Collusion in Auctions with Constrained Bids:
Theory and Evidence from Public Procurement*

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Abstract

We study the mechanics of cartel enforcement and its interaction with bidding constraints in the context of repeated procurement auctions. Under collusion, bidding constraints affect cartel behavior by limiting future pledgeable surplus. This yields a test of collusive behavior exploiting the counter-intuitive prediction that introducing minimum prices can lower the distribution of winning bids. The model’s predictions are borne out in procurement data from Japan, where we find considerable evidence that collusion is weakened by the introduction of minimum prices. An elementary theory of inference from observed bids lets us evaluate counterfactual policies.

Keywords: collusion, cartel enforcement, minimum prices, entry deterrence, procurement.

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1 Introduction

This paper studies the mechanics of cartel enforcement and its interaction with bidding constraints in the context of repeated procurement auctions. We emphasize the role of self-enforcement constraints in determining cartel behavior, and derive a test of collusion based on the behavior of the right tail of winning bids following the introduction of minimum prices. Minimum prices, which place a lower bound on the price at which procurement contracts can be awarded, are frequently used in public procurement. We show that the introduction of minimum prices makes cartel enforcement more difficult, yielding a first-order stochastic dominance drop in the distribution of winning bids to the right of the minimum price. The model’s predictions are borne out in procurement data from Japan, where the introduction of minimum prices indeed shifts down the right tail of winning bids. In addition, the effect is concentrated on bidders that are frequent participants, and auctions with a high reserve price, providing strong empirical evidence that cartel self-enforcement constraints are binding, and that minimum-prices undermine collusion. An elementary theory of inference from observed bids lets us evaluate counterfactual policies, as well as perform overidentification tests of the model.

We model firms repeatedly playing first-price procurement auctions. We assume that production costs are i.i.d., commonly observed among cartel members, and that firms are able to make transfers. Cartel behavior is limited by self-enforcement constraints: firms must be willing to follow bidding recommendations, as well as make equilibrium transfers. We provide an explicit characterization of optimal cartel behavior in this environment, and contrast its predictions with those obtained from non-collusive models under a range of information structures.

Our first set of results explores the effect of introducing minimum prices on the distribution of winning bids. In our repeated game environment, minimum prices weaken cartel discipline by limiting the impact of price wars. As a result we show that introducing
minimum prices causes a first-order stochastic dominance drop in the distribution of winning bids to the right of the minimum price: sustaining collusive bids above the minimum price becomes more difficult. Minimum prices have the opposite impact in environments without collusion: under complete information the right tail of winning bids is unchanged; under asymmetric information, minimum prices generate a first order stochastic dominance increase in the tail of winning bids.

Our second set of results provides a simple theory of inference from observed bids in the presence of collusion. We show that collusion limits the scope for inference, but that it is possible to partially identify the distribution of costs whenever the distribution of winning bids has a non-trivial support. Indeed, in the case where the distribution of winning bids has no atoms, the distribution of costs is identified up to a location parameter equal to the cartel’s pledgeable surplus. Using our model to map distribution of costs to pledgeable surplus, we obtain a fixed-point equation whose unique solution characterizes the distribution of costs.

We explore the empirical relevance of the mechanisms we study by using data from public procurement auctions occurring in four Japanese cities between 2007 and 2015. The introduction of minimum prices in one of our treatment cities in 2009 lets us use the difference-in-difference framework of Athey and Imbens (2006) to recover the counterfactual distribution of winning bids after the policy change. The data exhibits a large and significant drop in the distribution of winning bids to the right of the minimum price, implying that: (i) enforcement constraints limit the scope of collusion; (ii) minimum prices successfully weaken cartel discipline.

Richer data available from our treatment city lets us evaluate the channels through which the distribution of winning bids is affected using a single difference approach. We show that the effect of minimum prices is equally mediated by weakened entry deterrence, and weakened enforcement among cartel members. Motivated by the fact that 25% of bidders make up 80% of the (auction, bidder) pairs, we identify the top quartile of most active bidders as cartel members. Consistent with our theory, the effect of minimum prices is entirely concen-
trated on cartel members. In addition, the effect of minimum prices is disproportionately
concentrated on auctions with a high reserve price, suggesting that enforcement constraints
are more binding for large auctions.

Our paper lies at the intersection of several strands in the literature on collusion in
auctions. The seminal work of Graham and Marshall (1987) and McAfee and McMillan
(1992) studies static collusion in environments where bidders are able to contract. A key
take-away from their analysis is that the optimal response from the auctioneer should involve
setting more constraining reserve prices – in a procurement setting this means reducing the
maximum price that the auctioneer is willing to pay. We argue, theoretically and empirically,
that when bidders cannot contract and must enforce collusion through repeated game play,
minimum prices constraining bids on the other side may also benefit the auctioneer by
weakening cartel enforcement.

An important observation of McAfee and McMillan (1992) is that in the absence of cash
transfers the cartel’s ability to collude is severely limited even when commitment is available.
A recent strand of work takes seriously the idea that in repeated games, continuation values
may successfully replace transfers. Aoyagi (2003) studies bid rotation schemes and allows
for communication. Skrzypacz and Hopenhayn (2004) (see also Blume and Heidhues, 2008)
study collusion in environments without communication and show that while cartel members
may still be able to collude, they will remain bounded away from efficient collusion. Athey
et al. (2004) study collusion in a model of repeated Bertrand competition and emphasize that
information revelation costs will push cartel members towards rigid pricing schemes. Our
model simplifies away many of the important strategic issues considered in this body of work
by assuming complete information among cartel members and transferability.¹ As expected,
this allows us to provide a simple characterization of optimal collusion closely related to that
obtained in the relational contracting literature (Bull, 1987, Baker et al., 1994, 2002, Levin,

¹Importantly, we allow for asymmetric information when we study the impact of minimum prices in
competitive environments.
This lets us study the effect of price constraints on the distribution of winning bids, as well as estimate the distribution of costs from observed winning bids.

Several recent papers study the impact of the allocation format on collusion. Fabra (2003) compares the scope for tacit collusion in uniform and discriminatory auctions. Marshall and Marx (2007) study the role of bidder registration and information revelation procedures in facilitating collusion. Pavlov (2008) and Che and Kim (2009) consider settings in which cartel members can commit to mechanisms and argue that appropriate auction design can successfully limit the scope of collusion provided. Abdulkadiroglu and Chung (2003) make a similar point when bidders are patient.

More closely related to our work, Lee and Sabourian (2011) as well as Mezzetti and Renou (2012) study full implementation using dynamic mechanisms in a repeated environment. They show that implementation in all equilibria can be achieved by restricting the set of continuation values available to players to support repeated game strategies. The incomplete contract literature (see for instance Bernheim and Whinston, 1998, Baker et al., 2002) has suggested that the same mechanism used in the opposite direction provides foundations for optimally incomplete contracts: in order to sustain efficient cooperation, it may be optimal to keep contracts incomplete, thereby creating sufficient range in continuation play to enforce efficient behavior. We provide empirical evidence that this mechanism plays a significant role in practice, and can be meaningfully used to affect the level of collusion between parties.

argue that excess switching of second and third bidder across bidding rounds, compared to first and second bidder, is a smoking gun for collusive agreement.

An influential set of papers use structural methods to estimate deep payoff parameters. This allows to estimate the costs of collusion as well evaluate potential counter-collusion measures, such as raising reserve prices. Jofre-Bonet and Pesendorfer (2000, 2003), as well as Bajari et al. (2007) consider models of repeated collusion in which equilibrium bidding strategies have full support, and are Markovian with respect to observable states. In this setting, they show that it is possible to estimate payoff-relevant parameters using only the bidders’ incentive compatibility conditions without necessarily knowing which of many potential equilibrium they may be playing. Asker (2010) studies the behavior of a known cartel engaged in buying collectible stamps, for which extensive data is available. Using a precise theoretical model of the actual collusive scheme used by cartel members, he is able to estimate their values for items being auctioned. As expected the cartel causes inefficiencies, but interestingly the particular transfer scheme used by the cartel sometimes lead it to overbid. A consequence is that non-cartel members, rather than the auctioneer, suffer from the cartel’s existence.

The paper is structured as follows. Section 2 sets up our benchmark model of cartel behavior. Section 3 uses this model to derive a test of collusion as well as a simple theory of identification. Section 4 briefly extends these results in a setting with entry. Section 5 takes the model to data.

2 Self-Enforcing Cartels

Modeling strategy. McAfee and McMillan (1992)’s classic model of cartel behavior focuses on the constraints imposed by information revelation among cartel members. Instead we are interested in the enforcement of cartel recommendations through repeated play.
Viewed from the perspective of Myerson (1986), McAfee and McMillan (1992) focus on truthful revelation, while we focus on obedience constraints. The implications of the two frictions turn out to be different: McAfee and McMillan (1992) show that collusion makes lower maximum prices desirable (in the context of procurement); we argue that higher minimum prices may be helpful in weakening cartels.

This different emphasis is reflected in our modeling choices. Our analysis has three main goals:

(i) first, we want to provide clear intuition on how bidding constraints, here minimum prices, affect cartel behavior and the distribution of bids;

(ii) second, we want to assess whether enforcement constraints are a significant determinant of cartel behavior;

(iii) third, we want to provide a basic theory of inference from bids, permitting overidentification tests of the model, as well as counterfactuals.

Given those goals, we try to be as general as possible when modeling the environments we want to rule out, and go for simplicity and tractability when modeling environments of interest. Our preferred model of repeated game enforcement, assumes that costs are common knowledge among cartel members and monetary transfers are feasible. In contrast, our analysis of alternative competitive models studies a broader set of information structures.

2.1 The model

Players and payoffs. A buyer procures a single unit of a good at each period $t \in \mathbb{N}$ through a first-price auction described below. A set $N = \{1, \ldots, n\}$ of long-lived firms is present in the market. In each period a subset $\hat{N}_t \subset N$ of firms is able to participate in the auction. Participation is exogenous, i.i.d. over time, and cartel members are exchangeable. In other terms, for all subsets $J \subset N$ of cartel members, all permutations $\gamma : N \rightarrow N$ or
cartel member identities,

\[ \text{prob}(\hat{N}_t = J) = \text{prob}(\hat{N}_t = \sigma(J)). \]

We think of this set of participating firms as those potentially able to produce in the current period. Throughout the paper, participation is determined before production costs become known.\(^3\) In period \(t\), each participating firm \(i \in \hat{N}_t\) can deliver the good at a cost \(c_{i,t}\). Cost \(c_{i,t}\) is drawn i.i.d. across participants and time periods from a c.d.f. \(F\) with support \([c, \bar{c}]\) and density \(f\).

Firms are able to send transfers to each other, regardless of whether or not they participate in the auction. We denote by \(T_{i,t}\) the net transfer received or sent by firm \(i\). Let \(x_{i,t} \in \{0, 1\}\) denote whether firm \(i\) wins the procurement contract in period \(t\), let \(b_{i,t}\) denote her bid. We assume that firms have quasi-linear preferences, so that firm \(i\)'s overall stage payoff is

\[ \pi_{i,t} = x_{i,t}(b_{i,t} - c_{i,t}) + T_{i,t}. \]

Firms value future payoffs using a common discount factor \(\delta < 1\).

**The stage game.** The procurement contract is allocated according to a first price auction with constrained bids. Specifically, each participant must submit a bid \(b_i\) in the range \([p, r]\) where \(r\) is a maximum or reserve price, and \(p\) is a minimum price. Bids outside of this range are discarded. The winner is the lowest bidder, and ties are broken with a uniform draw. The winner then delivers the good at the price she bid.

