by maximizing this function.

The estimates for the risk parameters are extremely sensitive to the initial values used in the estimation procedure. In order to be sure that the estimates give a global maximum, a complex search algorithm must be used. I generated 50,000 random initial values within a reasonable range for the parameters estimates and evaluated the likelihood function at these parameter values. I then took the 10 initial values that gave the highest likelihood values and used these as the starting values in a linear search algorithm to maximize the function. The parameter estimates with these ten starting values that maximized the likelihood function are reported. The standard errors are calculated using the BHHH Estimator.\textsuperscript{11}

6. Results

6.1 Results for the Inflation Only Model

Table 5 contains the results of the estimation of the Inflation Only Models. The coefficient \( \delta_1 \) in short rate equations for both the Data-Based and Survey-Based Models

\textsuperscript{11} See Berndt, Hall, Hall, and Hausman (1974).
Table 5: Inflation Only Models

<table>
<thead>
<tr>
<th></th>
<th>The Data-Based Model</th>
<th>The Survey-Based Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>0.0081</td>
<td>0.0030</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>1.6482</td>
<td>1.6121</td>
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<tr>
<td></td>
<td>(0.208)</td>
<td>(0.141)</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>0.5237</td>
<td>0.2801</td>
</tr>
<tr>
<td></td>
<td>(1.461)</td>
<td>(0.815)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-28.1066</td>
<td>-22.6798</td>
</tr>
<tr>
<td></td>
<td>(605.522)</td>
<td>(120.096)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0023</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>0.8989</td>
<td>0.9353</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>0.0032</td>
<td>0.0020</td>
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<td></td>
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</tbody>
</table>

Estimates for the complete two factor models. Asymptotic standard errors are reported in parentheses.

The Data-Based Model uses the short rate equation: $r_t = \delta_0 + \delta_1 \pi_{t+1}$

The Survey-Based Model uses the short rate equation: $r_t = \delta_0 + \delta_1 E[\pi_{t+1}]$

displays the important “greater than one” principle of macroeconomic policy. In its reaction to higher inflation, the central bank raises the policy rate by more than increase in inflation. This is because if the central bank only raised the rate by the amount of inflation, it would adjust nominal rates to keep the real interest rate constant. However, the goal of the central bank is to combat higher inflation by raising the real interest rate, and it does so by raising the nominal rate by more than the change in the rate of inflation. The estimate for $\delta_1$ in the Data-Based Model (1.65) is approximately the same as the Survey-Based Model (1.61). I cannot reject the hypothesis that the estimates for $\delta_1$ are the equal in the Data-Based and Survey-Based Models. The R-squared in the Survey-Based Model (0.60) is much higher than the R-squared in the Data-Based Model (0.41), suggesting that the expected inflation in the survey data explains more of the variation in
the short rate than the actual data from the previous quarters. If we believe that the forecasts of the Survey of Professional Forecasters are on average similar to the forecasts used by the Federal Reserve, then this result is fairly natural. The Federal Reserve sets
policy based on its own forecast of several macro variables, so it we expect that a model using the surveys explains more of the variation than a model using historical data.

Analysis of the estimates of the parameters $\mu$ and $\Phi$ for the Inflation Only models reveals that even though the constants are statistically significant, they are so close to zero that they have very little economic importance. The autoregressive coefficients $\Phi$ on inflation for the two models are statistically significant. I cannot reject the hypothesis that they are equal to each other at the 95% significance level. The VAR estimates suggest that there is a great degree of persistence in both the actual inflation and the forecasts of inflation in the surveys. Part of this persistence is because the data is a four quarter moving average of inflation, so a large portion of the previous quarter’s value is used in the next quarters’ value. In addition, it is likely that the forecasters do not drastically adjust their forecasts of inflation from quarter to quarter. Rather, they probably adjust their previous quarter forecasts up or down slightly in response to any new information. The estimates for the Survey-Based Model serve as a robustness check for the estimates on the Data-Based Model. The similarity in results assures us that the parameter estimates are fairly accurate and robust across datasets.

