A Macroeconomic Model
with
Financial Panics*

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Abstract

This paper incorporates banks and banking crises within a conventional macroeconomic framework to analyze both qualitatively and quantitatively the dynamics of a financial crisis of the kind recently experienced. We are particularly interested in characterizing the sudden and discrete nature of the banking crisis as well as the circumstances that makes an economy vulnerable to such crises in some instances but not in others. Having a conventional macroeconomic model allows us to study the channels via which the crisis affects real activity and the effects of policies in containing crises.

*Very preliminary!
1 Introduction

As both Bernanke (2010) and Gorton (2010) argue, at the heart of the recent financial crisis was a series of banks runs that culminated in the precipitous demise of a number major financial institutions. During the period where the panics were most intense, in October 2008, all the major investment banks suddenly failed, the commercial paper market froze, and the Reserve Primary Fund (a major money market fund) experienced a run. The distress quickly spilled over to the real sector. Credit spreads rose to the Great Depression era levels. There was an immediate sharp contraction in economic activity. From 2008:Q4 through 2009:Q1 real output dropped eight percent, driven mainly by a forty percent drop in investment spending. Also relevant is that this sudden discrete contraction in financial and real economic activities occurred in the absence of any apparent large exogenous disturbance to the economy.

In this paper we incorporate banks and banking crises within a conventional macroeconomic framework - a New Keynesian model with capital accumulation. Our goal is to develop a model where it is possible to analyze both qualitatively and quantitatively the dynamics of a financial crisis of the kind recently experienced. We are particularly interested in characterizing the sudden and discrete nature of a banking crisis as well as the circumstances that make the economy vulnerable to such crises in some instances but not in others. Having a conventional macroeconomic model allows us to study the channels via which the crisis affects aggregate production and the effects of various policies in containing crises.

Our paper fits into a lengthy literature aimed at adapting core macroeconomic models to account for the recent financial crisis\textsuperscript{1}. Much of this literature emphasizes the role of balance sheets in constraining borrower from spending when financial markets are imperfect. Because balance sheets tend to strengthen in booms and weaken in recessions, financial conditions work to amplify fluctuations in real activity. Many authors have stressed that this kind of balance sheet mechanism played a central role in the crisis, particularly for banks and households, but also at the height of the crisis for non-financial firms as well. Nonetheless, as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2016) have emphasized, these models do not capture the highly nonlinear aspect of the crisis. The financial mecha-

\textsuperscript{1}See Gertler and Kiyotaki (2011) and Brunnermeier et. al (202x) for recent surveys.
nisms in these papers tend to amplify the effects of disturbances. They do not easily capture sudden discrete collapses. Nor do they tend to capture the run-like behavior associated with financial panics.

Conversely, beginning with Diamond and Dybvig (1983), there is a lengthy literature that models banking panics. An important common theme of this literature is how liquidity mismatch, i.e., partially illiquid long-term assets funded by short-term debt, opens up the possibility of runs. Most of the models in this literature, though, are partial equilibrium and highly stylized (e.g., three periods). They are thus limited for analyzing the interaction between the aggregate financial and real sectors.

Our paper builds on our earlier work - Gertler and Kiyotaki (2015) and Gertler, Kiyotaki and Prestipino (2016) - which analyzed bank runs in an infinite horizon endowment economy. These papers characterize runs as self-fulfilling rollover crises, following the Calvo (1988) and Cole and Kehoe (2001) models of sovereign debt crises. Both Gertler and Kiyotaki (2015) and Gertler, Kiyotaki and Prestipino (2016) emphasize the complementary nature of balance sheet effects and bank runs. Balance sheet conditions affect not only borrower access to credit but also whether the banking system is vulnerable to a run. In this way the model is able to capture the discrete highly nonlinear nature of a collapse: When bank balance sheets are strong, negative shocks do not push the financial system to the verge of collapse. When they are weak, the same size shock leads the economy into the danger zone in which bank run equilibrium exists. Given that these papers analyze bank runs in the context of an endowment economy, however, the focus is on the effects of anticipation and realization of panics on the behavior of assets prices and credit spreads. By extending the analysis to a conventional macroeconomic model, we can explicitly captures the interactions between a financial collapse and aggregate production.

Also related is important recent work on an occasionally binding borrowing constraints as a sources of nonlinearity in financial crises - Brunnermeier and Sannikov (2015), He and Krishnamurthy (2015) and Mendoza (2010). The general story is as follows: In good times the borrowing constraint is not binding and the economy behaves much the way it does with frictionless financial markets. However, a negative disturbance can move the economy into a region where the constraint is binding, amplifying the effect of the shock on the downturn.

Our approach also allows for occasionally binding financial constraints. However, in quantitative terms, bank runs provide the major source of non-
linearity. The effect of occasionally binding constraints is modest in our model for two reasons. First, instead of an exogenous maximum leverage ratio, we endogenize the maximum leverage ratio as the product of an agency problem. It turns out that the optimal maximum leverage ratio moves countercyclically, which has the effect of loosening the constraint in bad times, thus moderating the nonlinearity. Second, as noted in the literature, as the economy moves closer to the regions where the financial constraints bind, borrowers exhibit precautionary behavior (e.g. they reduce leverage) which has the effect of smoothing out the overall impact of the constraints. By contrast, as we show, bank runs generate discrete sharp contractions in both financial and real activities. In addition, the disturbances can significantly change the likelihood of bank run in the near future, which in turn can have a large effect on aggregate production, even if the run does not occur ex post.

Section 2 presents the behavior of households and banks, the sectors where the novel features of the model are introduced. Section 3 describes the features that are standard in the New Keynesian model: the behavior of firms, price setting, investment and monetary policy. Section 4 describes the calibration and then presents a variety of numerical exercises designed to illustrate the main features of the model. We conclude the section with an illustration of how the model can capture the dynamics of some of the main features of the recent financial crisis.

2 Households and banks

The baseline framework is a standard New Keynesian model with capital accumulation. In contrast to the conventional model, we allow for "bankers" who specialize in making loans and thus intermediate funds between households and productive capital. Households may also make these loans directly, but are less efficient in doing so than banks. On the other hand, banks may be constrained in their ability to obtain funds and also may be subject to runs. The net effect is that the cost of capital will depend on the endogenously determined flow of funds between intermediated and direct finance.

We distinguish between capital at the beginning of period $t$, $K_{t}$, and capital at the end of the period, $S_{t}$. The capital at the beginning of period is used in conjunction with labor to produce output at $t$. The capital at the end of period is the sum of newly produced capital and the amount of capital left after production:
\[ S_t = \Gamma \left( \frac{I_t}{K_t} \right) K_t + (1 - \delta) K_t, \]  
(1)

where \( \delta \) is the rate of depreciation. The quantity of newly produced capital, \( \Gamma(I_t/K_t)K_t \), depends upon investment \( I_t \) and the capital stock. We suppose that \( \Gamma(\cdot) \) is an increasing and concave function of \( I_t/K_t \) to capture convex adjustment costs.

A firm wishing to finance new investment as well as old capital issues a state-contingent claim on the earnings generated by the capital. Let \( S_t \) be the total number of claims outstanding at the end of period \( t \) (one claim per unit of capital), \( S^b_t \) the quantity intermediated by banks and \( S^h_t \) the quantity directly held by households. Then we have:

\[ S^b_t + S^h_t = S_t. \]  
(2)

Both the total capital stock and the composition of financing are determined in the equilibrium.

One way that \( K_{t+1} \) may differ from \( S_t \) is the presence of a multiplicative "capital quality" shock, \( \xi_{t+1} \), that randomly transforms the units of capital available at \( t + 1 \). A second way is that the composition of indirect bank versus direct household finance will matter. Following Brunnermeier and Sannikov (2015), we capture that households are less efficient in choosing and monitoring investment projects by assuming a "misallocation cost" that results in a reduced return to capital.