All firms belong to the cartel, and, importantly, firms in the cartel observe one another’s production costs. The assumption that costs are publicly observed by cartel members allows for a tractable framework in which to study the effect that minimum prices have on bidding behavior. Suppressing period index \(t\), the timing of information and decisions within the

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\(^3\)We consider the endogenous participation of entrants in Section 4.
stage game is as follows:

1. The set of participating firms $\hat{N}$ is drawn and observed by all cartel members.

2. The production costs $c = (c_i)_{i \in \hat{N}}$ of participating firms are publicly observed by cartel members.

3. Participating firms $i \in \hat{N}$ submit public bids $b = (b_i)_{i \in \hat{N}}$, and the procurement contract is publicly allocated, yielding public allocation $x = (x_i)_{i \in \hat{N}} \in \{0, 1\}^\hat{N}$.

4. Firms can make transfers $T_i$.

Positive transfers are always accepted and only negative transfers will be subject to an incentive compatibility condition. We require exact budget balance within each period at the overall cartel level, i.e. $\sum_{i \in \mathcal{N}} T_i = 0$.

Our model is intended to capture common features of public procurement, especially of procurement auctions for construction work. Governments running these auctions usually need to procure on a regular basis. Moreover, they face a small and stable set of firms that can potentially perform the work, a subset of which participates at each auction. Laws typically require governments to make bids and outcomes public after each auction is completed. The repeated nature of the interaction makes collusion a realistic concern. This motivates us to study a model of cartel behavior in which collusion is enforced through repeated play. To keep the model tractable and to focus on how enforcement constraints affect bidding behavior, we assume that firms’ costs are public information and that transfers are feasible.\(^4\)

Note that procurement auctions with minimum acceptable bids are frequently used in practice. For instance, auctions with minimum bids are used for procurement of public works in several countries in the European Union and by local governments in Japan. The common rationale for introducing minimum bids in the auction is to limit defaults and costly renegotiations from firms that win with very low bids.

\(^4\)The assumption that firms can transfer money is not unrealistic. Indeed, many known cartels used monetary transfers; see for instance Asker (2010) and Harrington and Skrzypacz (2011). In practice these transfers can be made in ways that make it difficult for authorities to detect them, like sub-contracting between cartel members or, in the case of cartels for intermediate goods, intra-firm sales.
**The repeated game.** Interaction is repeated and firms can use the promise of continued collusion to enforce obedient bidding and transfers. Formally, bids and transfers need to be part of a subgame perfect equilibrium of the repeated games among firms.

The history among cartel members at the beginning of time $t$ is

$$h^t = \{c_s, b_s, x_s, T_s\}_{s=0}^{t-1}.$$  

Let $\mathcal{H}^t$ denote the set of period $t$ public histories and $\mathcal{H} = \bigcup_{t \geq 0} \mathcal{H}^t$ denote the set of all histories.

Our solution concept is subgame perfect equilibrium (SPE), with strategies

$$\sigma_i : h_t \mapsto (b_{i,t}(c_t), T_{i,t}(c_t, b_t, x_t))$$

such that bids $b_{i,t}(c_t)$ and transfers $T_{i,t}(c_t, b_t, x_t)$ can depend on all public data available at the time of decision-making.

**Definition 1** (collusive and non-collusive environments). *We say that we are in a collusive environment if firms play a Pareto efficient SPE.*

*We say that we are in a competitive environment if firms play a subgame perfect equilibrium of the stage game.*

Clearly the hypothesis of collusive behavior is more restrictive than non-competitive behavior would require. We address the concern by allowing for more general information structures when evaluating alternative competitive models.
2.2 Optimal collusion

Denote by $\Sigma$ the set of SPE in the repeated stage game. Let

$$V(\sigma, h_t) = \mathbb{E}_\sigma \left[ \sum_{s \geq 0} \delta^s \sum_{i \in \hat{N}} x_{i,t+s}(b_{i,t+s} - c_{i,t+s}) | h_t \right]$$

denote the total surplus generated under equilibrium $\sigma$ conditional on history $h_t$. We denote by

$$V_p \equiv \sup_{\sigma \in \Sigma} V(\sigma, h_0)$$

denote the highest equilibrium surplus sustainable in equilibrium. We emphasize that this highest equilibrium value depends on minimum price $p$.

Given a history $h_t$ and a strategy profile $\sigma$, we denote by $\beta(c_t|h_t, \sigma)$ the bidding profile induced by strategy profile $\sigma$ at history $h_t$ as a function of realized costs $c_t$.

**Lemma 1** (stationarity). *If an equilibrium $\sigma$ attains $V_p$, then $\sigma$ delivers surplus $V(\sigma, h_t) = V_p$ after all on-path histories $h_t$.*

There exists a fixed bidding profile $\beta^*$ such that, in a Pareto efficient equilibria, firms bid $\beta(c_i|h_t, \sigma) = \beta^*(c_i)$ after all on-path histories $h_t$.

For any $i \in N$, let

$$V_i(\sigma, h_t) = \mathbb{E}_\sigma \left[ \sum_{s \geq 0} \delta^s x_{i,t+s}(b_{i,t+s} - c_{i,t+s}) | h_t \right]$$

denote the expected discounted payoff that firm $i$ gets in equilibrium $\sigma$ conditional on history $h_t$. Let

$$V_p \equiv \inf_{\sigma \in \Sigma} V_i(\sigma, h_0)$$

denote the lowest possible equilibrium payoff for a firm.

Given a bidding profile $\beta$, let us denote by $\beta^W(c)$ and $x(c)$ the induced winning bid and
allocation profile when realized costs are \( c \). For each firm \( i \), we define
\[
\rho_i(\beta^W, x)(c) \equiv 1_{\beta^W(c) > p} + \frac{1_{\beta^W(c) = p}}{\sum_{j \in \tilde{N}, j \neq i} 1_{x_j(c) > 0}} + 1.
\]

When \( \beta^W(c) > p \), \( \rho_i(\beta^W, x)(c) \) corresponds to a deviator’s likelihood of winning the contract by bidding below the equilibrium winning bid if \( \beta^W(c) > p \). Similarly, when \( \beta^W(c) = p \), \( \rho_i(\beta^W, x)(c) \) corresponds to a deviator’s likelihood of winning the contract by bidding \( p \).

**Lemma 2** (enforceable bidding). A winning bid profile \( \beta^W(c) \) and an allocation \( x(c) \) are sustainable in SPE if and only if for all \( c \),
\[
\sum_{i \in \tilde{N}} (\rho_i(\beta^W, x)(c) - x_i(c)) \left[ \beta^W(c) - c_i \right]^+ + x_i(c) \left[ \beta^W(c) - c_i \right]^ - \leq \delta(V_p - nV_p). \tag{1}
\]

As in Levin (2003), a bidding profile can be implemented in a SPE if and only if the sum of deviation temptations (both from bidders abstaining to bid above cost, and bidders bidding below cost) is less than or equal the total pledgeable surplus \( \delta(V_p - nV_p) \), i.e. the difference between the highest possible continuation surplus, and the sum of minimal surpluses guaranteed to a player in any equilibrium.

For each cost realization \( c \), let \( x^*(c) \) denote the efficient allocation; i.e., it allocates the procurement contract to the participating firm with the lowest cost (ties broken randomly). We define
\[
b_p^*(c) \equiv \sup \left\{ b \leq r : \sum_{i \in \tilde{N}} (1 - x_i^*(c)) \left[ b - c_i \right]^+ \leq \delta(V_p - nV_p) \right\}.
\]

For values of \( c \) such that \( b_p^*(c) > p \), this value is the highest enforceable winning bid when the cartel allocates the good efficiently.

**Proposition 1.** An optimal on-path bidding profile sets winning bid \( \beta_p^*(c) = \max\{b_p^*(c), p\} \) at every period. Moreover, the allocation is conditionally efficient: whenever \( \beta_p^*(c) > p \), the contract is allocated to the bidder with the lowest procurement cost.
Our next result characterizes the firm’s behavior in a competitive environment. We use the following notation: for any cost realization \( c \), we let \( c_{(2)} \) denote the second lowest cost.

**Corollary 1** (behavior under competition). *In a competitive environment, the cartel sets winning bid* \( \beta^*_p(c) = \max\{p, c_{(2)}\} \).

We now turn to study how minimum prices affect the set of payoffs that firms can sustain in a SPE. We use the following assumption.

**Assumption 1.** \( \tau - \xi \leq \delta(V_0 - nV_0) \).

Assumption 1, which holds whenever the firms’ discount factor is large enough, guarantees that the cartel has enough continuation surplus to provide incentives for participation.

**Lemma 3** (worst case punishment). *Under Assumption 1,*

(i) \( V_0 = 0 \), and

(ii) there exists \( \bar{p} > \xi \) such that, for all \( p \in (0, \bar{p}) \), \( V_p - nV_p \leq V_0 - nV_0 \). The inequality is strict whenever \( p > \xi \).

Lemma 3 (i) shows that, with no minimum price, the cartel can force a firm’s payoff down to its min-max value of 0. Lemma 3 (ii) establishes that the net surplus \( V_p - nV_p \) that the cartel can use to provide incentives decreases after introducing a minimum price. Intuitively, a minimum price \( p > \xi \) increases the lowest equilibrium value \( V_p \) and tightens the enforcement constraint (1). This in turn reduces the bids that the cartel can sustain in an optimal equilibrium, thereby reducing \( V_p \) and leading to a further tightening of (1).

### 3 Empirical implications

#### 3.1 The effect of minimum prices on the distribution of bids

We now turn to the empirical implications of our model. Our first result contrasts the effect that a minimum price has on the winning bid distribution under competition and collusion. 
Proposition 2 (the effect of minimum prices on bids). Fix \( p \in (0, \overline{p}) \), and consider \( q > p \).

(i) Under collusion, \( \text{prob}(\beta^*_p > q|\beta^*_p > p) \leq \text{prob}(\beta^*_0 > q|\beta^*_0 > p) \), the inequality being strict for some \( q > p \) whenever \( \text{prob}(\beta^*_0 < r) = 0 \).

(ii) Under competition, \( \text{prob}(\beta^*_p > q|\beta^*_p > p) = \text{prob}(\beta^*_0 > q|\beta^*_0 > p) \).

In words, under collusion introducing minimum prices induces a downward shift in the tail of winning bids to the right of the minimum price. In contrast, under competition, the introduction of minimum prices has no effects on the right tail of the winning bid distribution. Section 5 uses this result to detect collusion in procurement data from Japan.

Predictions under competition and asymmetric information. Our model assumes that procurement costs are publicly observable among cartel members. Under this assumption, Proposition 2 (i) shows that, in a competitive environment, the introduction of a minimum price leaves the right-tail of the distribution of winning bids unchanged. We now show how this result extends if firms have private information about their procurement costs.

Suppose it is common knowledge that the procurement cost of each firm \( i \in \hat{N} \) is drawn i.i.d. from c.d.f. \( F \) with support \([c, \overline{c}]\) and density \( f \). Each firm is privately informed about its own procurement cost. Let \( b^{AI} : [c, \overline{c}] \rightarrow \mathbb{R}_+ \) be the bidding function in an equilibrium of a first-price procurement auction with reserve price \( r \) and no minimum price.

Proposition 3. Under private information, a first-price auction with reserve price \( r \) and minimum price \( p < r \) has a unique symmetric equilibrium with bidding function \( b^{AI}_p \).