The estimates of the risk parameters, $\lambda_0$ and $\lambda_1$, are also fairly similar in the two models. Figure 4 plots the term structure of coefficients from the two models. From the plots, it is clear that the coefficients on inflation are almost the same in both models. This means that both inflation and the inflation forecast from the survey have approximately the same effect on yields over time. This is expected since the two series are highly correlated. The difference in the levels of the coefficients $A(\tau)$ between the
two models results from the higher value for $\delta_0$ in the Data-Based Model (0.008) compared to the Survey-Based Model (0.003). This slight difference, along with the small differences in the parameters of the risk premium create about a 65-70 basis point difference in the levels of yields between the two models. This difference seems to have a big effect on yields, but since neither of the constants is statistically significant in the estimation, I conclude that the two models are fairly similar in how they model yields. The parameters that are statistically significant in the estimation are all very close to each other. This assures us that the estimation of the models is robust.

The comparative statics of the one variable model are quite interesting. Based on the model, we predict that a 1% rise in inflation causes an overall rise in yields, but there is a greater increase at the short end of the yield curve than at the long end of the yield curve. The 1% rise in inflation causes the 1-year yield to rise by 2%, while it causes the 5-year yield to rise by only 1.5%. There is an intuitive explanation for why this is the case. With the rise in inflation, people still demand the same real rate of return, so the nominal yield must rise at least as much as the rate of inflation. Furthermore, people expect the Central Bank to respond to the higher inflation by raising its policy rate by more than the rate of inflation. The estimates predict that Central Bank raises the rates by approximately 60 basis points more than the 1% rise in inflation. Finally, the estimates from the VAR suggest that inflation is relatively persistent but mean reverting. Therefore, higher inflation is expected to persist for some time, but it slowly reverts back to its mean. Therefore, overtime, people expect the nominal rate not to have to adjust as much because inflation will eventually decrease. The time varying risk premium
associated with inflation also causes yields to rise. Therefore, a rise in inflation results in all yields rising, but short term yields rising more than long term yields, causing the yield curve to flatten. The estimates suggest that even a simple one factor model does a fairly good job of explaining the dynamics of the term structure of interest rates.

### 6.2 Results for the Complete Model with Inflation and Output Gap

The results of the estimation of the two-factor model with both inflation and output gap are reported in Table 6. In both models, the estimation of the short rate process reaffirms the “greater than one” principle of inflation targeting. The estimate of the coefficient on inflation in the monetary policy rule is estimated at 1.59 in the Data-Based Model and 1.58 in the Survey-Based Model. The coefficient of 1.5 which is assumed in practice is supported by these estimations. In the Data-Based Model, the estimate of the coefficient on output gap (0.47) is significant at the 95% significance level. This number is also very close to the 0.5 coefficient that is assumed in practice. This indicates that the two factor model does do a better job of describing the effects of monetary policy. The estimate for the coefficient on the output gap in the Survey-Based Model is not significant, and this is probably a result of the poor data on Real GDP forecasts in the survey. The output gap is not expected to contribute much to the dynamics of the term structure in the Survey-Based Model, and the estimates for the remaining parameters confirm this fact. I analyze the Data-Based Model in greater detail in order to evaluate the performance of the two factor model.
The estimates of the VAR in the Data-Based Model suggest that the autoregressive model is appropriate for these factors. The autoregressive coefficients are highly significant in the model. I also find that the lag of inflation is negatively correlated with the output gap. As a result, this model also captures the joint dynamics of inflation and the output gap. The estimate indicates that a 1% rise in inflation is associated with a 0.12% fall in the output gap in the next period. None of the estimates of the parameters of the risk premium are statistically significant. These estimates are extremely sensitive to the initial values used in the estimation, and are extremely difficult to pin down. The fact that they are not significant will make it difficult to estimate long term yields using this model, since it does not capture the risk premium very well.
The comparative statics for this model are most easily analyzed by looking at the coefficients $A(\tau)$ and $B(\tau)$. The constant $A(\tau)$ describes what the term structure would like if all the state variables were identically zero. The coefficients $B(\tau)$ describe how inflation and the output gap affect yields across the term structure. These coefficients can
be interpreted as the weights that determine the magnitude of changes in yields in response to changes in inflation and the output gap. The term structure of coefficients for the Data-Based and Survey-Based Models is plotted in Figure 5. The positive slope of the constant factor $A(\tau)$ implies that in “normal” economic times when inflation is low and the output gap is close to zero, the yield curve is upward sloping.