In particular, we suppose that the misallocation cost manifests itself as a loss \( \zeta(S^h_t, S_t) \) in the quantity of capital carried over to \( t + 1 \). As a result, the evolution of capital depends on the composition of bank versus household finance as follows:

\[ K_{t+1} = \xi_{t+1}[S^h_t + S^h_t - \zeta(S^h_t, S_t)] \]  
(3)

\[ = \xi_{t+1}[S_t - \zeta(S^h_t, S_t)]. \]  
(4)

We suppose further that \( \zeta(\cdot) \) is a piece-wise function as follows

\[ \zeta(S^h_t/S_t) = \begin{cases} \frac{1}{2}(S^h_t/S_t - \gamma)^2 S_t, & \text{if } \frac{S^h_t}{S_t} > \gamma \\ 0, & \text{if } \frac{S^h_t}{S_t} \leq \gamma \end{cases}. \]  
(5)

For \( S^h_t/S_t \leq \gamma \) there is no efficiency loss: Households are as efficient as banks in directly financing capital. Though we do not model it explicitly,
we capture the notion that certain types of investments are amenable to direct finance and do not require intermediation, e.g. investments in capital of mature and reputable publicly traded companies. As the share of direct finance exceeds $\gamma$, the efficiency cost $\zeta(\cdot)$ is increasing and convex in the share of direct household finance $S^h_t / S_t$. In this case, think of the marginal investments available to households that require increasing more expertise and resource to screen and monitor, e.g. investments of younger and more unknown firms. The convex form implies that the marginal efficiency losses rise with the size of the household’s direct capital holdings, capturing limits on its capacity to manage investments. Finally, for technical convenience, we assume that the efficiency costs is homogeneous in degree one in $S^h_t$ and $S_t$.

Both households and bankers can trade claims on capital in a competitive market. Let $Q_t$ be the market price of a claim on a unit of capital and $Z_t$ the rental price of capital. Then the gross rate of return on capital intermediated by banks, $R^{b}_{t+1}$, is given by

$$R^{b}_{t+1} = \xi_{t+1} \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_t}.$$  

(6)

Conversely, the marginal return on capital held by households, $R^{h}_{t+1}$, must factor in the increase in efficiency losses from raising the direct finance, as we characterize in the next section.

Given the efficiency losses from households financing capital, absent financial frictions banks will intermediate at least the fraction $1 - \gamma$ of the capital stock. Households in turn will save mostly in the form deposits at banks. However, when banks are constrained in their ability to obtain deposits, households will directly hold more than share $\gamma$ of the capital stock. As the constraints tighten in a recession, as will happen in our model, the share of capital held by households will expand. As we will show, in the general equilibrium the reallocation of capital holding from banks to households will raise the cost of capital, reducing investment and output. In the extreme event of a systemic bank run, which will become more likely in a recession, the contraction will become far more severe: As banks liquidate all their holdings, the household share of finance will temporarily rise to unity. In turn, there will be a sharp rise in the cost of credit, leading to an extreme contraction in investment and output.

\[ \text{\footnote{Think of the distribution of capital over its relative amenability to direct household finance as being invariant to the overall size of the capital stock.}} \]
In the rest of this section we characterize the behavior of households and banks, which are the non-standard parts of the model.

2.1 Households

Each household earns labor income, consumes and saves either by holding bank deposits or claims on capital directly.

Intermediary deposits at \( t \) are one period bonds that promise to pay a non-contingent gross nominal rate of return \( \bar{R}_{t+1} \) in the absence of default.\(^3\) In the event of default at \( t+1 \), depositors receive the fraction \( x_{t+1} \) of the promised return, where the recovery rate \( x_{t+1} \in [0,1) \) is the total liquidation value of bank assets per unit of promised deposit obligations. There are two reasons the bank may default: First, a sufficiently negative return on its portfolio may make it insolvent. Second, even if the bank is solvent at current market prices, the bank’s creditors may "run" forcing the bank to liquidate assets at resale prices. We describe each of these possibilities in detail in the next section. Let \( p_t \in [0,1] \) be the probability the bank defaults period in \( t+1 \). Given \( p_t \) and \( x_t \), we can express the gross rate of return on the deposit contract \( R_{t+1} \) as

\[
R_{t+1} = \begin{cases} 
\bar{R}_{t+1} & \text{with probability } 1 - p_t \\
x_{t+1} \bar{R}_{t+1} & \text{with probability } p_t 
\end{cases}
\]

As in the Cole and Kehoe (2001) model of sovereign default, a run in our model will corresponds to a panic failure of households to roll over deposits. This contrasts with the "early withdrawal" mechanism in the classic Diamond and Dybvig (1983) model. For this reason we do not need to impose a "sequential service constraint" which is necessary to generate runs in Diamond and Dybvig. Instead we make the weaker assumption that all households receive the same pro rata share of output in the event of default, whether it be due to insolvency or a run. Later we describe the conditions that lead to the existence of an equilibrium where a "failure to rollover" run is possible.

\(^3\)In the basic model, we assume the promised rate of returns of deposit is real, or indexed by the nominal price level. Later we will extend our model to the case in which the promised rate of return is noncontingent in terms of nominal as in most developed countries. This will lead to a possibility of bank run interacting with debt deflation as in the Great Depression.
Household utility $U_t$ is given by

$$U_t = E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( \ln C_{t+i}^h - \frac{1}{1 + \varphi (L_{t+i}^h)^{1+\varphi}} \right) \right],$$

where $C_t^h$ is household consumption, $L_t^h$ is labor supply, and $0 < \beta < 1$. Let $w_t$ denote the real wage. Then the household chooses consumption, labor supply, bank deposits $D_t$ and direct holdings of claims on capital $S_t^h$ to maximize expected utility subject to the budget constraint

$$C_t^h + D_t + Q_t S_t^h = w_t L_t^h + \Pi_t + R_t D_{t-1} + \xi_t[Z_t + (1 - \delta)Q_t][S_{t-1}^h - \varsigma(S_{t-1}^h, S_{t-1})],$$

where $\Pi_t$ are profits from monopolistically competitive firms (see below), and $\varsigma(S_t^h, S_t)$, given by equation (5), is the efficiency loss to the return on capital.

The first order condition for labor supply is standard:

$$w_t \frac{1}{C_t} = (L_t^h)\varphi. \quad (8)$$

The first order condition for bank deposits takes into account the possibility of default and is given by

$$1 = \{(1 - p_t)E_t[\Lambda_{t,t+1} | \text{no def.}] + p_t E_t[\Lambda_{t,t+1} x_{t+1} | \text{def.}]\} \cdot \overline{R}_{t+1} \quad (9)$$

where $E_t[\cdot | \text{no def.}]$ (and $E_t[\cdot | \text{def.}]$) are expected value of · conditional on no default (and default) at date $t+1$. The stochastic discount factor $\Lambda_{t,t+1}$ satisfies

$$\Lambda_{t,t+i} = \beta^i \frac{C_t^h}{C_{t+i}^h}. \quad (10)$$

Observe that the promised deposit rate $\overline{R}_{t+1}$ that satisfies equation (9) depends on the default probability $p_t$ as well as the recovery rate $x_{t+1}$.

Finally, the first order condition for capital holdings is given by

$$E_t(\Lambda_{t,t+1} R_{t+1}^h) = 1, \quad (11)$$

where $R_{t+1}^h$ is the marginal return on capital, given by

$$R_{t+1}^h = \left[ 1 - \frac{\partial \varsigma(S_t^h, S_t)}{\partial S_t^h} \right] \xi_{t+1} \frac{Z_{t+1} + (1 - \delta)Q_{t+1}}{Q_t}, \quad (12)$$
which can be expressed as

\[
R_{t+1}^h = \begin{cases} 
1 - \lambda \left( \frac{s_t^h}{s_t} - \gamma \right) & \text{if } \frac{s_t^h}{s_t} > \gamma \\
R_{t+1}^b & \text{if } \frac{s_t^h}{s_t} \leq \gamma 
\end{cases}
\]

The first order condition given by (11) will be key in determining the market price of capital. Observe that the market price of capital will tend to be decreasing in the share of capital held by households above the threshold \( \gamma \) since the efficiency cost \( \zeta(S_t^h, S_t) \) is increasing and convex. As will become clear, in a panic run banks will sell all their securities to households, leading to a sharp contraction in asset prices. The severity of the drop will depend on the curvature of the efficiency cost function given by (5).

2.2 Bankers

The banking sector we characterize corresponds best to the shadow banking system which was at the epicenter of the financial instability during the Great Recession. In particular, banks in the model are completely unregulated, hold long-term securities, issue short-term debt, and as a consequence are potentially subject to runs.