(i) If \( b^{AI}(c) \geq p \), then \( b^{AI}_p(c) = b^{AI}(c) \) for all \( c \in [c, \overline{c}] \);

(ii) If \( b^{AI}(c) < p \), there exists a cutoff \( \hat{c} \in (c, \overline{c}) \) with \( b^{AI}(\hat{c}) > p \) such that

\[
\begin{align*}
b^{AI}_p(c) &= \begin{cases} 
b^{AI}(c) & \text{if } c \geq \hat{c}, \\
p & \text{if } c < \hat{c}.
\end{cases}
\end{align*}
\]

Proposition 3 characterizes the equilibrium of a first-price auction with minimum price \( p \) when firms are publicly informed of their procurement costs. We note that, for \( p > b^{AI}(c) \), the bidding function \( b^{AI}_p \) has a discontinuity point at the threshold \( \hat{c} \).
For any cost vector $c$ of participating firms, we let $\beta^A_I(c) \equiv \min_i b^A_I(c_i)$ denote the winning bid.

**Corollary 2.** Fix $p > 0$. For all $q > p$, $\text{prob}(\beta^A_I > q \mid \beta^A_I > p) \geq \text{prob}(\beta^A_0 > q \mid \beta^A_0 > p)$.

### 3.2 Inferring the distribution of costs

Our next result shows that the c.d.f. of firms’ costs $F$ is identified from bidding behavior in auctions with two bidders. We start with a preliminary lemma.

**Lemma 4.** For any $b \in [p, r)$,

$$F(b - \delta(V_p - nV_p)) = \sqrt{\text{prob}(\beta^*_p \leq b \mid \hat{N} = 2)}.$$

Lemma 4 can be used to identify the c.d.f. of firms’ cost from bidding data. For simplicity, in the body of the text we focus on the case where the observed distribution of winning bids does not have a mass point at the reserve price $r$, which is true in our data – Appendix A.1 studies the more general. In addition, we maintain Assumption 1. By Lemma 4, for all $b \leq r$,

$$F(b - \delta V_0) = \sqrt{\text{prob}(\beta^*_0 \leq b \mid \hat{N} = 2)}.$$

This equation shows that the distribution of costs is identified from bidding data up to the location parameter $\delta V_0$. With knowledge of the firms’ discount factor, the location parameter $\delta V_0$ can be identified as follows. For any value $V \geq 0$, let $F_V$ denote the c.d.f. of firms’ costs identified from bidding data when $V_0 = V$; i.e., for any bid $b < r$, $F_V(b - \delta V) = \sqrt{\text{prob}(\beta^*_0 \leq b \mid \hat{N} = 2)}$. Let $W(V)$ denote the cartel’s expected discounted surplus from playing the optimal collusive equilibrium when $V_0 = V$ and the distribution of costs is $F_V$. Then, the total surplus $V_0$ is the solution to the fixed point equation $W(V) = V$. The following result summarizes this discussion.
Proposition 4 (inferring the distribution of values). Suppose that (i) Assumption 1 holds and (ii) the distribution of winning bids does not have atoms. Then, the distribution of costs is identified from the winning bid distribution in auctions with two bidders.

Finally, recall that the winning bid in a competitive environment is \( \beta_0^*(c) = c(2) \). Therefore, under competition, \( F(b) = \sqrt{\text{prob}(\beta_0^* \leq b | \hat{N} = 2)} \) for all \( b < r \); i.e., assuming competition when firms are colluding leads to an overestimation of procurement costs by a constant \( \delta V_0 \).

4 Entry

We now extend the model in Section 2 to allow for entry. We assume that, in addition to participating cartel members \( \hat{N} \), at each period a short-lived firm may also bid in the auction. To participate, the short-lived firm has to pay an entry cost \( k_t \). Entry cost \( k_t \) is drawn i.i.d. over time from distribution \( F_k \) with support \([k, \bar{k}]\). We let \( E_t \in \{0, 1\} \) denote the entry decision of the short-lived firm in period \( t \), with \( E_t = 1 \) denoting entry.

Upon paying the entry cost, the short-lived firm learns its cost \( c_{e,t} \) of delivering the good, which is drawn i.i.d. from a c.d.f. \( F_e \) with support \([c, \bar{c}]\) and density \( f_e \). Finally, we assume that the short-lived firm’s entry cost \( k_t \), her entry decision and her procurement cost \( c_{e,t} \) are publicly observed.

The timing of information and decisions within the stage game is as follows:

1. The short-lived firm’s entry cost \( k \) is drawn and publicly observed. The short-lived firm makes her participation decision, which is observed by cartel members.

2. The set of participating cartel members is drawn and observed by cartel members and short-lived firm.

3. The production costs \( c \) of participating firms are drawn and publicly observed by all firms.
4. Participating firms submit public bids and the procurement contract is publicly allocated.

5. Cartel members can make transfers $T_i$.

The public history at the beginning of time $t$ is now $h^t = \{k_t, E_t, c_t, b_t, x_t, T_s\}^t_{s=0}$, and is observed by both cartel members and entrants. Let $\mathcal{H}^t$ denote the set of period $t$ public histories and $\mathcal{H} = \bigcup_{t \geq 0} \mathcal{H}^t$ denote the set of all histories. Our solution concept is subgame public equilibrium, with strategies

$$
\sigma_i : h_t \mapsto (b_{i,t}(k_t, E_t, c_t), T_{i,t}(k_t, E_t, c_t, b_t, x_t))
$$

for cartel members and strategies

$$
\sigma_e : h_t \mapsto (E_t(k_t), b_{e,t}(k_t, c_t))
$$

for the short-lived firms.

The analysis of this model is essentially identical to that of the model of Section 2 except that now the cartel must deter entry in addition to enforcing collusive bidding. For concision, we focus on salient empirical features of this model. Appendix A provides details on optimal cartel behavior.

**Proposition 5** (the effect of minimum prices on bids). Fix $p \in (0, \overline{p})$, and consider $q > p$.

(i) Under collusion, $\text{prob}(\beta^*_p > q | \beta^*_p > p, E = 0) \leq \text{prob}(\beta^*_0 > q | \beta^*_0 > p, E = 0)$. The inequality is strict for some $q > p$ whenever $\text{prob}(\beta^*_0 < r | E = 0) > 0$.

(ii) Under competition, $\text{prob}(\beta^*_p > q | \beta^*_p > p, E = 0) = \text{prob}(\beta^*_0 > q | \beta^*_0 > p, E = 0)$.

## 5 Empirical Analysis

The mechanism we delineate in Sections 2 and 4 is only effective if punishment is a binding constraint on cartel behavior. There are also two channels by which price constraints may
affect cartel behavior: the first is greater entry of new firms, the second is worse enforcement within the cartel. This begs the questions: are price constraints a relevant way to limit cartel power? and, what is the relative importance of different channels in limiting cartel power?

We provide empirical answers to these questions using auction data from four Japanese cities located in the Ibaraki prefecture: Hitachiomiya, Tsuchiura, Tsukuba and Ushiku. The data covers public work projects auctioned off between May 2007 and March 2015, corresponding to 4358 auctions, including 1565 for the treatment city alone.

Throughout the period, all cities use first-price auctions. On October 28th 2009, the city of Tsuchiura implemented a policy change, moving from a zero minimum price to a strictly positive minimum price ranging between 70% and 85% of the reserve price. The remaining cities use first-price auctions with no minimum price throughout the period. This lets us explore the effect of minimum prices on bidder behavior using a differences-in-differences approach, using Tsuchiura as our treatment, and the three remaining cities as controls.

5.1 Some facts about the data

Sample selection. The sample of cities was selected as follows. In a study of paving auctions, Ishii (2008) notes the use of minimum prices in Japanese procurement auctions. The author was able to point us to data from Ibaraki Prefecture exhibiting required variation. We then proceeded to search for all publicly available data from the 10 most populous cities in the prefecture. We kept all cities that had public data available covering the relevant policy change period. This left us with the four cities included in the study. The cities are broadly comparable: their population ranges from 48K to 215K, with Tsuchiura at 143K. They are located within 75km of one another, and within 150km of Tokyo. None of our results is sensitive to dropping one of the treatment cities.

5Tsuchiura also happens to be a sister city of Palo Alto, CA.
Policy change. The minimum prices used in our treatment city are chosen by a formal rule and should not be interpreted as having any signalling content. Minimum prices range between 70% to 85% of the reserve price, with the 25th, 50th and 75th quantiles respectively at 80%, 82% and 84%. There is no evidence that the policy change was triggered by city specific factors also affecting the distribution of bids. This is supported by the fact that all of the control cities had switched to minimum prices by the end of 2014. Our understanding is that minimum prices were introduced to remove bidders’ incentives to bid excessively low.

Descriptive statistics. Some facts about our sample of auctions are worth noting. The first is that although all auctions include a reserve price, these reserve prices are not set to extract greater surplus by the city along the lines of Myerson (1981) or Riley and Samuelson (1981). Rather, consistent with recorded practice, reserve prices are engineering estimates (Ohashi, 2009, Tanno and Hirai, 2012, Kawai and Nakabayashi, 2014), that provide an upper-bound to the range of possible costs for the project. This is corroborated by the fact that 99.7% of auctions have a winner. This lets us treat reserve prices as an exogenous scaling parameter and use it to normalize the distribution of bids to $[0, 1]$:

$$\text{norm\_winning\_bid} = \frac{\text{bid}}{\text{reserve\_price}}.$$

The distribution of winning bids is closely concentrated near reserve prices. Indeed, the mean cost savings from running an auction rather than using the reserve price as a take-it-or-leave-it offer are equal to 4.9%. This could be because reserve prices are obtained through very precise engineering estimates, but this provides justifiable concern that collusion may be going on. It is also worth observing that the $10^{th}$ quantile of the distribution of normalized winning bids is equal to 83% of the reserve price. This means that minimum prices (set

\hfill \footnote{Table 7 suggests that the distribution of reserve prices is unaffected by treatment.}
within 70% and 85% of reserve prices) are in the low quantile of the distribution of winning bids (the median minimum price is in the first decile of the distribution of winning bids). Propositions 2 and 5 suggest that absent collusive dynamics, this should lead to a weakly positive first-order stochastic dominance increase in the right tail of winning bids.

5.2 The impact of minimum prices on the distribution of winning bids

Figure 1 plots distributions of winning bids in treatment and control cities before and after treatment. The data shows a clear pattern making it highly suitable for a differences in differences approach. The distribution of bids in the control cities is unchanged, while the distribution of the treatment city to the right of reserve prices experiences a significant first-order stochastic dominance drop.

This observation vindicates the mechanism that we analyze in Sections 2, 3 and 4. There is collusion in the data, and the sustainability of collusion is limited by price constraints. This visual assessment can be made formal using the differences-in-differences framework of Athey and Imbens (2006).

Differences-in-differences. Following Athey and Imbens (2006) we use the differences in differences setup to compute counterfactual estimates of the distribution of winning bids in our treatment city, absent minimum prices. We report here results using Hitachiomiya and Ushiku as a control: data from Hitachiomiya, Ushiku and Tsuchuria are all available by May 2008, whereas data from Tsukuba only become available after May 2009. We provide results including Tsukuba in Appendix A: the main findings are unchanged.

Propositions 2 and 5 make clear predictions: if there is no-collusion the introduction of a minimum price should not change the right tail of winning bids; if there is collusion, we anticipate a drop in the right tail of winning bids. The actual and counterfactual quantiles
The data is unequivocal. There is collusion. The cartel is constrained by enforcement constraints. These enforcement constraints are worsened by the introduction of minimum prices.