The coefficients $B(\tau)$ on inflation in the complete Data-Based Model are very different from the coefficients in the Inflation Only version of this model. In the Inflation Only model, the coefficients describe inflation as having a large effect on the slope of the yield curve. The large downward sloping nature of the term structure of coefficients implies a sharp flattening of the yield curve when inflation rises. The complete Data-Based Model predicts a higher level effect but a smaller slope effect in response to higher inflation. A 1% rise in inflation is associated with a 2.22% rise in short term yields and a 2.13% rise in long term yields. Once again, a rise in inflation causes short term yields to increase more than long term yields, but the flattening effect is much smaller than in the Inflation Only model. This result does confirm the widely held belief that tightening of monetary policy, which occurs when inflation is high, causes a flattening of the yield curve. In this full model, the output gap is the factor that has a strong effect on the slope of the yield curve. A 1% rise in the output gap is associated with a 40 basis point rise in the 1-year yield but a 7 basis point fall in the 5-year yield. This leads to a slope effect where the yield curve flattens by 47 basis points. This also affirms the belief that when the Federal Reserve tightens monetary policy in response to a high output gap, it is accompanied by the flattening of the yield curve. The role of inflation as a strong factor
in determining the level of the yield curve confirms the results of Ang and Piazzesi (2003) who found that inflation was related to the “level” factor that Litterman and Scheinkman (1991) found. Furthermore, the small slope effect of inflation also confirms the results of Ang and Piazzesi (2003) that inflation loaded strongly into Litterman and Scheinkman’s (1991) “slope” factor. However, this paper adds another piece to the puzzle by showing that the output gap factor helps predict the slope of the yield curve. Ang and Piazzesi were unable to capture this effect with their “real activity” factor.

In the Survey-Based Model, yields are less sensitive to both inflation and output. A 1% rise in the forecast of inflation in the surveys corresponds to a 1.67% rise in the 1-year yield and only a 0.80% rise in the 5-year yields. This implies that over the long run,
people expect inflation to decrease, so 5-year yield rises less than the increase in inflation. The Survey-Based Model also predicts a flatter yield curve, but the levels are lower than those predicted by the Data-Based Model. As expected, the coefficients on output gap are almost zero, so a change in the forecast of output gap has almost no effect on yields in this model. As a result, the dynamics of this two variable model are similar to the Inflation Only Survey-Based Model.

6.3 Goodness of Fit

There are several ways to measure the goodness-of-fit of the predicted yields to the actual data. First, in Figure 6, I plot the actual 1-month yield along with the 1-month yields predicted by the monetary policy rules in both the Data-Based and Survey-Based Models. By looking at the graph, it is clear that although the monetary policy rule does a good job of predicting the short rate at most times, it does not do very well during business cycle contractions. In particular, during the period between Q3 1990 and Q4 1991 as well as the contraction from Q1 2001 to Q4 2001, the model underestimates the magnitude of the fall in yields. This can probably be explained by the fact that the output gap factor does not have a very large coefficient in the monetary policy rule. As a result, even though the output gap is negative during these periods, the effect of inflation outweighs the effect of the output gap in the model. During periods of recession, the central bank may temporarily be more concerned with output than inflation, and as a result, set policy with more discretion than a simple monetary policy rule might suggest.
Figure 7: Goodness of Fit of Models

- **1-Month Yield**
- **1-Year Yield**
- **2-Year Yield**
- **3-Year Yield**
- **4-Year Yield**
- **5-Year Yield**

*Legend:*
- **Actual**
- **Survey-Based Model**
- **Data-Based Model**
In such a scenario, it would be natural to see actual yields differ from those predicted by the monetary policy rule.

Figure 7 plots the actual yields and those predicted by the model for all maturities. The result for the goodness-of-fit of these models is mixed. The model behaves poorly during the times of economic recession. It underestimates the fall in yields during the early 1990’s and the early 2000’s for all maturities. This occurs because the estimates of the model place a much higher emphasis on inflation than on the output gap. In these two periods of recession, inflation was about 2%, which is close to its average level, while the output gap was -1%, far below its average level (See Figures 2 and 3). However, the coefficient on inflation is almost five times as high as that on the output gap for short term yields, and even higher for long term yields. As a result, even though the central bank and economy are reacting more to output than inflation during times of recession, the model places a large degree of emphasis on inflation. A model that would accurately predict yields during these periods must have some variable that is highly correlated with a recession, and assign it a large coefficient in order to improve the goodness-of-fit.