2.2.1 Bankers optimization problem

Each banker manages a financial intermediary. Bankers fund capital investments by issuing short term deposits \( d_t \) to households as well as by using their own equity, or net worth, \( n_t \). Due to financial market frictions, bankers may be constrained in their ability to obtain deposits.

We assume that bankers have a finite expected lifetime: Specifically, each banker has an i.i.d. probability \( \sigma \) of surviving until the next period and a probability \( 1 - \sigma \) of exiting. The expected lifetime of a banker is then \( \frac{1}{1 - \sigma} \). We introduce finite expected lifetimes for bankers to preclude them from saving their way out of the financing constraint.

To replace each exiting banker, a new banker enters. We assume that in the entry period each new banker earns "start up equity" by supplying \( l^* \) units of labor inelastically. From that point on, the banker accumulates equity via retained earnings.
In particular, we assume that bankers are risk neutral and enjoy utility from consumption in the period they exit.\footnote{We could generalize to allow active bankers to receive utility that is linear in consumption each period. So long as the banker is constrained, it will be optimal to defer all consumption until the exit period.} The expected utility of a continuing banker at the end of period $t$ is given by

$$V_t = E_t \left[ \sum_{i=1}^{\infty} \beta^i (1 - \sigma) \sigma^{i-1} c_{t+i}^b \right],$$

where $(1 - \sigma) \sigma^{i-1}$ is probability of exiting at date $t + i$, and $c_{t+i}^b$ is terminal consumption if the banker exits at $t + i$.

During each period $t$, a continuing bank (either new or surviving) finances asset holdings $Q_t s_t^b$ with newly issued deposits and net worth:

$$Q_t s_t^b = d_t + n_t.$$ \hspace{1cm} (13)

We assume that banks can only accumulate net worth via retained earnings and do not issue new equity. While this assumption is a reasonable approximation of reality, we do not explicitly model the agency frictions that underpin it.

The net worth of "surviving" bankers, accordingly, is the gross return on assets net the cost of deposits, as follows:\footnote{In data, net worth here corresponds to the mark-to-market difference between assets and liabilities of the bank balance sheet. It is different from the book value often used in the official report, which is slow in reacting to market conditions. Also the bank assets here are securities and loans to non-financial sector, which exclude those to the other financial intermediaries. In data, the net mark-to-market leverage multiple of the financial intermediation sector - the ratio of securities and loans to the nonfinancial sector to the net worth of the aggregate financial intermediaries - tends to move counter-cyclically, even though the gross leverage multiple - the ratio of book value total assets (including securities and loans to the other intermediaries) to the net worth of some individual intermediaries may move procyclically. See He, Khang and Krishnamurthy (2010) and Adrian and Shin (2010).}

$$n_t = R_t^b Q_{t-1} s_{t-1}^b - R_t d_{t-1}.$$ \hspace{1cm} (14)

So long as $n_t$ is positive the bank does not default. In this instance it pays its creditors the promised rate $\overline{R}_t$. If $n_t$ turns negative (due either to a run or
simply a bad realization of returns), the bank defaults. It then pays creditors
the product of recovery rate $x_t$ and $R_t$, where $x_t$ is given by.

$$x_t = \frac{R_b^t Q_{t-1}^b s_{t-1}^b}{R_t d_{t-1}} < 1.$$  \hspace{1cm} (15)

For new bankers at $t$, net worth simply equals labor income earned at the
time of entry:

$$n_t = w_t b.$$ \hspace{1cm} (16)

Meanwhile, exiting bankers no longer operate banks and simply use their net
worth to consume:

$$c_t^b = n_t.$$ \hspace{1cm} (17)

Observe that the equity withdrawals by the exiting bankers correspond to
dividend payouts.

To motivate a limit on the bank’s ability to issue deposits, we introduce
the following moral hazard problem: After accepting deposits and buying
assets at the beginning of $t$, but still during the period, the banker decides
whether to operate "honestly" or to divert assets for personal use. To operate
honestly means holding assets until the payoffs are realized in period $t + 1
and then meeting deposit obligations. To divert means selling a fraction $\theta$
of assets secretly on a secondary market in order to obtain funds for personal
use. We assume that the process of diverting assets takes time: The banker
cannot quickly liquidate a large amount assets without the transaction being
noticed. To remain undetected, he can only sell up to a fraction $\theta$ of the
assets and he can only sell these assets slowly. For this reason the banker
must decide whether to divert at $t$, prior to the realization of uncertainty at
$t + 1$. The cost to the banker of the diversion is that the depositors can force
the intermediary into bankruptcy at the beginning of the next period.

The banker’s decision at $t$ boils down to comparing the franchise value of
the bank $V_t$, which measures the present discounted value of future payouts
from operating honestly, with the gain from diverting funds, $\theta Q_t k_t^b$. In this
regard, rational depositors will not lend funds to the banker if he has an
incentive to cheat. Accordingly, any financial arrangement between the bank
and its depositors must satisfy the following incentive constraint:

$$\theta Q_t s_t^b \leq V_t.$$ \hspace{1cm} (18)

Given that bankers simply consume their net worth when they exit, we
can restate the bank’s franchise value recursively as the expected discounted
value of the sum of net worth conditional on exiting and the value conditional on continuing as:

\[ V_t = E_t[\beta (1 - \sigma) n_{t+1} + \beta \sigma V_{t+1}] \]

\[ = \text{Prob}(n_{t+1} > 0) \cdot E_t\{\beta \Omega_{t,t+1} n_{t+1} | n_{t+1} > 0\}, \]

where the bank uses the stochastic discount factor \( \beta \Omega_{t,t+1} \) to value net worth realized in \( t+1 \), and \( \Omega_{t,t+1} \) is the banker’s shadow value of a unit of net worth at \( t + 1 \), averaged across the likelihood of exit and the likelihood of survival, given by

\[ \Omega_{t,t+1} = 1 - \sigma + \frac{V_{t+1}}{n_{t+1}}. \]

With probability \( 1 - \sigma \) the banker exits, implying a unit of net worth equals unity (the number of consumption goods it can purchase). With probability \( \sigma \) the banker survives implying the marginal value of \( n_{t} \) is \( \frac{V_{t+1}}{n_{t+1}} \), the franchise value of the bank per unit of net worth. As will become clear, to the extent an additional unit of net worth relaxes the financial market friction, \( \frac{V_{t+1}}{n_{t+1}} \) in general will exceed unity, provided that the bank does not default.

We can simplify the maximization problem a bit further. Let \( \phi_t \) denote the bank’s ratio of assets to net worth, \( Q_t s_t^b / n_t \), which we will call the "leverage multiple." Then, combining the flow of funds constraint (14) and the balance sheet constraint (13) yields the following expression for the evolution of net worth for a surviving bank:

\[ n_{t+1} = (R_{t+1}^b - \overline{R}_{t+1}) Q_t s_t^b + \overline{R}_{t+1} n_t \]

\[ = \left[ (R_{t+1}^b - \overline{R}_{t+1}) \phi_t + \overline{R}_{t+1} \right] n_t. \]

Net worth at \( t \) for a surviving banks depends positively on the excess return on its assets and the leverage multiple.

Let \( 1 - p_t = \text{Prob}(n_{t+1} > 0) \) be probability of no default and \( p_t \) be probability of default. Combining equation (21) with the objective (19) and dividing through by \( n_t \) permits us to restate the objective as to maximize \( V_t/n_t \), as follows:

\[ \frac{V_t}{n_t} = \max_{\phi_t} (1 - p_t) E_t\{\beta \Omega_{t,t+1} \left[ (R_{t+1}^b - \overline{R}_{t+1}) \phi_t + \overline{R}_{t+1} \right] | \text{no def.} \} \]
subject to the incentive constraint (obtained from equation (18)): 

$$\theta \phi_t \leq \frac{V_t}{n_t}$$  \hspace{1cm} (23)$$

and the deposit rate constraint (obtained from equations (9) and (15)):

$$\overline{R}_{t+1} = \left[ \{(1 - p_t)E_t \{\Lambda_{t,t+1} | \text{no def}\} + p_t E_t \{\Lambda_{t,t+1} x_{t+1} | \text{def}\} \right]^{-1}$$  \hspace{1cm} (24)$$

where $x_{t+1}$ is the following function of $\phi_t$:

$$x_{t+1} = \frac{\phi_t}{\phi_t - 1} \frac{R_t}{\overline{R}_{t+1}}.$$  

Given the linearity in the bank’s portfolio decision problem, the optimal choice of $\phi_t$ is independent of $n_t$.