7The results are unchanged if we consider the distribution of normalized winning bids conditional on prices being above .75, .82, or .85 of the reserve price, or if we use raw winning bids. See Appendix A for details.
quantile 0.113 0.268 0.371 0.526 0.681 0.784 0.938  
actual  0.832 0.877 0.937 0.962 0.973 0.979 0.986  
counterfactual 0.913 0.949 0.95 0.966 0.975 0.979 0.984  
diff  -0.082*** -0.071*** -0.013*** -0.003 -0.002 -0.0 0.002  
(0.025) (0.013) (0.012) (0.005) (0.003) (0.002) (0.002)  

***, ** and * respectively denote effects significant at the .1, .05 and .01 level.

Table 1: quantiles of the conditional actual and counterfactual distributions of normalized winning bids (> .8)

![Conditional actual and counterfactual c.d.f. of normalized winning bids (> .8)](image)

Figure 2: conditional actual and counterfactual c.d.f. of normalized winning bids (> .8)

A difference-in-difference analysis applied to the entire data lets us measure the impact of minimum prices on the average winning bid. The effect is negative but insignificant \((-0.0026, (0.014))\). While there is strong evidence that minimum prices reduce collusion, excessively high minimum-prices may obviously increase average bids. Estimating the distribution of costs using Proposition 4 would permit a counterfactual analysis of alternative minimum prices.
Single city regression. Our analysis going forward focuses on better understanding the channels by which price constraints affect the distribution of winning bids. For this purpose, we must rely on data from our treatment city alone. This is due to data restrictions: public data available from our treatment city provides detailed information about individual auctions, including the names of bidders and their bids.

We use a regression discontinuity design and begin by replicating the results from our differences-in-differences framework. We define variables

$$\text{window} = \mathbf{1}_{\text{date} \in \{\text{October 28th 2009} \pm 12 \text{ months}\}}$$

$$\text{policy\_change} = \text{window} \times \mathbf{1}_{\text{date} \geq \text{October 28th 2009}}$$

and perform both OLS and quantile regressions on the linear model

$$\text{norm\_winning\_bid} \sim \beta_0 + \beta_1 \text{window} + \beta_2 \text{policy\_change} + \beta_\text{controls} \quad (2)$$

where \(\text{controls}\) (used throughout the analysis) include Japanese log GDP as well as the current year.

<table>
<thead>
<tr>
<th>norm_winning_bid</th>
<th>mean</th>
<th>25\textsuperscript{th} quantile</th>
<th>50\textsuperscript{th} quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>window</td>
<td>0.001</td>
<td>-0.002</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.01)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>policy_change</td>
<td>-0.021***</td>
<td>-0.077***</td>
<td>-0.011***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.008)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>lnGDP</td>
<td>0.434***</td>
<td>0.457***</td>
<td>0.107***</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.101)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>year</td>
<td>0.005***</td>
<td>0.003*</td>
<td>0.003**</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Table 2: the effect of minimum prices on winning bids

While the results are not precisely identical, these magnitudes match those of our differences-in-differences design, which gives us some confidence that we control for enough time-
varying covariates to justify a single-city analysis. Note that the drop in normalized winning bids obtained from this regression, $-2.1\%$, is large given that the mean cost saving from running an auction rather than using reserve-prices as take-it-or-leave-it offers is roughly $5\%$.

5.3 The Impact of minimum prices on entry and cartel behavior

We now wish to better understand the channels through which minimum prices affect the distribution of winning bids. Specifically, we are interested in understanding how the effect of minimum prices breaks down along greater entry, and worse collusion among cartel members, keeping entry constant.

Consistent with the theory, we define cartel members and entrants according to the frequency with which they participate in auctions. Our treatment city exhibits considerable heterogeneity in the degree of bidder activity over the seven years spanned by our data. The median number of auctions a bidder participates in is 4, whereas the average is at 22. As a result, the 25\% most active bidders make up 80\% of the auction$x$bidder data. Accordingly, we define as cartel members this 25\textsuperscript{th} quantile of the most active bidders (58 out of 234 total number of bidders). We define entrants as non-cartel-members.

Greater entry vs. worse collusion. We assess the relative importance of greater entry and worst enforcement by first assessing the impact of minimum prices on entry, and second assessing the impact of minimum prices on winning bids, controlling for entry. For greater robustness, we report regressions using both the number of entrants, and the total number of bidders to measure broader participation by cartel members.

As expected, minimum prices increase both entry and participation. Table 3 reports the

8This reduction in normalized winning bids underestimates aggregate cost-savings: we show below that the bulk of the effect of minimum prices is on high-cost projects.
results from OLS estimation of the following linear models:

\[ \text{num\_entrants} \sim \beta_0 + \beta_1 \text{window} + \beta_2 \text{policy\_change} + \beta_{\text{controls}} \]  

\[ \text{num\_bidders} \sim \beta_0 + \beta_1 \text{window} + \beta_2 \text{policy\_change} + \beta_{\text{controls}} \]  

\[ \sim \beta_0 + \beta_1 \text{window} + \beta_2 \text{policy\_change} + \beta_3 \text{num\_entrants} + \beta_{\text{controls}}. \]  

<table>
<thead>
<tr>
<th></th>
<th>num_entrants</th>
<th>num_bidders</th>
<th>num_bidders</th>
</tr>
</thead>
<tbody>
<tr>
<td>window</td>
<td>-0.495***</td>
<td>-0.781***</td>
<td>-0.552***</td>
</tr>
<tr>
<td>policy_change</td>
<td>0.434***</td>
<td>0.873***</td>
<td>0.673***</td>
</tr>
<tr>
<td>lnGDP</td>
<td>-5.031***</td>
<td>-2.18</td>
<td>.144</td>
</tr>
<tr>
<td>year</td>
<td>-0.042*</td>
<td>-0.394***</td>
<td>-0.375***</td>
</tr>
<tr>
<td>num_entrants</td>
<td></td>
<td>.462***</td>
<td>(0.031)</td>
</tr>
</tbody>
</table>

Table 3: the effect of minimum prices on entry and participation

The introduction of minimum prices has a significant effect on both entry and participation by cartel members, adding on average .43 entrants and .87 bidders to auctions. These numbers are large given that the mean and median number of participants per auction are respectively 3.8 and 4. Note that participation increases even controlling for new entrants, suggesting that participation by cartel members is an endogenous decision, which is not included in our model.

Next, we examine the effect of minimum prices on winning bids controlling for participation, using the linear model

\[ \text{norm\_winning\_bid} \sim \beta_0 + \beta_1 \text{window} + \beta_2 \text{policy\_change} + \beta_3 \text{num\_bidders} + \beta_{\text{controls}}. \]  

25
whose estimates are reported in Table 4. Regression (6) assigns similar shares of the drop in normalized winning bids ($-2.1\%$, Table 2) to the greater-entry channel ($-1.2\% \times 0.87 = 1.04\%$) and the worst-collusion channel ($-1\%$). This suggests that the total effect of minimum prices on winning bids is mediated in roughly equal shares through greater entry, and worst enforcement among cartel members.

**Who does the policy change affect?** We obtain further vindication of the mechanism analyzed in Sections 2, 3 and 4 by distinguishing the effect of minimum prices on cartel members and entrants. Our theory predicts that the price paid by winning cartel members should go down, but not the price paid by winning entrants. We assess this hypothesis by estimating the linear model

\[
\text{norm\_winning\_bid} \sim \beta_0 + \beta_1 \text{window} + \beta_2 \text{cartel\_winner} + \beta_3 \text{policy\_change} + \beta_4 \text{cartel\_winner} \times \text{policy\_change} + \beta_{\text{controls}}
\]

whose estimates are reported in Table 5. The findings are entirely consistent with the theory. Absent minimum prices, cartel winners obtain contracts at higher prices. The introduction of minimum prices reduces winning bids only when a cartel member is the winner.

<table>
<thead>
<tr>
<th>norm_winning_bid</th>
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<th>25\text{th} quantile</th>
<th>50\text{th} quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>window</td>
<td>-0.008</td>
<td>-0.009</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.011)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>policy_change</td>
<td>-0.01**</td>
<td>-0.055***</td>
<td>-0.01***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.009)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>num_bidders</td>
<td>-0.012**</td>
<td>-0.011***</td>
<td>-0.01***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>lnGDP</td>
<td>0.408**</td>
<td>0.392***</td>
<td>0.096**</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.105)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>year</td>
<td>0.0</td>
<td>-0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Table 4: the effect of minimum prices on winning bids, controlling for participation

26
Auction size. The mechanism we develop in Sections 2, 3 and 4 argues that minimum prices affect the sustainability of collusion by reducing enforcement capabilities, or in other terms by reducing the pledgeable surplus across cartel members. This mechanism should be stronger when obedience constraint (1) binds. If auction size varies over time, the deviation temptation will vary over time whereas continuation values will remain stable. Hence, if our mechanism is the correct one, we should expect minimum prices to have a larger effect on larger auctions.

The scale of projects, measured by their reserve price, exhibits sufficient heterogeneity to implement this test: the 25th, 50th and 75th quantiles being respectively at ¥5M, ¥13M, and ¥29M. We define an auction as large whenever its reserve price is above the 75th quantile of reserve prices and estimate relationships of the form

\[
\text{normalized\_winning\_bid} \sim \beta_0 + \beta_2 \text{cartel\_winner} + \beta_3 \text{cartel\_winner} \times \text{policy\_change} + \beta_4 \text{window} + \beta_5 \text{policy\_change} + \beta_{controls}
\]  (8)
Table 6: the effect of minimum prices for large and small auctions

<table>
<thead>
<tr>
<th></th>
<th>if large</th>
<th>if not large</th>
</tr>
</thead>
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<tr>
<td>cartel_winner</td>
<td>0.026**</td>
<td>0.025***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>cartel_winner × policy_change</td>
<td>-0.059***</td>
<td>-0.056***</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>window</td>
<td>-0.011</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>policy_change</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>lngdp</td>
<td>0.177</td>
<td>0.396***</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.107)</td>
</tr>
<tr>
<td>year</td>
<td>0.006***</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>num_bidders</td>
<td>-0.016***</td>
<td>-0.012***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Table 6: the effect of minimum prices for large and small auctions conditional on auction size.

Table 6 shows that the effect of minimum prices is indeed concentrated on cartel members participating in large auctions. The effect of the policy on cartel members is either insignificant or much smaller for small auctions, the difference in coefficients (−5.9% for large auctions, versus −1.9% for smaller auctions) being significant at the 5% level.

6 Discussion

Summary. We provide a tractable framework to analyze the effect of price constraints on repeated collusion. Our model delivers a simple intuition: price constraints limit the range of continuation equilibria, making cartel enforcement and entry deterrence more difficult. Our model also yields a simple empirical prediction: under collusion, the introduction of minimum prices should yield a first order stochastic dominance drop in the distribution of winning bids. In addition, we can use our characterization of optimal bidding to infer costs from the distribution of bids.

Our test finds strong evidence of collusion in procurement data from Japan, validating the
hypothesis that enforcement constraints are binding, and that price constraints can weaken enforcement.

**Limits.** Some limits of our framework are worth highlighting. One is that we do not model participation by cartel members. We simply treat it as exogenous. However it may well be optimal to limit the set of auctions that firms can participate in. We also elude collusion under asymmetric information, and heterogeneity among bidders. While this limits the scope of our inference results, this does not affect the meaningfulness of our test of collusion. In competitive environments, whether or not there is asymmetric information, the introduction of minimum prices leads to a first-order stochastic dominance increase in the right tail of the distribution of bids.