Finally, I evaluate the goodness-of-fit of the various models estimated in the paper using the Root Mean Squared Errors (RMSE) and Mean Absolute Errors (MAE) criteria. Table 7 gives these values by maturity. It also provides an average RMSE and MAE value for each model to see its goodness-of-fit across the entire yield curve. The results are once again mixed. Using both RMSE and MAE criteria, the Survey-Based Model had the smallest pricing errors and fit the yields the best, and the Data-Based Model had the largest pricing errors and fit the yields the worst.
Regardless of whether I use RMSE or MAE, the models that used survey data fit the yields better than the models using historical data. This implies that forecasts generated using rational expectations and historical data lack some important component that is captured in the surveys. Furthermore, this difference is being recognized by market participants and affecting the term structure of interest rates. Table 7 also shows that within any particular model, short term yields are predicted more accurately than long term yields. This finding is in line with results of other papers that have shown that macroeconomic factors tend to have a much higher effect in the short term than over the long term. This suggests that while inflation and the output gap are strong drivers of
short term interest rates, there are other variables that influence longer term rates that are not captured in this simple model. Furthermore, the dynamics of the state variables may not be captured by a VAR with only one lag. This could affect future forecasts of the state variables, and thereby decrease the ability of the model to fit long term yields.

It is surprising that the Data-Based Model does worse than the Inflation Only model using historical data, but that is most likely a result of the estimates for the parameters of the risk premium. The estimation yields values for the parameters that are very large and estimated with a lot of error. The fact that none of the parameters were statistically significant creates problems for pricing the risk premium correctly. As a result, the fit of this model is not as good as it could be, and it is much worse for long term yields than short term ones. There are probably starting values at which the fit of this model would be better than the Inflation Only model. In fact, we should expect that the fit of the Data-
Based Model should be even better than the Survey-Based Model since the output gap data was much better in the Data-Based Model. On the other hand, the fact that both the one and two variable Survey-Based Models performed better than their corresponding Data-Based Models is encouraging because it confirms the hypothesis that forecasts that come from surveys are better at explaining the dynamics of the yield curve than looking only at historical data. Rational expectations models of macro variables may not capture all the information about the state of the economy. Rather, surveys contain information that explains dynamics of yields that is not found in a simple one period rational expectations forecasting model. I expect the goodness-of-fit of the Survey-Data model would improve even more if better survey data for the output gap was available.

6.4 Impulse Responses

While the comparative statics explain what would happen immediately to yields if there was a shock in the economy, we are also interested in analyzing how economic shocks and their effects on yields propagate through time. I use impulse responses to analyze the dynamics of the yield curve over time when there is an economic shock. Figure 9 plots the impulse response to a one standard deviation shock to inflation in the first period. The first chart in Figure 9 shows what happens to inflation and the output gap over time. Both factors are mean reverting, and they both begin to fall immediately. The factors keep falling until they converge to their respective long run means.

The second chart shows the effect of the state variable dynamics on the 1-year, 3-
year, and 5-year yields. The shock to the economy causes yield across the curve to rise. Furthermore, since both inflation and output gap shocks affect the slope of the yield curve, the shock is associated with an immediate flattening of the yield curve. A rise in
inflation and the output gap causes the all yields to rise, but it causes short term yields to increase more than long term yields. This can be seen in Figure 10 which graphs the slope of the yield curve after the shock. This slope is the difference between the 5-year and the 1-year yields. The curve is initially flat (46 bps). For the first few periods, the curve actually becomes steeper (50 bps). This is because of the negative feedback of inflation into the output gap. Since the output gap has a strong slope effect, the fall in the output gap causes the curve to become temporarily steeper. As time goes on and the macroeconomic factors adjust back towards their means, yields fall across the board and the curve begins to become flatter. In this case, once the output gap goes back towards its mean level, the slope effect of inflation takes over and causes the yield curve to flatten.
a little bit. From this impulse response, one can see how a shock that was larger than one standard deviation could cause the model to predict an inverted yield curve because of the curve flattening effect of both inflation and the output gap.