2.2.2 Banker’s decision rules

We next proceed to characterize the bank’s optimal choice of the leverage multiple $\phi_t$. Given equation (22), we can express $V_t$ as

$$\frac{V_t}{n_t} = \mu_t \phi_t + \nu_t,$$  \hspace{1cm} (25)$$

where

$$\mu_t = (1 - p_t) E_t \{\beta \Omega_{t,t+1} \{(R_t - \overline{R}_{t+1}) | \text{no def}\} \}$$  \hspace{1cm} (26)$$

$$\nu_t = (1 - p_t) E_t \{\beta \Omega_{t,t+1} \overline{R}_{t+1} | \text{no def}\}.$$  

We can think of $\mu_t$ as the expected discounted excess return on banks assets relative to deposits and $\nu_t$ as the expected discounted cost of a unit of deposits.

Next define $\mu^*_t$ as the expected discounted \textit{marginal excess return to bank assets},

\footnote{Note that, although the default probability $p_t$ depends upon $\phi_t$, the marginal effect of $\phi_t$ on firm value $V_t$ through the change of $p_t$ is zero. This is because at the borderline of default, $n_{t+1} = 0$ and thus $V_{t+1} = 0$. Thus a small shift in the probability mass from the no-default to the default region has no impact on $V_t$. Similarly, the promised deposit rate $\overline{R}_t$ does not change since at the borderline of default, the recovery rate $x_t$ is unity. See Appendix for details. Important to the argument is the absence of deadweight loss associated with default.}
\begin{equation}
\mu_t^r = \mu_t - (\phi_t - 1) \frac{\nu_t}{R_{t+1}} \frac{dR_{t+1}}{d\phi_t} < \mu_t. \tag{27}
\end{equation}

The second term on the right of equation (27) reflects the effect of the increase in $R_{t+1}$ that arises as the bank increases $\phi_t$. An increase in $\phi_t$ reduces the recovery rate, forcing $R_{t+1}$ up to compensate depositors, as equation (24) suggests. The term $(\phi_t - 1) \frac{\nu_t}{R_{t+1}}$ then reflects the reduction in the bank franchise value that results from each unit increase in $R_{t+1}$. Due to the marginal effect on $R_{t+1}$ from expanding $\phi_t$, the marginal excess return $\mu_t^r$ is below the average excess return $\mu_t$.

The solution for $\phi_t$ depends on whether or not the incentive constraint (23) is binding. In the case where it binds, $\phi_t$ is given by

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}, \text{ if } \mu_t^r > 0. \tag{28}$$

In this instance, even though the marginal excess return is positive, the incentive constraint limits the bank from increasing leverage to acquire more assets. The constraint (28) limits the leverage multiple to the point where the bank’s gain from diverting funds per unit of net worth $\theta \phi_t$ is exactly balanced by the cost per unit of net worth of losing the franchise value, which is measured by $V_t/n_t = \mu_t \phi_t + \nu_t$. Note that the excess return $\mu_t$ tends to move countercyclically since the excess return $E_t R_{t+1}^h - R_{t+1}$ widens as the borrowing constraint tightens in recessions. As a result, $\phi_t$ tends to move countercyclically.

Conversely, when the constraint is not binding, the bank expands leverage and assets to the point where the marginal excess return is zero.

$$\mu_t^r = 0, \text{ if } \phi_t < \frac{\nu_t}{\theta - \mu_t}. \tag{29}$$

Even if the constraint does not bind, the bank may still choose to limit the leverage multiple, so long as there is a possibility that the incentive constraint could bind in the future. In this instance, as in Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2015), banks have a precautionary motive for scaling back their respective leverage multiples.\footnote{One difference from these papers is that because default is possible, the bank’s decision over its leverage multiple also affects to promised deposit rate, which affects the cost of funds at the margin. This effect provides an additional motive for the bank to reduce its leverage multiple.}

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precautionary motive is reflected by the presence of the discount factor \( \Omega_{t,t+1} \) in the measure of the discounted excess return. The multiplier \( \Omega_{t,t+1} \), which reflect the shadow value of net worth, tends to vary countercyclically given that borrowing constraints tighten in downturns. By reducing their leverage multiples, banks reduce the risk of taking losses when the shadow value of net worth is high - even though the bank’s preference is risk neutral.

In either case, as we conjectured, the franchise value of the bank \( V_t \) is proportionate to \( n_t \) by a factor that in independent of bank-specific factors: When the incentive constraint is binding (see equation (23)),

\[
V_t = \theta \phi_t \cdot n_t
\]

and it is not currently binding, \( \mu^* = 0 \), implying

\[
V_t = \left\{ \left[ (\phi_t - 1) \frac{\nu_t}{R_{t+1}} \frac{dR_{t+1}(\phi_t)}{d\phi_t} \right] \phi_t + \nu_t \right\} \cdot n_t
\]

as equations (25) and (27) suggest.

An important corollary is that the bank cannot operate with zero net worth. In this instance \( V_t \) falls to zero, implying that the incentive constraint (18) would always be violated if the bank tried to issue deposits. As we show, a necessary condition for a bank run is that banks cannot operate with zero net worth.

### 2.2.3 Aggregation of the financial sector absent default

Given that individual bank portfolio decisions are homogenous in net worth, the optimal leverage multiple \( \phi_t \) is independent of bank-specific factors. Accordingly, we can sum across banks to obtain the following relation between aggregate bank asset holdings \( QK_t^b \) and the aggregate quantity of net worth in the banking sector:

\[
\frac{QK_t^b}{N_t} = \phi_t.
\]

Adding across both surviving and entering bankers yields the following expression for the evolution of net worth

\[
N_t = \sigma [(R_t^b - \bar{R}_t)\phi_{t-1} + \bar{R}_t]N_{t-1} + W_t^b
\]
where $W_t^h = (1 - \sigma)w^t l^h$ is the total endowment of entering bankers. The first term is the total net worth of bankers that operated at $t - 1$ and survived until $t$.

Conversely, exiting bankers consume the fraction $1 - \sigma$ of net earnings on assets:

$$C_t^b = (1 - \sigma)[(R_t^b - \overline{R}_t)\phi_{t-1} + \overline{R}_t]N_{t-1}.$$ 

### 2.3 Bank runs versus insolvency and the probability of default

In this section we describe bank runs and the condition for a bank run equilibrium to exist. We also distinguish a run equilibrium from insolvency, a second source of default. In the process, a sharp distinction will arise in our model between illiquidity and insolvency. Finally we characterize how the overall default probability is determined. Within our calibrated model, the possibility of runs will significantly increase the likelihood of default. Further, both runs and insolvency are more likely in recessions, which has important implications for model behavior.

#### 2.3.1 Conditions for a bank run equilibrium

As in Diamond and Dybvig (1983), the runs we consider are runs on the entire banking system and not an individual bank. A run on an individual bank will not have aggregate effects as depositors simply shuffle their funds from one bank to another. As we noted earlier, though, we differ from Diamond and Dybvig though in that runs reflect a panic failure to roll over deposits as opposed to early withdrawal.