**Design.** Using minimum prices to reduce collusion requires thoughtfully calibrating the level of the minimum price to ensure that it is not excessively high. However, minimum prices below the observed distribution of winning bids can only help. One subtlety worth emphasizing is that the minimum prices we explore are fixed, and not indexed on bids. In some settings (e.g. Italy) minimum bids are set as an increasing function of other bidders’ bids (e.g. a quantile of submitted bids, Conley and Decarolis (2011), Decarolis (2013)). We expect such minimum price schemes to be less effective than fixed minimum prices in deterring collusion: by coordinating low bids, cartel bidders can bring minimum prices down off equilibrium.
Appendix

A Additional Results

A.1 Inferring the distribution of costs – general case

This appendix extends the results in Section 3.2 to the case in which the observed distribution of winning bids has a mass point at the reserve price. We show that in this case we can obtain bounds on $V_0$. For simplicity, we maintain Assumption 1. We also assume that the support $[c, \bar{c}]$ of the cost distribution $F$ is such that $\bar{c} \leq r$, which is true in our data.\(^9\)

By Lemma 4, for all $c < r - \delta V_0$

$$F(c) = \sqrt{\text{prob}(\beta^*_0 \leq c + \delta V_0 | \hat{N} = 2)}.$$  \hspace{1cm} (9)

Since the reserve price is an upper bound to procurement costs,

$$\text{prob}_F(c \in [r - \delta V_0, r]) = 1 - \sqrt{\text{prob}(\beta^*_0 < r | \hat{N} = 2)}.$$  \hspace{1cm} (10)

For any $V \geq 0$, let $\mathcal{F}_V$ be the set of all c.d.f. that are consistent with (9) and (10) when $V_0 = V$. For any $F \in \mathcal{F}_V$, let $W(F, V)$ be the cartel’s expected discounted surplus from playing the optimal collusive equilibrium when $V = V$ and the distribution of cost is $F$. Let $W^{\max}(V) = \sup_{F \in \mathcal{F}_V} W(F, V)$ and $W^{\min}(V) = \inf_{F \in \mathcal{F}_V} W(F, V)$. Finally, let $V^{\max}$ and $V^{\min}$ be, respectively, the solutions to the fixed point equations $V^{\max} = W^{\max}(V^{\max})$ and $V^{\min} = W^{\min}(V^{\min})$, so that $V^{\max} > V^{\min}$. Then, the cartel’s total surplus $V_0$ is be bounded by $V^{\min}$ and $V^{\max}$; i.e., $V_0 \in [V^{\min}, V^{\max}]$.

\(^9\)Indeed, 99.7% of all the auctions in our dataset have a winner.
A.2 Collusion under threat of entry

This appendix analyzes the model with entry in Section 4. We let \( \hat{N}_e \) denote the set of all participants in the auction; i.e., \( \hat{N}_e = \hat{N} \) when \( E = 0 \), and \( \hat{N}_e = \hat{N} \cup \{e\} \) when \( E = 1 \).

Given a history \( h_t \) and an equilibrium \( \sigma \), we let \( \beta(k,c|h_t,\sigma) \) be the bidding profile of cartel members and short-lived firm induced by \( \sigma \) at history \( h_t \) as a function of realized entry cost \( k \) and procurement costs \( c = (c_i)_{i \in \hat{N}_e} \).\(^{10}\) Our first result generalizes Lemma 1 to the current setting.

\[ \text{Lemma A.1 (stationarity – entry). If an equilibrium } \sigma \text{ attains } V_p, \text{ then } \sigma \text{ delivers surplus } V(\sigma,h_t) = V_p \text{ after all on-path histories } h_t. \]

There exists a fixed bidding profile \( \beta^* \) such that, in a Pareto efficient equilibrium, firms bid \( \beta(k,c|h_t,\sigma) = \beta^*(k,c) \) after all on-path histories \( h_t \).

Given a bidding profile \( \beta \), we let \( \beta^W(c) \) be the winning bid and \( x(c) = (x_i(c))_{i \in \hat{N}_e} \) be the induced allocation when realized costs are \( c = (c_i)_{i \in \hat{N}_e} \). As in Section 2, for all \( i \in \hat{N}_e \) we let

\[ \rho_i(\beta^W,x)(c) \equiv 1_{\beta^W(c) > p} + \frac{1_{\beta^W(c) = p}}{\sum_{j \in \hat{N}_e, j \neq i} 1_{x_j(c) > p} + 1}. \]

\[ \text{Lemma A.2 (enforceable bidding – entry). A winning bid profile } \beta^W(c) \text{ and an allocation } x(c) \text{ are sustainable in SPE if and only if, for } E \in \{0,1\} \text{ and for all } c, \]

\[ \sum_{i \in \hat{N}} \{(\rho_i(\beta^W,x)(c) - x_i(c))[\beta^W(c) - c_i]^+ + x_i(c)[\beta^W(c) - c_i]^-. \} \leq \delta(V_p - nV_p). \quad (11) \]

\[ E \times \{(\rho_e(\beta^W,x)(c) - x_e(c))[\beta^W(c) - c_e]^+ + x_e(c)[\beta^W(c) - c_e]^-. \} \leq 0. \quad (12) \]

For any bidding profile \( \beta \) that is sustainable in a SPE, let \( \pi^e(\beta) \) denote the expected payoff that a short-lived bidder gets when it enters the auction and participating firms bid

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\(^{10}\)Since the vector of costs \( c \) includes the cost of the short-lived firm in case of entry, the cartel’s bidding profile can be different depending on whether the short-lived firm enters the auction or not.
according to $\beta$. Let $\pi^e_p \equiv \inf\{\pi^e(\beta) : \beta \text{ satisfies equations (11) and (12)}\}$. Note that the cartel can guarantee that all firms with entry cost larger than $\pi^e_p$ don’t participate in the auction by playing a sustainable bidding profile that attains $\pi^e_p$ when the short-lived firm’s entry cost is larger than $\pi^e_p$.

Our next result shows that minimum prices reduce the cartel’s ability to deter entry.

**Lemma A.3** (entry deterrence). For all $p > 0$, $\pi^e_p \geq \pi^e_0 = 0$, with strict inequality if $p > c$.

Recall that

$$b^*_p(c) = \sup \left\{ b \leq r : \sum_{i \in \hat{N}} (1 - x^*_i(c)) [b - c_i]^+ \leq \delta(V_p - nV_p) \right\}.$$ 

We use the following notation: for any cost vector $c$, we let $c(1) = \min_{i \in \hat{N}} c_i$ be the lowest cost among participating cartel members.

**Proposition A.1.** In an optimal equilibrium, the on-path bidding profile is such that:

(i) if $E = 0$, the cartel sets winning bid $\beta^*_p(c) = \max\{b^*_p(c), p\}$;

(ii) if $E = 1$, a cartel member wins the auction only if $c(1) \leq \max\{c_e, p\}$; the winning bid is $\beta^*_p(c) = \max\{p, \min\{c_e, b^*_p(c)\}\}$ when a cartel wins the auction, and is $\beta^*_p(c) = \max\{c(1), p\}$ when the entrant wins the auction.

A short-lived firm enters the auction if and only if $k \leq \pi^e_p$.

Proposition A.1 characterizes bidding behavior under an optimal equilibrium. In periods in which the short-lived firm does not participate, the cartel’s bidding behavior is the same as in Section 2. Entry by a short-lived firm reduces the cartels profits in two ways: (i) the cartel loses the auction whenever the entrant’s procurement cost is low enough, and (ii) entry leads to weakly lower winning bids when the cartel wins the auction.

For any cost vector $c = (c_i)_{i \in \hat{N}_e}$, let $\hat{c}(2)$ be the second lowest cost among participating firms.
Corollary A.1. In a competitive environment, for any $c$ the winning bid is $\beta^*(c) = \max\{\hat{c}(2), p\}$.

Our last result in this section extends Lemma 3 to the current setting.

Lemma A.4 (worse case punishment – entry). Under Assumption 1,

(i) $V_0 = 0$;

(ii) there exists $\bar{p} > c$ such that, for all $p \in (0, \bar{p})$, $V_p - nV_p \leq V_0 - nV_0$. The inequality is strict whenever $p > c$. 

B Robustness of Empirical Findings

We test whether reserve prices are affected by the policy change by running the regression

$log\_reserve\_price \sim \beta_0 + \beta_1 window + \beta_3 policy\_change + \beta controls.$

The results, summarized in Table 7, suggest that this is by and large not the case, except perhaps at the higher quantiles. Furthermore the effect, if any, seems to be a reduction in reserve prices, which would tend to strengthen our results: minimum prices can bring normalized winning bids down even starting from lower reserve prices.

<table>
<thead>
<tr>
<th>log_reserve_price</th>
<th>mean</th>
<th>25\textsuperscript{th} quantile</th>
<th>50\textsuperscript{th} quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>window</td>
<td>-0.025</td>
<td>-0.015</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.169)</td>
<td>(0.159)</td>
</tr>
<tr>
<td>policy_change</td>
<td>-0.132</td>
<td>-0.073</td>
<td>-0.374***</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.135)</td>
<td>(0.127)</td>
</tr>
<tr>
<td>lnGDP</td>
<td>-1.778</td>
<td>0.403</td>
<td>0.707</td>
</tr>
<tr>
<td></td>
<td>(1.136)</td>
<td>(1.665)</td>
<td>(1.559)</td>
</tr>
<tr>
<td>year</td>
<td>0.086***</td>
<td>0.048*</td>
<td>0.088***</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.028)</td>
<td>(0.026)</td>
</tr>
</tbody>
</table>

Table 7: The impact of treatment on reserve-prices
C  Proofs

C.1 Proofs for Section 2

This appendix contains the proofs of Section 2. We assume for now that the lowest SPE payoff $V_p$ can be attained. Lemma 3 provides sufficient conditions under which this is true.

We start with a few preliminary observations. Fix a SPE $\sigma$ and a history $h_t$. Let $\beta(c)$ and $T(c, b, x)$ be the bidding and transfer profile that firms play in this equilibrium after history $h_t$, and let $x(c)$ be the allocation induced by bidding profile $\beta(c)$. Let $h_{t+1} = h_t \cup (c, b, x, T)$ be the concatenated history composed of $h_t$ followed by $(c, b, x, T)$, and let $\{V(h_{t+1})\}_{i \in N}$ be the vector of continuation payoffs after history $h_{t+1}$. For each $c$, let $\beta^W(c)$ and $x(c)$ be, respectively, the winning bid and the allocation. Recall that $\rho_i(\beta^W, x)(c) = 1_{\beta^W(c) > p} + \sum_{j \in \hat{N}, j \neq i} 1_{x_j(c) > 0} + 1$.

To economize on notation, we let $h_{t+1}(c) = h_t \cup (c, \beta(c), x(c), T(c, \beta(c), x(c)))$ denote the on-path history that follows $h_t$ when current costs are $c$. Note that the following inequalities must hold:

(i) for all $i \in \hat{N}$ such that $c_i \leq \beta^W(c)$,

$$x_i(c)(\beta^W(c) - c_i) + T_i(c, \beta(c), x(c)) + \delta V_i(h_{t+1}(c)) \geq \rho_i(\beta^W, x)(c)(\beta^W(c) - c_i) + \delta V_p.$$  \hfill (13)

(ii) for all $i \in \hat{N}$ such that $c_i > \beta^W(c)$,

$$x_i(c)(\beta^W(c) - c_i) + T_i(c, \beta(c), x(c)) + \delta V_i(h_{t+1}(c)) \geq \delta V_p.$$  \hfill (14)

(iii) for all $i \in N$,

$$T_i(c, \beta(c), x(c)) + \delta V_i(h_{t+1}(c)) \geq \delta V_p.$$  \hfill (15)

The inequality in (13) must hold since a firm with cost below $\beta^W(c)$ can obtain a payoff at least as large as the right-hand side by undercutting the winning bid when $\beta^W(c) > p$, or,
by bidding \( p \) when \( \beta^W(c) = p \). Similarly, the inequality in (14) must hold since firms with cost larger than \( \beta^W(c) \) can obtain a payoff at least as large as the right-hand side by bidding more than \( \beta^W(c) \). Finally, the inequality in (15) must hold since otherwise firm \( i \) would not be willing to make the required transfer.