7. Inverted Yield Curves

Although the complete model only has mixed results in predicting the levels of yields, it does a much better job predicting the slope of the yield curve. The actual and predicted slopes of the yield curve, defined as the 5-year yield minus the 2-year yield, are graphed in Figure 8. This relationship is robust across different measures of the slope of the yield curve. The Data-Based model tracks the value of the slope as well as the direction of the changes in slope fairly well. It performs much better than the Survey-Based model which does a poor job of mimicking the slope of the yield curve. The inclusion of the output gap variable is responsible for this improved performance. The graph of the coefficients in Figure 5 reveals that in the Data-Based model, inflation generally affects the level of the yield curve, while the output gap is responsible for changes in the slope. This is why the inclusion of the output gap variable improves the fit of the slope of the term-structure.

The most interesting feature that we notice Figure 8 is that the graph reveals how well the model predicts an inverted yield curve. The yield curve is inverted when short term interest rates are higher than long term ones. A widely held belief is that an inverted yield curve predicts a recession because the inversion of the curve implies that people expect the central bank to have to reduce interest rates in the future in order to stimulate
output. In fact, other than when the yield curve inverted for a few days in 1998, every other inversion of the yield curve has been followed by a recession. This has led to
speculation that the inversion of the curve in the early part of 2006 has predicted a recession or at least an economic slowdown in the upcoming quarters.

To analyze inverted yield curves, I look at instances when the curve was inverted for a prolonged period of time within my sample. There are two such instances. The first of these was in 1989 when the curve was not entirely inverted, but rather, it was hump shaped, with intermediate term yields higher than both long and short term yields. Figure 8 shows that the model predicts that the slope between the two and five year yields would be negative at approximately the same time as the event occurred. The first chart in Figure 11 plots the actual one through five year yields during this period as well as the yields predicted by the Data-Based model. Even though the model does not capture the level of the yield curve very well, it does capture the inversion of the yield curve. A second period of yield curve inversion was the middle of the year 2000 before the dot-com bubble burst. Figure 8 shows that the model predicts the inversion of the yield curve at almost exactly the same time as it actually occurred. The fact that a simple two variable model is able to capture an event as rare as an inverted yield curve is quite remarkable. Although the model does not predict the level of the yield curve correctly, once again, it does predict the inversion of the yield curve.

A look at the macroeconomic data reveals that the first period of yield curve inversion in 1989 was characterized by 4% inflation and a 1% output gap. Both of these variables were at levels that were more than a standard deviation above their means. This is a very rare occurrence. The correlation of inflation and the output gap is very low. Furthermore, the estimates from the VAR of the Data-Based model indicate that high
inflation tends to persist in the next period, and is also associated with a decrease in the output gap in the next period. Therefore, a period of high inflation and a high output gap is quite rare. When it does occur, however, the model predicts the inversion of the yield curve. There was another time in the sample period that was characterized by similar values for the macroeconomic variables. Early in the year 2000, inflation was close to its mean level of about 2.5%, but the output gap was at its maximum of 1.9%. Inflation and output gap both contribute to the slope of the yield curve, and the output gap is especially important in determining the slope. A 2% rise in the output gap is associated with about a 1% flattening of the yield curve. In both of these time periods, a high output gap caused the yield curve to flatten considerably, and higher inflation made the flattening more pronounced, leading to the inversion of the curve.

The intuition for why high inflation and output gap would cause the yield curve to invert is fairly straightforward. When output is far above its potential level, people begin to realize that such a high rate of growth cannot be sustainable. Eventually, the economy will revert to its potential level of output. However, when there is already a high output gap, this implies that the economy will slow down in the upcoming quarters in order to return to its potential levels. Therefore, a very high output gap can be looked at as an economic bubble that will eventually burst. If this is coupled with high inflation, then people expect the Federal Reserve to fight the inflation by raising its policy rate. As previously discussed, higher inflation causes a rise in yields across the term structure. Since macroeconomic models of aggregate demand propose a negative relationship between the interest rate and output, the higher level of interest rates caused by inflation
increase concerns that output will decline. As a result, people expect future rates to decline amidst a recession or economic slowdown, and the yield curve inverts. Based on the estimates of this model, I believe that this is what occurred during 1989 and 2000 when the yield curve was inverted.