Consider the behavior of a household that acquired deposits at $t - 1$. Suppose further that the banking system is solvent at the beginning of time $t$ : i.e. net worth is positive, implying that assets valued at current market prices exceed liabilities. The household must then decide whether to roll over deposits at $t$. A self-fulfilling "run" equilibrium exists if the household perceives that in the event all other depositors run, thus forcing the banking system into liquidation, the household will lose money if it rolls over its deposits individually. Note that this condition is satisfied if the liquidation makes the banking system insolvent, i.e. drives aggregate bank net worth to zero. Given the moral hazard problem, a household that deposits in a zero net worth bank will simply lose its money (as the bank runs away with it.)
The condition for a bank run equilibrium at $t$, accordingly, is that in the event of liquidation following a run, bank net worth goes to zero. Recall that earlier we defined the depositor recovery rate, $x_t$, as the ratio of the value of bank assets in liquidation to promised obligations to depositors. Accordingly, the condition for a bank run equilibrium is simply that the recovery rate conditional on a run, $x_t^R$, is less than unity:

$$x_t^R = \frac{\xi_t[(1-\delta)Q_t^* + Z_t]S_t^b}{R_tD_{t-1}} < 1$$

where $R_t^b$ is the return on bank assets conditional on liquidation and $Q_t^*$ is the asset liquidation price. Note that in general that the liquidation price $Q_t^*$ is below the market price $Q_t$, implying that a run may occur even if the bank is solvent at beginning of period market prices. Further, as discussed in Gertler and Kiyotaki (2015), given $\frac{R_t^b}{R_t}$ is procyclical and $\phi_{t-1}$ is countercyclical, the likelihood of a bank run equilibrium existing is greater in recessions than in booms.

### 2.3.2 The liquidation price

Key to the condition for a bank run equilibrium is the behavior of the liquidation price $Q_t^*$. A depositor run at $t$ induces all the existing banks to liquidate their assets by selling them to households. We suppose that new banks enter only one period after the panic. Accordingly in the wake of the run:

$$S_t^b = S_t.$$  

(33)

The banking system then rebuilds itself over time as new banks enter. The evolution of net worth following the run at $t$ is given by

$$N_{t+1} = W_{t+1}^b.$$  

(34)

$$N_{t+i} = \sigma[(R_{t+i}^b - R_{t+i})\phi_{t+i-1} + R_{t+i}]N_{t+i-1} + W_{t+i}^b, \text{ for all } i \geq 2.$$  

To obtain $Q_t^*$, we invert the household Euler equation to obtain:

$$Q_t^* = E_t \left\{ \sum_{i=1}^{\infty} \Lambda_{t+i} \prod_{j=0}^{i} \left[ \left( 1 - \lambda Max \left( \frac{S_{t+j}^b}{S_{t+j}^b} - \gamma, 0 \right) \right) \xi_{t+i}(1-\delta) \right] \cdot Z_{t+i} \right\}.$$  

(35)
After a bank run at $t$, we have $\frac{S_h}{S_t} = 1$, implying

$$1 - \lambda \max\left(\frac{S_h}{S_t} - \gamma, 0\right) = 1 - \lambda(1 - \gamma).$$

The liquidation price is thus equal to the expected discounted stream of dividends net the marginal efficiency losses from household portfolio management. Since marginal efficiency losses are at a maximum when $S_h = S_t$, $Q_t^*$ is at a minimum, given the expected future path of $S_t^h$. Further, the longer it takes the banking system to recover (so $S_t^h$ to falls back to its steady state value) the lower will be $Q_t^*$. Finally, note that $Q_t^*$ will vary positively with the expected path of $\xi_{t+1} Z_{t+1}$ and with the stochastic discount factor $\Lambda_{t,t+1}$.

### 2.3.3 The default probability and illiquidity versus insolvency

In the run equilibrium we just described, banks default even though they are solvent at beginning of period market prices: It is the forced liquidation at firesale prices with run that pushes these banks into bankruptcy. Thus, in the context of our model, a bank run can be viewed as a situation of illiquidity. By contrast, default is also possible if banks enter period $t$ insolvent at current market prices.

Accordingly, the total probability of default in the subsequent period, $p_t$, is the sum of the probability of a run $p_t^R$ and the probability of insolvency $p_t^I$:

$$p_t = p_t^R + p_t^I. \quad (36)$$

We begin with $p_t^I$. By definition, banks are insolvent if the ratio of assets valued at current market prices is less than liabilities. In our economy, the only exogenous shock to the aggregate economy is shock to the capital quality $\xi_t$. Accordingly, define $\xi_{t+1}^I$ as the value of the shock to the return on capital, $\xi_{t+1}$, that makes the depositor recovery rate at current market prices, $x(\xi_{t+1})$, equal to unity.

$$x(\xi_{t+1}^I) = \frac{\xi_{t+1}^I (Z_{t+1}(\xi_{t+1}^I) + (1 - \delta)Q_{t+1}(\xi_{t+1}^I)]S_t^h}{R_t D_t} = 1. \quad (37)$$

For values of $\xi_{t+1}^I$ below $\xi_{t+1}$, the bank will be insolvent and must default. Accordingly, the probability of default due to insolvency is given by
\[ p_t^I = \text{prob}_t \left( \xi_{t+1} < \xi_{t+1}^I \right), \]  

(38)

where \( \text{prob}_t (\cdot) \) is the expected value of \( \cdot \) conditional on date \( t \) information.

We next turn to the determination of the run probability. Let \( \chi_{t+1} \) be a binary sunspot variable that takes on a value of 1 with probability \( \pi \) and a probability of 0 with probability \( 1 - \pi \). In the event of \( \chi_{t+1} = 1 \), depositors coordinate on a run if a bank run equilibrium exists. Accordingly, a bank run arises at \( t + 1 \) if (i) a bank run equilibrium exists at \( t + 1 \) and (ii) \( \chi_{t+1} = 1 \). Let \( \omega_t \) be the probability at \( t \) that a bank run equilibrium exists at \( t + 1 \). Then the probability \( p_t^R \) of a run at \( t + 1 \) is given by

\[ p_t^R = \omega_t \cdot \pi. \]  

(39)

To find the value of \( \omega_t \), let us define \( \xi_{t+1}^R \) as the value of \( \xi_{t+1} \) that makes the recovery rate conditional on a run \( x_{t+1}^R \) unity when evaluated at the firesale liquidation price \( Q_{t+1}^s \):

\[ x(\xi_{t+1}^R) = \frac{\xi_{t+1}^R [(1 - \delta)Q^*(\xi_{t+1}^R) + Z(\xi_{t+1}^R)]S^h_t}{R_tD_t} = 1. \]  

(40)

Accordingly, for values of \( \xi_{t+1} \) below \( \xi_{t+1}^R \), \( x_{t+1}^R \) is below unity, leading to the possibility of a bank run equilibrium. The probability of a bank run equilibrium existing is accordingly the probability that \( \xi_{t+1} \) lies in the interval below \( \xi_{t+1}^R \) but above the threshold for insolvency \( \xi_{t+1}^I \). In particular,

\[ \omega_t = \text{prob}_t \left( \xi_{t+1}^I \leq \xi_{t+1} < \xi_{t+1}^R \right). \]  

(41)

Given equation (41), we can distinguish regions of \( \xi_{t+1} \) where insolvency emerges \( (\xi_{t+1} < \xi_{t+1}^I) \) from regions where an illiquidity problem may emerge \( (\xi_{t+1}^I \leq \xi_{t+1} < \xi_{t+1}^R) \).

Within the numerical exercises we present the probability of a fundamental shock that induces an insolvent banking system is negligible. However, the probability of a shock that induces a bank run equilibrium is non-trivial. It follows that the probability of a run varies inversely with the expected recovery rate \( E_t x_{t+1} \). The lower the forecast of the depositor recovery rate, the higher \( \omega_t \) and thus the higher \( p_t \). In this way the model captures that an expected weakening of the banking system raises the likelihood of a run. As we show next, there is an interesting feedback: a rise in the run probability will weaken the banking system in turn.
Finally, comparing equations (38) and (41) makes clear that the possibility of a run equilibrium expands the set of realizations where default is possible. That is, the possibility of runs significantly expands the chances for a banking collapse, beyond the probability that would arise simply from default due to insolvency. In this way the possibility of runs make the system more fragile.

3 Production sector, resource constraints and policy

The rest of the model is fairly standard. There is a production sector consisting of final goods, intermediate goods and capital goods producing firms. Prices are sticky in the intermediate goods sector. In addition there is a central bank that conducts monetary policy.

3.1 Final and intermediate goods firms

As noted, there are two types of goods producing firms: final and intermediate. There is a continuum of measure unity of each type. Final goods firms make a homogenous good $Y_t$ that may be consumed or used as input to produce new capital goods. Each intermediate goods firm $f \in [0,1]$ makes a specialize good $Y_t(f)$ that is used in the production of final goods. Intermediate goods firms also face costs of adjusting nominal prices.