Conversely, suppose there exists a winning bid profile \( \beta^W(c) \), an allocation \( x(c) \), a transfer profile \( T \) and equilibrium continuation payoffs \( \{V_i(h_{t+1}(c))\}_{i \in N} \) that satisfy inequalities (13)-(15). Then, \( (\beta^W, x, T) \) can be supported in a SPE as follows. For all \( c \), firms \( i \in \hat{N} \) with \( x_i(c) > 0 \) bid \( \beta^W(c) \), and firms \( i \in \hat{N} \) with \( x_i(c) = 0 \) bid \( \beta_i(c) > \beta^W(c) \). If no firm deviates at the bidding stage, firms make transfers \( T_i(c, \beta(c), x(c)) \). If no firm deviates at the transfer stage, in the next period firms play an SPE that gives payoff vector \( \{V(h_{t+1}(c))\}_{i \in N} \). If firm \( i \) deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm \( i \) a payoff of \( V_p \); if firm \( i \) deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm \( i \) a payoff of \( V_p \) (deviations by more than one firm go unpunished). Since (13) holds, under this strategy profile no firm has an incentive to undercut the winning bid \( \beta^W(c) \). Since (14) holds, no firm with \( c_i > \beta^W(c) \) and \( x_i(c) > 0 \) has an incentive to bid above \( \beta^W(c) \) and lose.\(^{11}\) Finally, since (15) holds, all firms have an incentive to make their required transfers.

**Proof of Lemma 1.** Let \( \sigma \) be a SPE that attains \( V_p \). Towards a contradiction, suppose there exists an on-path history \( h_t = h_{t-1} \sqcup (c, \beta(c), x(c), T(c, \beta(c), x(c))) \) such that \( \sum_i V_i(\sigma, h_t) = V(\sigma, h_t) < V_p \). Let \( \{V_i\}_{i \in N} \) be an equilibrium payoff vector with \( \sum_i V_i = V_p \).

Consider changing the continuation equilibrium at history \( h_t \) by an equilibrium that delivers payoff vector \( \{V_i\}_{i \in N} \), and changing the transfers after history \( h_{t-1} \sqcup (c, \beta(c), x(c)) \) as follows. First, for each \( i \in N \), let \( \hat{T}_i \) be such that \( \hat{T}_i + \delta V_i = T_i(c, \beta(c), x(c)) + \delta V_i(\sigma, h_t) \). Note that

\[
\sum_i \hat{T}_i = \sum_i \{T_i(c, \beta(c), x(c)) + \delta(V_i(\sigma, h_t) - V_i)\} < 0,
\]

\(^{11}\)Upward deviations by a firm \( i \) with \( c_i < \beta^W(c) \) who bids \( \beta^W(c) \) and wins with probability 1 can be deterred by having the lowest cost firm who losses the auction randomize over an interval \([\beta^W(c), \beta^W(c) + \epsilon]\).
where we used $\sum_i V_i = \nabla_p > \sum_i V_i(\sigma, h_i)$ and $\sum_i T_i(c, \beta(c), x(c)) = 0$. For each $i \in N$, let $\tilde{T}_i = \hat{T}_i + \frac{\epsilon}{n}$, where $\epsilon > 0$ is such that $\sum_i \tilde{T}_i = \sum_i \hat{T}_i + \epsilon = 0$. Replacing $T_i(c, \beta(c), x(c))$ by $\tilde{T}_i$ relaxes constraints (13)-(15) and increases the total expected discounted surplus that the equilibrium generates. Therefore, if $\sigma$ attains $\nabla_p$, it must be that $\nabla(\sigma, h_i) = \nabla_p$ for all on-path histories $h_i$.

We now prove the second statement in the Lemma. Fix an optimal equilibrium $\sigma$, and let $\{V_i\}_{i \in N}$ be the equilibrium payoff vector that this equilibrium delivers, with $\sum_i V_i = \nabla_p$. Let $\beta$ be the bidding profile that firms use in the first period under $\sigma$, and let $x(c)$ denote the allocation induced by bidding profile $\beta$. It follows that

$$\nabla_p = \mathbb{E} \left[ \sum_{i \in N} x_i(c)(\beta_i(c) - c_i) \right] + \delta \nabla_p \iff \nabla_p = \frac{1}{1 - \delta} \mathbb{E} \left[ \sum_{i \in N} x_i(c)(\beta_i(c) - x(c)) \right].$$

We show that there exists an optimal equilibrium in which firms use bidding profile $\beta$ after all on-path histories. For any $(c, b, x)$, let $T_i(c, b, x)$ denote the transfer that firm $i$ makes at the end of the first period under equilibrium $\sigma$ when first period bids, bids and allocation are given by $c$, $b$ and $x$. Let $V_i(h_1(c))$ denote firm $i$'s continuation payoff under equilibrium $\sigma$ after first period history $h_1(c) = (c, \beta(c), x(c))$. By our arguments above, $\sum_i V_i(h_1(c)) = \nabla_p$ for all $c$. Since $\sigma$ is an equilibrium, it must be that $\beta(c)$, $x(c)$ and $V_i(h_1(c))$ satisfy (13)-(15).

Consider the following strategy profile. Along the equilibrium path, at each period $t$ firms bid according to $\beta$. For any $(c, \beta(c), x(c))$, firm $i$ makes transfer $\tilde{T}_i(c, \beta(c), x(c))$ such that $\tilde{T}_i(c, \beta(c), x(c)) + \delta V_i = T_i(c, \beta(c), x(c)) + \delta V_i(h_1(c))$. Note that

$$\sum_i \tilde{T}_i(c, \beta(c), x(c)) = \sum_i \{T_i(c, \beta(c), x(c)) + \delta(V_i(h_1(c)) - V_i)\} = 0,$$

where we used $\sum_i T_i(c, \beta(c), x(c)) = 0$ and $\sum_i V_i(h_1(c)) = \nabla_p = \sum_i V_i$. If firm $i$ deviates at the bidding stage or transfer stage, then firms revert to an equilibrium that gives firm $i$ a payoff of $\nabla_p$. Clearly, this strategy profile delivers total payoff $\nabla_p$. Moreover, firms have
weakly stronger incentives to bid according to $\beta$ and make their required transfers than under the original equilibrium $\sigma$. Hence, no firm has an incentive to deviate and this strategy profile can be supported as an equilibrium. ■

Proof of Lemma 2. Suppose there exists a SPE $\sigma$ and a history $h_t$ in which firms bid according to a bidding profile $\beta$ that induces winning bid $\beta^W(c)$ and allocation $x(c)$. Let $T_i(c, \beta(c), x(c))$ be firm $i$’s transfers at history $h_t$ when costs are $c$ and all firms play according to the SPE $\sigma$. Let $h_{t+1}(c) = h_t \sqcup (c, \beta(c), x(c), T(c, \beta(c), x(c)))$ be the on-path history that follows $h_t$ when costs are $c$, and let $V_i(h_{t+1}(c))$ be firm $i$’s equilibrium payoff at history $h_{t+1}(c)$. Since the equilibrium must satisfy (13)-(15), it follows that for all $c$,

$$\sum_{i \in \hat{N}} \left\{ (\rho_i(\beta^W, x)(c) - x_i(c)) \left[ \beta^W(c) - c_i \right]^+ + x_i(c) \left[ \beta^W(c) - c_i \right]^- \right\}$$

$$\leq \sum_{i \in N} T_i(c, \beta(c), x(c)) + \delta \sum_{i \in N} (V_i(h_{t+1}(c)) - V_p) \leq \delta (V_p - nV_p),$$

where we used $\sum_{i \in \hat{N}} T_i(c, \beta(c), x(c)) = 0$ and $\sum_{i \in N} V_i(h_{t+1}(c)) \leq V_p$.

Next, consider a winning bid profile $\beta^W(c)$ and an allocation $x(c)$ that satisfies (1) for all $c$. We now construct a SPE that supports $\beta^W$ and $x$ in the first period. Let $\{V_i\}_{i \in N}$ be an equilibrium payoff vector with $\sum_i V_i = V_p$. For each $i \in N$ and each $c$, we construct transfers $T_i(c)$ as follows:

$$T_i(c) = \begin{cases} 
-\delta(V_i - V_p) + (\rho_i(\beta^W, x)(c) - x_i(c))(\beta^W(c) - c_i) + \epsilon(c) & \text{if } i \in \hat{N}, c_i \leq \beta^W(c), \\
-\delta(V_i - V_p) + x_i(c)(\beta^W(c) - c_i) + \epsilon(c) & \text{if } i \in \hat{N}, c_i > \beta^W(c), \\
-\delta(V_i - V_p) + \epsilon(c) & \text{if } i \notin \hat{N}, 
\end{cases}$$
where $\epsilon(c) \geq 0$ is a constant to be determined below. Note that, for all $c$,

$$\sum_{i \in N} T_i(c) - n\epsilon(c) = -\delta(V_p - nV_p) + \sum_{i \in \hat{N}} \left\{ (\rho_i(\beta^W, x)(c) - x_i(c)) \left[ \beta^W(c) - c_i \right]^+ + x_i(c) \left[ \beta^W(c) - c_i \right]^- \right\} \leq 0,$$

where the inequality follows since $\beta^W$ and $x$ satisfy (1). We set $\epsilon(c) \geq 0$ such that transfers are budget balance; i.e., such that $\sum_{i \in N} T_i(c) = 0$.

The SPE we construct is as follows. At $t = 0$, for each $c$, firms with $x_i(c) > 0$ bid $\beta^W(c)$ and firms with $x_i(c) = 0$ bid $\beta_i > \beta^W(c)$. If no firm deviates at the bidding stage, firms exchange transfers $T_i(c)$. If no firm deviates at the transfer stage, from $t = 1$ onwards they play a SPE that supports payoff vector $\{V_i\}$. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play a SPE that gives firm $i$ a payoff $V_p$ (if more than one firm deviates, then firms punish the lowest indexed firm that deviated). One can check that this strategy profile satisfies (13)-(15), and so $\beta^W$ and $x$ are implementable. ■

**Proof of Proposition 1.** By Lemma 1, if there exists an optimal equilibrium, then there exists an optimal equilibrium in which firms use the same bidding profile $\beta$ at every on-path history. For each cost vector $c$, let $\beta^W(c)$ and $x(c)$ denote the winning bid and the allocation induced by this bidding profile under cost vector $c$.