8. Understanding the Conundrum

On February 16, 2005, then Federal Reserve Chairman Alan Greenspan testified before the U.S. Senate committee on Banking, Housing, and Urban Affairs. At that point, the Federal Reserve had raised the Federal Funds rate by 150 basis points. At the same time, however, long term interest rates, including the 10-year yield were trending downwards. Figure 12 shows a graph of this trend around the time when Chairman Greenspan gave his testimony. Economic theory would have predicted that a rise in the short term interest rates would raise the forward rates and cause the long term yields to rise. However, at the time, the rise in short term forward rates was being offset by a greater fall in long term forward rates. This led to a severe flattening of the yield curve, which is evident in Figure 12. Chairman Greenspan, unable to explain why this was occurring, referred to the phenomenon as a “conundrum”.

I use the estimation of the two-variable Data-Based Model to predict yields in the out of sample period that Chairman Greenspan was referring to. The Data-Based model has mixed success in predicting the type of changes in yields that during the period. If

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12 This chart was taken from a presentation by Monika Piazzesi at Stanford University
Figure 12

Selected interest rates

<table>
<thead>
<tr>
<th>Ten-year Treasury</th>
<th>Percent</th>
</tr>
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<tr>
<td>6</td>
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<tr>
<td>5</td>
<td></td>
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</table>

Note: The data are daily and extend through July 13, 2005. The ten-year Treasury rate is the constant-maturity yield based on the most actively traded securities. The dates on the horizontal axis are those of FOMC meetings.

Source: Department of the Treasury and the Federal Reserve.
we consider the “conundrum” to be the change in the slope of the yield curve where short
term yields rose much more than long term yields, then the model does pick up this
effect. During the period, inflation was slightly over 2.5% and the output gap was less
than 1% but rising. As a result, the model predicts the flattening of the yield curve,
which did occur, but it does not predict the falling of long term yields. This is primarily
because the model does not have anything to say about changes in the real interest rate.
If the fall in yields at the long end of the curve is related to a lower real interest rate, we
would not expect this model to pick up the effects. The Data-Based model does offer
some interesting insights into why it would exist in the first place.

After looking at the implications of rising inflation and output gap, we should see
the “conundrum” not as an event unto itself, but rather as an intermediate stage in the
process of the yield curve inverting. This period of yield curve flattening was followed
by the inversion of the yield curve in 2006. The Data-Based model has a lot to say about
why such an inversion would occur. An important finding is that a rise in the output gap
causes the 5-year yield to fall. This implies that the model does have a variable which is
capable of causing short term yields to rise and long term yields to fall. This effect
occurs exactly when the output gap is rising, as it was during this period.

An augmented model could go beyond the slope effect and also capture the
contributes significantly to the “curvature” factor in the yield curve. This curvature
factor represents a twisting of the yield curve, causing rates to rise at one end of the yield
curve while the fall at another end. The graph of the coefficients for output gap in Figure
5 showed some of this effect when the coefficient for long term yields is negative. However, if an additional variable was added which explained the curvature of the yield curve, it is very likely that a simple model could predict the “conundrum” entirely. A factor such as volatility, which is known to be related to the convexity of the yield curve, may be enough to estimate such a model. I leave this for future research.

One way to test whether the conundrum was caused by changes in expectations about future output is to use survey data. If one could find a good series of forecasts of the level of real GDP, then by extracting the expected output gap, one could test whether the fall in long term yields was caused by a change in expectations about the output gap. This paper has already argued that survey data contains some additional information about the yields that is not contained in forecast models of rational expectations. Policymakers who look at survey data can reduce the uncertainty of market response to policy actions by incorporating those surveys into their models and their policy decisions.

9. Conclusion

Dynamics of the term structure of interest rates can be explained using macroeconomic data. A simple two variable model that uses inflation and the output gap can describe the dynamics of yields with a reasonable degree of precision. This paper combines the macro literature with the finance literature and shows that the term structure can be modeled without the explicit introduction of latent factors. The term structure of interest rates reacts to forward-looking data about inflation and output. By directly incorporating forward-looking information into a dynamic term structure model through
the use of survey data, this paper explores a topic that has not been addressed in the current term structure literature. Both a one and two variable model using survey data outperform their relative Data-Based Models in fitting yields. The incorporation of forecasts directly from surveys eliminates the need to include additional lags. Further research in this field should focus on estimating the term structure dynamics without the use of latent variables and by incorporating other forward-looking information from surveys or from market data.