The production function that final goods firms use to transforms intermediate goods into final output is given by the following CES aggregator:

$$Y_t = \left[ \int_0^1 Y_t(f) \frac{\varepsilon-1}{\varepsilon} df \right]^\frac{1}{\varepsilon-1},$$

where $\varepsilon > 1$ is the (constant) elasticity of substitution between intermediate goods.

Let $P_t(f)$ be the nominal price of intermediate good $f$. Then cost minimization yields the following demand function for each intermediate good $f$ (after integrating across the demands of by all final goods firms):

$$Y_t(f) = \left[ \frac{P_t(f)}{P_t} \right]^{-\varepsilon} Y_t,$$

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where $P_t$ is the price index as

$$P_t = \left[ \int_0^1 P_t(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}.$$ 

There is a continuum of intermediate good firms owned by consumers, indexed by $f \in [0, 1]$. Each produces a differentiated good and is a monopolistic competitor. Intermediate goods firm $f$ uses both labor $L_t(f)$ and capital $K_t(f)$ to produce output according to:

$$Y_t(f) = A_t K_t(f)^\alpha L_t(f)^{1-\alpha},$$

where $A_t$ is a technology parameter and $0 > \alpha > 1$ is the capital share.

Both labor and capital are freely mobile across firms. Firms rent capital from owners of claims to capital (i.e. banks and households) in a competitive market on a period by period basis. Then from cost minimization, all firms choose the same capital labor ratio, as follows

$$\frac{K_t(f)}{L_t(f)} = \frac{\alpha w_t}{1-\alpha Z_t} = \frac{K_t}{L_t},$$

where, as noted earlier, $w_t$ is the real wage and $Z_t$ is the rental price of capital. The first order conditions from the cost minimization problem also imply that the minimized marginal cost equals the average cost and is given by

$$MC_t = 1 \frac{1}{A_t} \left( \frac{W_t/P_t}{1-\alpha} \right)^{1-\alpha} \left( \frac{Z_t}{\alpha} \right) \alpha.$$ 

Observe that marginal cost is independent of firm-specific factors.

Following Rotemberg (1982), each monopolistically competitive firm $f$ faces quadratic costs of adjusting prices. Let $\rho^r$ ("r" for Rotemberg) be the parameter governing price adjustment costs. Then each period, it chooses $P_t(f)$ and $Y_t(f)$ to the expected discounted value of profit:

$$E_t \left\{ \sum_{i=0}^\infty \Lambda_{t,t+i} \left[ \left( \frac{P_{t+i}(f)}{P_{t+i}^*} - MC_{t+i} \right) Y_{t+i}(f) - \frac{\rho^r}{2} Y_{t+i} \left( \frac{P_{t+i}(f)}{P_{t+i-1}^*} - 1 \right)^2 \right] \right\},$$

subject to the demand curve (43). Here we assume that the adjustment cost is loss of utility rather than the use of resources and is proportional to the aggregate demand $Y_{t+i}$.

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Taking the firm’s first order condition for price adjustment and imposing symmetry implies the following forward looking Phillip’s curve:

\[
\left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} = \frac{\varepsilon}{\rho'} \left( MC_t - \frac{\varepsilon - 1}{\varepsilon} \right) + E_t \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t}.
\]

(48)

3.2 Capital producers

There is a continuum of measure unity competitive capital goods firms. Each produces new investment goods that it sells at the competitive market price \( Q_t \). By "investing" units of final goods output, firm \( j \) can produce \( \Gamma(I_t(j)/K_t) \cdot K_t \) new capital goods, with \( \Gamma' > 0, \Gamma'' < 0 \), and where \( K_t \) is the aggregate capital stock.\(^8\)

The decision problem for capital producer \( j \) is accordingly

\[
\max_{I_t(j)} Q_t \Gamma \left( \frac{I_t(j)}{K_t} \right) K_t - I_t(j).
\]

(49)

Given symmetry for capital producers \((I_t(j) = I_t)\), we can express the first order condition as the following "Q" relation for investment:

\[
\text{Here capital stock depreciates only after used twice for producing intermediate goods and new capital goods and thus}
\]

\[
\int_0^1 K_t(j) dj = K_t.
\]

From the first order condition with respect to \( I_t(j) \) and \( K_t(j) \) imposing symmetry, we obtain (50) in the text and

\[
Q_t^0 = 1 - \delta + \Gamma(I_t/K_t) - (I_t/K_t)\Gamma'(I_t/K_t).
\]

Because production function is constant returns to scale, profit of capital goods producers equals zero in equilibrium. See Lorenzoni and Walentin (2007) for a similar formulation.

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\(^8\)Aggregate capital stock may enters into production function as an externality, for an example. Alternatively, each capital goods producer buys capital stock \( K_t(j) \) (after being used to produce intermediated goods) at price \( Q_t^0 \) and combine with final goods \( I_t(j) \) to produce total capital stock \( \Gamma(I_t(j)/K_t) \cdot K_t(j) + (1 - \delta)K_t(j) \). Taking the prices as given, they choose \( I_t(j) \) and \( K_t(j) \) to maximize the profit

\[
Q_t [\Gamma(I_t(j)/K_t(j))K_t(j) + (1 - \delta)K_t(j)] - I_t(j) - Q_t^0 K_t(j).
\]

Here capital stock depreciates only after used twice for producing intermediate goods and new capital goods and thus

\[
\int_0^1 K_t(j) dj = K_t.
\]
\[ Q_t = \left[ \Gamma' \left( \frac{I_t}{K_t} \right) \right]^{-1} \]  

which yields a positive relation between \( Q_t \) and investment.

### 3.2.1 Monetary Policy

Let \( \Theta_t \) be a measure of cyclical resource utilization, i.e., resource utilization relative to the flexible price equilibrium. Next let \( R = \beta^{-1} \) denote the real interest rate in the deterministic steady state with zero inflation. We suppose that the central bank sets the nominal rate, \( R^n_t \) according to the following Taylor rule:

\[ R^n_t = \max \left\{ \frac{1}{\beta} \left( \frac{P_t}{P_{t-1}} \right)^{\kappa_\pi} (\Theta_t)^{\kappa_\pi}, 1 \right\} \]  

with \( \kappa_\pi > 1 \). Note that the rule imposes a zero lower bound: the net nominal rate cannot go below zero, which implies that the gross nominal rate must be greater than or equal to unity.

A standard way to measure \( \Theta_t \) is to use the ratio of actual output to a hypothetical output of flexible price equilibrium. Computational considerations lead us to use a measure which similarly captures the cyclical efficiency of resource utilization but is much easier to handle numerically. Specifically, we take as our measure of cyclical resource utilization the ratio of the desired markup, \( 1 + \mu = \varepsilon/(\varepsilon - 1) \) to the current markup \( 1 + \mu_t \)\(^9\)

\[ \Theta_t = \frac{1 + \mu}{1 + \mu_t} \]  

with

\[ 1 + \mu_t = MC_t^{-1} = \frac{(1 - \alpha)(Y_t/L_t)}{L_t^c C_t} . \]  

The markup corresponds to the ratio of the marginal product of labor to the marginal rate of substitution between consumption and leisure (i.e. the labor market wedge.) It thus provides measures of labor market inefficiency. The inverse markup ratio \( \Theta_t \) thus isolates the cyclical movement in the efficiency of the labor market, specifically the component that is due to nominal rigidities.

\(^9\)In the case of consumption goods only, our markup measure of efficiency corresponds exactly to the output gap.
Finally, one period bonds which have a riskless nominal return have zero net supply. (Banks deposits have default risk). Nonetheless we can use the following household euler equation to price the nominal interest rate of these bonds \( R_{t+1}^{n} \) as

\[
E_{t} \left\{ A_{t,t+1}R_{t+1}^{n} \frac{P_{t}}{P_{t+1}} \right\} = 1. \tag{54}
\]

3.2.2 Resource constraints and equilibrium

Total output is divided between consumption and investment

\[
Y_{t} = C_{t} + I_{t}. \tag{55}
\]

Given a symmetric equilibrium, we can express total output as the following function of aggregate capital and labor:

\[
Y_{t} = A_{t}K_{t}^{\alpha}L_{t}^{1-\alpha}. \tag{56}
\]

Finally, labor market must clear, which implies that the total quantity of labor demanded must equaled the total amount supply by households and new bankers:

\[
L_{t} = L_{t}^{h} + l^{b}. \tag{57}
\]

This completes the description of the model.