We first show that $\beta^W(c) = b^*_p(c)$ for all $c$ such that $b^*_p(c) > p$. Towards a contradiction, suppose there exists $c$ with $\beta^W(c) \neq b^*_p(c) > p$. Since $x^*(c)$ is the efficient allocation, the procurement cost under allocation $x(c)$ is at least as large as the procurement cost under allocation $x^*(c)$. Since bidding profile $\beta$ is optimal, it must be that $\beta^W(c) > b^*_p(c) > p$. Indeed, if $\beta^W(c) < b^*_p(c)$, then the cartel would strictly prefer to use a bidding profile that allocates the contract efficiently and has winning bid $b^*_p(c)$ under cost vector $c$ than to use
bidding profile $\beta(c)$. By Lemma 2, $\beta^W(c)$ and $x(c)$ must satisfy

$$\delta(V_p - nV_p) \geq \sum_{i \in \hat{N}} \left\{ (1 - x_i(c)) \left[ \beta^W(c) - c_i \right]^+ + x_i(c) \left[ \beta^W(c) - c_i \right]^- \right\}$$

$$\geq \sum_{i \in \hat{N}} (1 - x_i^*(c)) \left[ \beta^W(c) - c_i \right]^+,$$

which contradicts $\beta^W(c) > b_p^*(c) > p$. Therefore, $\beta^W(c) = b_p^*(c)$ for all $c$ such that $b_p^*(c) > p$.

Next, we show that $\beta^W(c) = p$ for all $c$ such that $b_p^*(c) \leq p$. Towards a contradiction, suppose there exists $c$ with $b_p^*(c) \leq p$ and $\beta^W(c) > p$. By Lemma 2, $\beta^W(c)$ and $x(c)$ satisfy

$$\delta(V_p - nV_p) \geq \sum_{i \in \hat{N}} \left\{ (1 - x_i(c)) \left[ \beta^W(c) - c_i \right]^+ + x_i(c) \left[ \beta^W(c) - c_i \right]^- \right\}$$

$$\geq \sum_{i \in \hat{N}} (1 - x_i^*(c)) \left[ \beta^W(c) - c_i \right]^+,$$

which contradicts $\beta^W(c) > p \geq b_p^*(c)$. Therefore, $\beta^W(c) = p$ for all $c$ such that $b_p^*(c) \leq p$. Combining this with the arguments above, $\beta^W(c) = \beta_p^*(c) = \max\{p, b_p^*(c)\}$.

Finally, we characterize the allocation in an optimal equilibrium. Note first that under an optimal bidding profile the cartel must allocate the procurement contract efficiently whenever $\beta_p^*(c) > p$. Indeed, by construction, the optimal allocation is sustainable whenever the winning bid is $\beta_p^*(c) > p$. Therefore, if the allocation was not efficient for some $c$ with $\beta_p^*(c) > p$, the cartel could strictly improve its profits by using a bidding profile with winning bid $\beta_p^*(c)$ that allocates the good efficiently.

Consider next a cost vector $c$ such that $\beta_p^*(c) = p$. In this case, the cartel’s bidding profile in an optimal equilibrium induces the most efficient allocation consistent with (1). For each $k = 1, ..., \hat{N}$, let $x^k(c)$ be the allocation such that each of the $k$ firms with the lowest cost gets the contract with probability $1/k$. For each cost vector $c$ with $\beta_p^*(c) = p$, let $\hat{k}(c)$ be the lowest integer $k \in \{1, ..., \hat{N}\}$ such that $x^k(c)$ is consistent with (1) when the winning bid is $p$. Then, in an optimal equilibrium, for all $c$ with $\beta_p^*(c) = p$ the cartel’s bidding profile
is such that the $\hat{k}(c)$ firms with the lowest costs bid $p$, and each of them gets the contract with probability $1/\hat{k}(c)$. ■

Fix a minimum price $p$. For every value $V \geq nV_p$ and every $c$, let

$$b_p(c; V) \equiv \sup \left\{ b : \sum_i (1 - x^*_i(c)) (b - c_i)^+ \leq \delta (V - nV_p) \right\},$$

and let $\beta_p(c; V) = \max\{b_p(c; V), p\}$. Note that $\beta_p(c; V)$ would be the winning bid if the cartel’s total surplus were $V$. Let $x^p(c; V)$ be the allocation under an optimal equilibrium when the cartel’s total surplus is $V$. For every $V \geq nV_p$, let

$$W_p(V) \equiv \frac{1}{1 - \delta} \mathbb{E} \left[ \sum_{i \in \hat{N}} x^p_i(c; V) (\beta_p(c, V) - c_i) \right],$$

be the total surplus generated under a bidding profile that induces winning bid $\beta_p(c; V)$ and allocation $x^p(c; V)$. The winning bid and allocation in an optimal equilibrium are $\beta^*_p(c) = \beta_p(c; V_p)$ and $x^p(c; V_p)$, and so $V_p = W_p(V_p)$. Let

$$\overline{W}_p \equiv \sup\{V \geq nV_p : V \leq W_p(V)\}.$$

Since $\beta_p(c; V)$ is continuous and increasing in $V$ for all $c$, $W_p(V)$ is also continuous and increasing in $V$. Therefore, $W_p(\overline{W}_p) = \overline{W}_p$.

**Lemma C.1.** $\overline{V}_p = \overline{W}_p$.

**Proof.** Since $\overline{V}_p = W_p(\overline{V}_p)$, it follows that $\overline{W}_p \geq \overline{V}_p$. We now show that $\overline{W}_p \leq \overline{V}_p$. Let $V = \frac{\overline{W}_p}{n}$, and consider the following strategy profile. For all on-path histories, cartel members use a bidding profile $\beta$ inducing winning bid $\beta_p(c; \overline{W}_p)$ and allocation $x^p(c; \overline{W}_p)$. If firm $i$ deviates at the bidding stage, there are no transfers and in the next period firms play an equilibrium that gives firm $i$ a payoff of $\overline{V}_p$ (if more than one firm deviates, firms play an
equilibrium that gives $V_p$ to the lowest indexed firm that deviated). If no firm deviates at the bidding stage, firms make transfers $T_i(c)$ given by

$$T_i(c) = \begin{cases} 
-\delta(V - V_p) + (\rho_i(\beta^W, x)(c) - x^p_i(c; W_p))(\beta_p(c; W_p) - c_i) + \epsilon(c) & \text{if } i \in \hat{N}, c_i \leq \beta_p(c), \\
-\delta(V - V_p) + \epsilon(c) & \text{otherwise},
\end{cases}$$

where $\epsilon(c) \geq 0$ is a constant to be determined.\(^{12}\) Note that

$$\sum_i T_i(c) - n\epsilon(c) = -\delta(W_p - nV_p) + \sum_i ((\rho_i(\beta^W, x)(c) - x^p_i(c; W_p))(\beta_p(c; W_p) - c_i) + \epsilon(c)) \leq 0,$$

where the inequality follows since $\beta_p(c; W_p)$ and $x^p_i(c; W_p)$ are the winning bid and the allocation under an optimal equilibrium when the cartel’s total surplus is $W_p$. We set $\epsilon(c) \geq 0$ such that $\sum_i T_i(c) = 0$. If firm $i$ deviates at the transfer stage, in the next period firms play an equilibrium that gives firm $i$ a payoff of $V_p$ (if more than one firm deviates, firms play an equilibrium that gives $V_p$ to the lowest indexed firm that deviated). Otherwise, in the next period firms continue playing the same strategy as above. This strategy profile generates total surplus $W_p > V_p$ for the cartel. Since firms play symmetric strategies, it gives a payoff $V = \frac{W_p}{n}$ to each cartel member. One can check that no firm has an incentive to deviate at any stage, and so this strategy profile constitutes an equilibrium. Hence, it must be that $W_p \leq V_p$. \ \[\blacksquare\]

**Proof of Lemma 3.** We first establish part (i). Suppose $p = 0$ and fix equilibrium payoffs $\{V_i\}_{i \in N}$ with $V_i = V = \frac{V_0}{n}$. Consider the following strategy profile. At $t = 0$, all firms $i \in \hat{N}, i \neq k$ bid $c_k$ if $k \in \hat{N}$, and bid according to the static equilibrium of the game if $k \notin \hat{N}$. Firm $k$ bids $b > c_k$ if $k \in \hat{N}$. If all firms bid according to this profile, firm $k$’s transfer is $T_k = -\delta V$ at the end of the period regardless of whether $k \in \hat{N}$ or $k \notin \hat{N}$. If $k \notin \hat{N}$, $^{12}$Recall that $x^0(c; W_p)$ is the allocation under an optimal equilibrium when continuation payoff is $W_p$. Therefore, $x^0(c; W_p)$ is such that $x^p_i(c; W_p) = 0$ for all $i$ with $c_i > \beta_p(c; W_p)$.
the transfer of firm \( i \neq k \) is \( T_i = \frac{1}{n-1} \delta V \) at the end of the period. If \( k \in \hat{N} \), the transfer of firm \( i \notin \hat{N} \) is \( T_i = - \delta V \), and the transfer of firm \( i \in \hat{N}, i \neq k \) is \( T_i = \frac{n-(\hat{N}-1)}{\hat{N}-1} \delta V \). Note that \( \sum_i T_i = 0 \). If no firm deviates at the bidding or transfer stage, at \( t = 1 \) firms play the equilibrium that delivers payoffs \( \{V_i\} \). If firm \( i \) deviates at the bidding stage, there are no transfers and at \( t = 1 \) firms play the strategy just described with \( i \) in place of \( k \). If no firm deviates at the bidding stage and firm \( i \) deviates at the transfer stage, at \( t = 1 \) firms play the strategy just described with \( i \) in place of \( k \) (if more than one firm deviates at the bidding or transfer stage, from \( t = 1 \) firms play the equilibrium that delivers payoffs \( \{V_i\}_{i \in N} \)). Note that this strategy profile gives player \( k \) a payoff of 0.

We now show that, under Assumption 1, this strategy profile is a SPE. Note first that firm \( k \) does not have an incentive to deviate: in the current period she is playing a best response to the bidding profile of the other firms, and she weakly prefers to pay transfer \( T_k \) than to be punished next period. Firms \( j \notin \hat{N} \) weakly prefer to pay \( T_j \) than to be punished next period. Finally, firm \( i \in \hat{N}, i \neq k \) with cost \( c_i \) finds it optimal to bid \( c_k \) if and only if

\[
(c_k - c_i) \frac{1}{\hat{N} - 1} + \frac{n - (\hat{N} - 1)}{\hat{N} - 1} \delta V + \delta V \geq 0.
\]

Note that this inequality is satisfied for all \( c_i, c_k \in [\underline{c}, \overline{c}] \) whenever is satisfied for \( c_i = \overline{c} \) and \( c_k = \underline{c} \). Fixing \( c_i = \overline{c} \) and \( c_k = \underline{c} \) and rearranging yields

\[
\underline{c} - \overline{c} + \delta V_0 \geq 0,
\]

which holds whenever Assumption 1 holds.

We now turn to part (ii). Consider an auction with minimum price \( p > \underline{c} \), and note that

\[
V_p \geq \nu_p \equiv \frac{1}{1 - \delta} \mathbb{E} \left[ \frac{1}{\hat{N}} \mathbf{1}_{c_i \leq p} (p - c_i) \right] > V_0 = 0,
\]

where the first inequality follows since \( \nu_p \) is the minimax payoff for a firm in an auction
with minimum price $p$. Note further that $b_0 \equiv \inf_c \beta_0^*(c) = c + \frac{\delta V_0}{n-1} > c.$\(^{13}\) We show that $V_p - nV_p < V_0$ for any minimum price $p \in [c, b_0)$. Suppose that $V_p \geq V_0 + nV_p > V_0$. This implies $V_0 < V_p = W_p(V_p) = W_0(V_p) \leq V_0$, a contradiction. Therefore, $V_p - nV_p < V_0$ for any minimum price $p \in [c, b_0)$. \(\blacksquare\)

**Proof of Proposition 2.** Consider first a collusive environment. By Proposition 1 and Lemma 3, $\beta_\ast^p(c) \leq \beta_\ast^0(c)$ for all $c$ such that $\beta_\ast^0(c) > p$, with strict inequality if $\beta_\ast^0(c) < r$. Therefore, $\Pr(\beta_\ast^p > q | \beta_\ast^p > p) \leq \Pr(\beta_\ast^0 > q | \beta_\ast^0 > p)$, and the inequality is strict for some $q > p$ whenever $\Pr(\beta_\ast^0 < r) > 0$. This proves part (i).