Inflation and output gap describe different aspects of the dynamics of the term structure. Inflation helps describe changes in levels of yields, while the output gap is very good at fitting changes in the slope of the term structure. On the whole, the macro variables are much better at describing dynamics of short term yields. Long term yields tend to have more variation that cannot be explained by these two factors alone. Investors demand both a constant and a time varying risk premium for holding bonds with long maturities. Inflation is the factor that dominates the time varying risk premium. As a result, inflation has a strong effect on the overall level of the yield curve. The low time varying risk premium associated with the output gap implies that for longer maturities, a rise in the output gap has a very small effect. This creates a slope effect of the output gap where short term yields are affected more than long term yields.

An interesting application of dynamic term structure models is to analyze periods when the yield curve behaved in ways that are not “normal”. The “conundrum” of 2004 and 2005 and the inverted yield curves in 1989 and 2000 are interesting cases to study. This paper finds that the two-variable Data-Based model is extremely good at predicting
the inversion of the yield curve. It also explains why the yield curve inverts – the macroeconomic policy response to high inflation coupled with the slope effect of high output gap cause short term yields to rise substantially while long term yields rise by much less or fall in some circumstances. The combination of tightening monetary policy and the expectation of the output gap falling in the future create an environment where yields are expected to fall in the future, explaining the inversion yield curve.

The yield curve has a lot of information about what people expect to happen in the economy. If policymaker who considers the implications of monetary policy on the entire term structure of interest rates rather than just one rate is able to craft monetary policy more precisely to achieve the desired goals. Understanding the joint dynamics of asset prices, interest rates, and macroeconomic variables in one simple model allows policy makers and market participants to understand the economy in richer ways. Exploring and exploiting these dynamics will help policy makers achieve price stability and stable output while keeping a better pulse on the economy.
References


Appendix

A. Recursive Bond Prices

I show that bonds can be priced recursively with the coefficients $A$ and $B$ as defined in equation (8). I first begin by noting that the price of a one period bond is

\[ p_1 = E_t[m_{t+1}] = \exp\{-r_t\} = \exp\{-\delta_0 - \delta_1 x_t\} \]  

(A.1)

Here $m_{t+1}$ is defined by (5). Matching coefficients to (1) leads to

\[ \bar{A} = -\delta_0 \]
\[ \bar{B} = -\delta_1 \]  

(A.2)

This gives us the initial conditions for the recursive equations. Now I proceed by induction. Suppose that the price of an n period bond is given by

\[ p^n_t = \exp\{A_n + B_n x_t\} \]

The goal is to show that the price of the n+1 period bond also follows exponential form:

\[
P_{n+1}^{n+1} = E_t[m_{t+1}p_{n+1}^n]
= E_t\left[\exp\left\{-r_t - \frac{1}{2}\lambda_t^2 + A_n + B_n x_t\right\} \right]
= E_t\left[\exp\left\{-r_t - \frac{1}{2}\lambda_t^2 + A_n\right\}E_t\left[\exp\left\{-\lambda_t^2 + B_n x_t\right\} \right]\right]
= E_t\left[\exp\left\{-r_t - \frac{1}{2}\lambda_t^2 + A_n\right\}E_t\left[\exp\left\{-\lambda_t^2 + B_n (\mu + \Phi x_t + \Sigma e_{t+1})\right\} \right]\right]
= \exp\left\{-\delta_0 + A_n + B_n \mu + \left(B_n \Phi - \delta_1^2\right)x_t - \frac{1}{2}\lambda_t^2\right\} \times E_t\left[\exp\left\{-\lambda_t^2 - B_n \Sigma e_{t+1}\right\} \right]
\]

(A.3)

14 This is the same derivation that is provided in Ang and Piazzesi (2003), but the sign error in their derivation is corrected.
\[
= \exp\left\{ -\delta_0 + \bar{A}_n + \bar{B}_n (\mu - \Sigma \lambda_0) + \frac{1}{2} \bar{B}_n \Sigma \Sigma \bar{B}_n - \delta_i X_i + \bar{B}_n \Phi X_i - \bar{B}_n \Sigma \lambda_i X_i \right\}
\]

The last equality is a result of \( \epsilon_i \) being IID normal with \( E[\epsilon_i] = 0 \) and \( \text{var}(\epsilon_i) \) being a degenerate matrix. This, along with the assumptions about the parameterization of the risk premium implies that \( \lambda_i \lambda_i = \lambda_i \text{var}(\epsilon_i) \lambda_i \). Finally, collecting like terms and matching coefficients result in the recursive bond pricing relations that are given in Equation (8).