4 Numerical exercises

4.1 Calibration

Table 1 lists the choice of parameter values for our model. Overall there are seventeen parameters. Ten are conventional in the respect that they appear in standard New Keynesian DSGE models. The other seven govern the financial sector and are specific to our model.

We begin with the conventional parameters. For the discount rate \( \beta \), the inverse Frisch elasticity \( \phi \), the elasticity of substitution between goods \( \varepsilon \), and the capital share \( \alpha \) we use standard values in the literature. Three additional
parameters \((\eta, a, b)\) involve the investment technology, which we express as follows:

\[
\Gamma \left( \frac{I_t}{K_t} \right) = a \left( \frac{I_t}{K_t} \right)^{1-\eta} + b.
\]

We set \(\eta\), which corresponds to the elasticity of the price of capital with respect to investment rate, equal to 0.25, a value in line with panel data estimates. We then choose \(a\) and \(b\) to hit to targets: first, a ratio of quarterly investment to the capital stock of 2.5\% and a value of the price of capital \(Q\) equal to unity in the risk-adjusted steady state. Next we choose the cost of price adjustment parameter \(\rho^{fr}\) to generate an elasticity of inflation with respect to marginal cost equal to 0.01, which is roughly in line with the estimates. Finally, we set the feedback parameters in the Taylor rule, \(\kappa_\pi\) and \(\kappa_y\) equal to 1.5 and 0.20 respectively.

We now turn to the financial sector parameters. There are five parameters that directly affect the evolution of banks net worth and credit spreads: bankers survival probability \(\sigma\); initial labor endowment of entering bankers \(l^e\) which determines their initial equity endowment, \(W^e = (1-\sigma)w_tl^e\); the asset diversion parameter \(\theta\); the threshold share for costless direct household financing of capital, \(\gamma\); and the parameter governing the convexity of the efficiency cost of direct financing, \(\lambda\). We choose these parameters to hit the following five targets in the risk adjusted steady state: (i) the arrival rate of systemic bank run is 2 percent annually, corresponding to a frequency of banking panics of once every 50 years; (ii) a value of bank leverage equal 10; (iii) excess return on bank assets relative to deposits equals 150 basis points; (iv) a share of bank intermediated assets of \(1/2\); (v) a drop in investment following a bank run that matches the evidence.

Finally, we suppose that the capital quality shock obeys the following first order process:

\[
\log \xi_{t+1} = \rho_\xi \log \xi_t + \epsilon_{t+1}
\]

with \(0 < \rho_\xi < 1\) and where \(\epsilon_{t+1}\) an normally distributed i.i.d. random variable with mean zero and standard deviation \(\sigma_\xi\). We choose the serial correlation of the capital quality \(\rho_\xi\) and and the standard deviation of the innovations \(\sigma_\xi\) to match the observed standard deviation of investment and output. Table 2 shows unconditional standard deviation for some key macroeconomic variables in the model and in the data.
4.2 Experiments

In this section we perform several experiments that are meant to illustrate the behavior of our model economy and gauge its ability to capture the macroeconomic dynamics observed during the recent financial crisis. We show the response of the economy to a capital quality shock with and without runs trying to isolate the different sources of amplification. We then turn to an experiment that shows how the model can replicate salient features of the recent financial crisis.

4.2.1 Response to capital quality shock: without vs with runs

Figure 1 shows the response of the economy to a negative one standard deviation (1%) shock to the quality of capital. Poor asset returns cause bank equity to decrease by about 20%. The direct effects of a capital quality worsening on banks balance sheets are amplified by the familiar financial accelerator mechanism of Bernanke Gertler and Gilchrist (1999) and Kiyotaki Moore (1997). As net worth declines, incentive constraints tighten and banks decrease their demand for assets causing the price of capital to drop. The drop in asset prices feeds back into lower bank networth, an effect that is magnified by the extent of bank leverage.

As financial constraints tighten and asset prices decline, excess returns rise by 100 basis points which allows banks to increase their leverage by about 15%. Overall, a one percent decline in the quality of capital results in a drop in investment by 8 percent and a drop in output by 2.5 percent. Notice that the decline in output is mostly driven by a contraction of employment induced by the decline of investment demand in the presence of nominal rigidities. We now turn to the effects of a run.

In our model there is also a second channel through which fluctuations in banks balance sheets amplify the effects of shocks on investment demand. This channel works through variations in the run probability. As shown by equation (40), an increase in banks’ leverage raises the threshold level of capital quality below which a run equilibrium exists in the subsequent period. The the dashed line in the first panel of Figure 1 shows the response of the shock needed to push the economy in the run region, \( \xi_{t+1}^R - E_t \xi_{t+1} \). This threshold goes up by almost half a percent on impact resulting in a doubling of the run probability that goes from about 60 basis points in steady state to more than 1.2 percent after the shock (in the second panel). An
increase in the run probability raises the probability that assets will be traded at liquidation prices in the next period so that demand for assets by both households and banks declines. Banks demand is further reduced by a fall in the bank net worth (in the third panel) associated with a higher probability of failure in subsequent periods.

Figure 2 quantifies the effect of variation in the run probability on the behavior of the economy. We compare the response of our baseline economy in the solid line, to an economy in which the sunspot probability is set to zero in the dashed line. We start both economies from the risk adjusted steady state of our baseline. The drop in investment is twice as large in the baseline as the increase in the run probability interacts with decreasing bank net worth to generate a substantial increase in spreads. Moreover, when agents anticipate the possibility of a run in the future they decrease their leverage and increase their net worth for precautionary reasons. The figure shows how in the long run, bank net worth is higher and leverage is lower in the economy in which runs are possible (solid lines) than the economy without run (dashed lines). Also long-run investment and output are lower in the baseline economy compared to the economy without the run.

As discussed above, a negative shock to the quality of capital induces an increase in the run probability. However, the level of the run probability after a one percent shock starting from the steady state is still modest, i.e. below 1.5%, and a run can occur only after another rather large shock to the quality of capital. To isolate the effect of runs, in Figure 3 we assume that the economy is hit by a sequence of three equally size negative shocks that push the economy to the run threshold. The solid line in the figure shows the response of the economy starting from period two onwards under the assumption that the economy experiences a run with arrival of a sunspot in period three. For comparison, the dashed line shows the response of the economy to the same exact capital quality shocks but assuming that no sunspot is observed and so no run happens.

As shown in panel 1 the size of the threshold innovation of capital quality shock is $-1.2\%$. The serial correlation coefficient of the log level of capital quality shock $\rho_\xi$ equal 0.66. After the first two innovation of $-1.2\%$ (so that the capital shock equals $-1.2 + (-1.2) \cdot 0.66 = -2\%$), the run probability is about 3% and another $-1.2\%$ innovation pushes the economy into the run region (where capital quality shock equals $-1.2 + (-2) \cdot 0.66 = -2.4\%$). When the sunspot is observed bank net worth is wiped out forcing households to absorb the entire capital stock. Households relative inefficiency at intermedi-
ating assets leads excess returns to spike and investment to collapse. When the run occurs, investment drops an additional 20% resulting in an overall drop of 35%. Comparing with the case without the arrival of sunspot clarifies that only a small proportion of this additional drop is due to the capital quality shock itself. The collapse in investment demand causes inflation to decrease and induces monetary policy to accommodate, bringing the policy rate slightly below zero. However, monetary accommodation is not sufficient to insulate output which drops more than 10%.

As new bankers enter the economy, bank net worth is slowly rebuilt and the economy returns to the steady state. This recovery is slowed down by a persistent increase in the run probability following a banking panic.

In the previous experiment we employed an interest rate feedback rule that had the interest rate be less sensitive to output than the standard parametrization would suggest in order to keep the nominal rate from going significantly below the zero lower bound. To get a sense of the role that nominal rigidities in conjunction with the zero lower bound are playing, Figure 4 describes the effect of bank runs in the economy with flexible prices. For comparison, with the analogous experiment in our baseline (in Figure 3) we hit the flex price economy with the same sequence of shocks that would take the baseline to the run threshold.\footnote{However, since in the flex price economy there is much less amplification, the ex-post run that we consider is actually off equilibrium. As the first panel in the figure shows, the run threshold after two shocks of 1.2% is still very large in the flex price economy.} There are two main takeaways from Figure 4. First, bank runs endogenously generate a steep decline in the natural rate of interest by inducing a collapse in investment demand. Second, the amplification effects associated with bank runs do not depend crucially on the presence of nominal rigidities. In fact, the investment drop with a bank run is still twice as big as the one that would take place absent a run. The reason the drop in output is muted relative to the baseline case is that the real interest rate drop roughly five hundred basis points below zero, something that would not be feasible with nominal rigidities and a zero lower bound. The steep drop in real interest rate actually pushes consumption demand positive, but cannot offset the effect of the banking collapse on investment.
4.2.2 Non-Linearities: occasionally binding constraints vs bank runs

We now turn to nonlinearity within our baseline model. We will start by considering the effects of occasionally binding constraints. Figure 5 shows the behavior of the economy when it transits from slack to binding financial constraints. Starting from the risk adjusted steady state we consider how the economy responds to variations in capital quality. If the shock to capital quality is positive the constraint is slack, while it becomes binding with negative capital quality shocks. Notice that the behavior of investment is very similar in the two regions. The effect of occasionally binding constraints is modest for two reasons. First, even within the constrained region banks increase leverage with a worsening of capital quality. This is because the rise in excess returns relaxes the incentive constraint. This has the effect of smoothing the kink in the policy function for leverage. Second, as the economy moves closer to the regions where the financial constraints bind, banks expectations of being constrained in the future leads them to require higher excess returns so that even in the unconstrained region excess returns will move countercyclically. This contrasts with the case in which financial frictions are absent altogether and banks require no excess return on their asset holdings as in the case discussed above.\footnote{Similarly, in the case of perfect foresight excess returns equal zero in the unconstrained region.}

Next we consider bank runs. Figure 6 shows the response of the economy to a capital quality shock starting from the same initial state considered in Figure 2. The dashed line depicts the response in the case in which no sunspot is observed and the solid one shows the case in which a sunspot is observed. As long as capital quality shocks are above the run threshold the responses are identical. In fact, in this region a run is not an equilibrium because banks would have positive networth even if assets traded at liquidation prices and the occurrence of the sunspot is irrelevant. Below the run threshold banking panics become an equilibrium. In this region, when agents observe a sunspot they run on financial institutions pushing the economy to an equilibrium in which banks are forced to liquidated assets at fire sale prices and their net worth is completely wiped out as the proceeds from asset sales are not enough to satisfy depositors. The effects are as discussed above: excess returns spike and investment collapses as households are forced to absorb the entire capital stock. Expectations of depressed consumption in the future
push interest rates down, but this effect is more than offset by the increase in excess returns.

4.2.3 Crisis experiment: model versus data

Figure 7 illustrates how the model can replicate some salient features of the recent financial crisis. We assume that the first shock hits the economy in 2007 Q4, around the time of the first credit crunch following Bear Stearns’ losses on its MBS portfolios, (even though the shadow banking sector started experiencing the strain from summer of 2007). We pick the size of the capital quality shocks from 2007Q4 until 2008Q3 to match the observed decline in investment during this period (first panel). We assume that a run happens in 2008Q4, the quarter in which Lehman failed and the shadow banking system collapsed. When the run happens with an arrival of a sunspot, there is a sudden spike in excess returns and a drop in investment, output, consumption and employment of similar magnitudes as those observed during the crisis.

5 Conclusion and Literature

To be completed.
References


TABLE 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
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<tbody>
<tr>
<td>( \beta )</td>
<td>Impatience</td>
<td>.99</td>
<td>Risk Free Rate</td>
</tr>
<tr>
<td>( \eta^{fr} )</td>
<td>Inverse Frish</td>
<td>.5</td>
<td>Literature</td>
</tr>
<tr>
<td>( \eta^q )</td>
<td>Elasticity of q to i</td>
<td>.25</td>
<td>Literature</td>
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<tr>
<td>( \eta^k )</td>
<td>Capital Share</td>
<td>.33</td>
<td>Capital Share</td>
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<tr>
<td>( \delta )</td>
<td>Depreciation</td>
<td>.0249</td>
<td>( \frac{I}{K} = .025 )</td>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
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<tbody>
<tr>
<td>Financial Intermediation Parameters</td>
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<td></td>
</tr>
<tr>
<td>( \lambda )</td>
<td>HH Intermediation Costs</td>
<td>.04</td>
<td>%( \Delta I ) in crisis ( \approx 35% )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>HH Intermediation Costs</td>
<td>.41</td>
<td>( \frac{S^b}{X} = .5 )</td>
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<tr>
<td>( \sigma )</td>
<td>Banker Survival rate</td>
<td>.93</td>
<td>( ER^b - R = 1.5% ) Annual</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Share of assets divertible</td>
<td>.2</td>
<td>Leverage ( \frac{Q^b}{N} = 10 )</td>
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<tr>
<td>( W )</td>
<td>New Bankers Endowment as a share of Capital</td>
<td>.1%</td>
<td>Run Probability 2% Annual</td>
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</table>

Nominal Rigidity and Monetary Policy Parameters

<table>
<thead>
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<th>Parameter</th>
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<td>( \rho^{ir} )</td>
<td>Price adj costs</td>
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<td>( \varepsilon )</td>
<td>Elasticity of subst across varieties</td>
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<td>Markup 10%</td>
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<td>( \kappa_y )</td>
<td>Policy Response to Output</td>
<td>.05</td>
<td>Policy path in crisis</td>
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<td>( \kappa_\pi )</td>
<td>Policy Response to Inflation</td>
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<td>Literature</td>
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</table>

Parameters Governing Exogenous Process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<td>( \sigma (\phi^c) )</td>
<td>std of innovation to capital quality</td>
<td>1%</td>
<td>std Output</td>
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<tr>
<td>( \rho^2 )</td>
<td>serial correlation of capital quality</td>
<td>.66</td>
<td>std Investment</td>
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Table 2: Standard Deviations Data vs. Model (1949q1-today)

<table>
<thead>
<tr>
<th></th>
<th>Data*</th>
<th>Model</th>
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<tr>
<td>Y</td>
<td>6.0</td>
<td>6.5</td>
</tr>
<tr>
<td>I</td>
<td>13.0</td>
<td>13.5</td>
</tr>
<tr>
<td>C</td>
<td>4.0</td>
<td>6.5</td>
</tr>
<tr>
<td>L</td>
<td>4.5</td>
<td>5.0</td>
</tr>
</tbody>
</table>

All values in percentages.
* Linear trend calculated from 1949q1-today.
Figure 1. Response to a Capital Quality Shock: No Run Case
Figure 2. Amplification due to Anticipating a Run
Figure 3. Response to a Sequence of Shocks: Run VS No Run

- **Capital Quality**
  - Red dashed line: Capital Quality
  - Blue line: Run Threshold
  - Diamond line: Initial Threshold

- **Run Probability**

- **Bank Net Worth**

- **Leverage Multiple: $\phi$**

- **Investment**

- **Output**

- **Excess Return: ER$^b$ - R$^{free}$**

- **Policy Rate**

- **Inflation**

Levels are shown on the y-axis, with percentages from SS (% Δ from SS), levels, excess return, policy rate, and inflation measured in annual basis points, and quarters indicated on the x-axis.
Figure 4. Response to a Sequence of Shocks in Flex Price Economy: Run VS No Run
Figure 5. Non-Linearity (or lack thereof) due to Occasionally Binding Constraints
Figure 6. Non-linearities from Runs

- Investment
- Price of Capital
- Real Interest Rate: $R^{\text{free}}$
- Net Worth
- Leverage
- Excess Returns: $ER^{b-R}$

- No Sunspot
- Sunspot

Run Threshold: $\epsilon^r = -1.2\%$
Figure 7. Financial Crisis: Model vs. Data

1. Investment

2. Financial Equity

3. Spreads (BAA-AAA)

4. GDP

5. Labor (hours)

6. Consumption