Consider next a competitive environment. By Corollary 1, under competition $\Pr(\beta_\ast^\tilde{p} > q | \beta_\ast^\tilde{p} > p) = \Pr(c(2) > q | c(2) > p)$ for all $q > p$ and all $\tilde{p} \leq p$. This proves part (ii). \(\blacksquare\)

**Proof of Lemma 4.** Consider an auction in which only two bidders participate. Note that for all $c$, $\beta_\ast^p(c) = \max\{p, b_\ast^p(c)\} < r$ and $b_\ast^p(c) = c(2) + \delta(V_p - nV_p)$. Then, for all $b < r$, $\Pr(\beta_\ast^p \leq b | \hat{N} = 2) = \Pr(c(2) \leq b - \delta(V_p - nV_p) | \hat{N} = 2) = F^2(b - \delta(V_p - nV_p))$.\(\blacksquare\)

**Proof of Proposition 3.** We first show that there exists a symmetric equilibrium as described in the statement of the proposition, and then we show uniqueness.

Suppose first that $p \leq b^{Al}(c)$. Clearly, in this case all firms using the bidding function $b^{Al}(\cdot)$ is a symmetric equilibrium of the auction with minimum price $p$.

Consider next the case in which $b^{Al}(c) < p$. For any $c \in [c, \bar{c}]$, define

$$P(c) = \sum_{j=0}^{\hat{N}-1} \binom{\hat{N} - 1}{j} \frac{1}{j+1} F(c)^j (1 - F(c))^{\hat{N}-j-1}. $$

$P(c)$ is the probability with which a firm with cost $c' \leq c$ wins the auction if all firms use a

\(^{13}\)Indeed, $\beta_0^*(c)$ attains its lowest value when all cartel members participate in the auction and costs are $c = (c)_{i \in N}$ (i.e., all firms have cost $c$). For this cost vector, $\beta_0^*(c) = c + \frac{\delta V_0}{n-1}$. 

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bidding function $\beta(\cdot)$ with $\beta(c') = b \geq p$ for all $c' \leq c$ and $\beta(c') > b$ for all $c' > c$.

Let $\hat{c} \in (\underline{c}, \overline{c})$ be the unique solution to $P(\hat{c})(p - \hat{c}) = (1 - F(\hat{c}))^{\hat{N} - 1}(b^{AI}(\hat{c}) - \hat{c})$. Let $b^{AI}_p(\cdot)$ be given by

$$b^{AI}_p(c) = \begin{cases} 
    b^{AI}(c) & \text{if } c \geq \hat{c}, \\
    p & \text{if } c < \hat{c}.
\end{cases}$$

Note that if all firms bid according to bidding function $\beta^{AI}_p(\cdot)$, the probability with which a firm with cost $c < \hat{c}$ wins the auction is $P(\hat{c})$. We now show that all firms bidding according to $b^{AI}_p(\cdot)$ is an equilibrium.

Suppose that all firms $j \neq i$ bid according to $b^{AI}_p(\cdot)$. Note first that it is never optimal for firm $i$ to bid $b \in (p, b^{AI}_p(\hat{c}))$. Indeed, if $c_i < b^{AI}_p(\hat{c})$, bidding $b \in (p, b^{AI}_p(\hat{c}))$ is strictly dominated by bidding $b^{AI}_p(\hat{c})$: in both cases firm $i$ wins with probability $(1 - F(\hat{c}))^{\hat{N} - 1}$, but by bidding $b^{AI}_p(\hat{c})$ the firm gets a strictly larger payoff in case of winning. If $c_i > b^{AI}_p(\hat{c})$, bidding $b \in (p, b^{AI}_p(\hat{c}))$ gives firm $i$ a strictly lower payoff than bidding $b^{AI}_p(\hat{c})$.

Suppose that $c_i \geq \hat{c}$. Since $b^{AI}_p(x) = b^{AI}(x)$ for all $x \geq \hat{c}$, firm $i$ with cost $c_i$ gets a larger payoff bidding $b^{AI}_p(c_i)$ than bidding $b^{AI}_p(x)$ with $x \in [\hat{c}, \overline{c}]$. If $c_i = \hat{c}$, firm $i$ is by construction indifferent between bidding $p$ and bidding $b^{AI}_p(\hat{c})$. Moreover, for all $c_i > \hat{c}$,

$$
(1 - F(c_i))^{\hat{N} - 1}(b^{AI}_p(c_i) - c_i) \geq (1 - F(\hat{c}))^{\hat{N} - 1}(b^{AI}_p(\hat{c}) - \hat{c}) + (1 - F(\hat{c}))^{\hat{N} - 1}(\hat{c} - c_i) \\
= P(\hat{c})(p - \hat{c}) + (1 - F(\hat{c}))^{\hat{N} - 1}(\hat{c} - c_i) \\
> P(\hat{c})(p - \hat{c}) + P(\hat{c})(\hat{c} - c_i),
$$

where the strict inequality follows since $P(\hat{c}) > (1 - F(\hat{c}))^{\hat{N} - 1}$ and $c_i > \hat{c}$. Therefore, firm $i$ finds it optimal to bid $b^{AI}_p(c_i)$ when her cost is $c_i \geq \hat{c}$.

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14Note first that such a $\hat{c}$ always exists whenever $b^{AI}(\underline{c}) < p$. Indeed, in this case $P(\underline{c})(p - \underline{c}) = p - \underline{c} > b^{AI}(\underline{c}) - \underline{c}$ while $P(\overline{c})(p - \overline{c}) = \frac{1}{\overline{N}}(p - \overline{c}) < 0 = (1 - F(\overline{c}))^{\hat{N} - 1}(b^{AI}(\overline{c}) - \overline{c})$. By the Intermediate value Theorem, there exists $\hat{c} \in (\underline{c}, \overline{c})$ such that $P(\hat{c})(p - \hat{c}) = (1 - F(\hat{c}))^{\hat{N} - 1}(b^{AI}(\hat{c}) - \hat{c})$. Moreover, for all $c \leq p$, \[ \frac{\partial}{\partial c} P(c)(p - c) = -P(c) + P'(c)(p - c) \leq -P(c) < -(1 - F(c))^{\hat{N} - 1} = \frac{\partial}{\partial c} (1 - F(c))^{\hat{N} - 1}(b^{AI}(\hat{c}) - c), \] so $\hat{c}$ is unique.
Finally, suppose that \( c_i < \hat{c} \). Firm \( i \)'s payoff from bidding \( b_p^{AI}(c_i) = p \) is \( P(\hat{c})(p - c_i) \). Note that, for all \( c \geq \hat{c} \),

\[
P(\hat{c})(p - c_i) = P(\hat{c})(p - \hat{c}) + P(\hat{c})(\hat{c} - c_i)
\geq (1 - F(\hat{c}))\hat{N}^{-1}(b_p^{AI}(c) - \hat{c}) + P(\hat{c})(\hat{c} - c_i)
> (1 - F(\hat{c}))\hat{N}^{-1}(b_p^{AI}(c) - c_i),
\]

where the first inequality follows since \( P(\hat{c})(p - \hat{c}) = (1 - F(\hat{c}))\hat{N}^{-1}(b_p^{AI}(c) - \hat{c}) \geq (1 - F(\hat{c}))\hat{N}^{-1}(b_p^{AI}(c) - \hat{c}) \) for all \( c \geq \hat{c} \), and the second inequality follows since \( P(\hat{c}) > (1 - F(\hat{c}))\hat{N}^{-1} \) for all \( c \geq \hat{c} \) and since \( c_i < \hat{c} \). Therefore, firm \( i \) finds it optimal to bid \( b_p^{AI}(c_i) = p \) when her cost is \( c_i < \hat{c} \).

Next we establish uniqueness. We start with a few preliminary observations. Fix an auction with minimum price \( p > 0 \) and let \( b_p \) be the bidding function in a symmetric equilibrium. By standard arguments (see, for instance, Maskin and Riley (1984)), \( b_p \) must be weakly increasing; and it must be strictly increasing and differentiable at all points \( c \) such that \( b_p(c) > p \). Lastly, \( b_p \) must be such that \( b_p(c) = \overline{c} \).

Consider a bidder with cost \( c \) such that \( b_p(c) > p \), and suppose all of her opponents bid according to \( b_p \). The expected payoff that this bidder gets from bidding \( b_p(\hat{c}) > p \) is \( (1 - F(\hat{c}))\hat{N}^{-1}(b_p(c) - \hat{c}) \). Since bidding \( b_p(c) > p \) is optimal, the first-order conditions imply that \( b_p \) solves

\[
b_p'(c) = \frac{f(c)}{1 - F(c)}(\hat{N} - 1)(b_p'(c) - c),
\]

with boundary condition \( b_p(\overline{c}) = \overline{c} \). Note that bidding function \( b^{AI} \) solves the same differential equation with the same boundary condition, and so \( b_p(c) = b^{AI}(c) \) for all \( c \) such that \( b_p(c) > p \).

Consider the case in which \( p < b^{AI}(\overline{c}) \), and suppose that there exists a symmetric equilib-
rium \( b_p \neq b_A^I \). By the previous paragraph, \( b_p(c) = b_A^I(c) \) for all \( c \) such that \( b_p(c) > p \). Therefore, if \( b_p \neq b_A^I \) is an equilibrium, there must exist \( \tilde{c} > c \) such that \( b_p(c) = p \) for all \( c < \tilde{c} \), and \( b_p(c) = b_A^I(c) \) for all \( c \geq \tilde{c} \). For this to be an equilibrium, a bidder with cost \( \tilde{c} \) must be indifferent between bidding \( b_A^I(\tilde{c}) \) or bidding \( p \): \( P(\tilde{c})(p - \tilde{c}) = (1 - F(\tilde{c}))^{\tilde{N}} - (b_p^I(\tilde{c}) - \tilde{c}) \). But this can never happen when \( p < b_A^I(\tilde{c}) \) since \( P(\tilde{c})(p - \tilde{c}) = p - \tilde{c} < b_p^I(\tilde{c}) - \tilde{c} \), and for all \( c \in [\tilde{c}, p] \), \( \frac{\partial}{\partial c} P(c)(p - c) = -P(c) + P'(c)(p - c) \leq -P(c) < -(1 - F(c))^{\tilde{N}} - 1 = \frac{\partial}{\partial c} (1 - F(c))^{\tilde{N}} - (b_A^I(c) - c) \). Therefore, in this case the unique symmetric equilibrium is \( b_A^I \).

Consider next the case with \( p > b_A^I(\tilde{c}) \). By the arguments above, any symmetric equilibrium \( b_p \) must be such that \( b_p(c) = b_A^I(c) \) for all \( c \) with \( b_p(c) > p \). Therefore, in any symmetric equilibrium, there exists \( \tilde{c} > c \) such that \( b_p(c) = p \) for all \( c < \tilde{c} \), and \( b_p(c) = b_A^I(c) \) for all \( c \geq \tilde{c} \). Moreover, \( \tilde{c} \) satisfies \( P(\tilde{c})(p - \tilde{c}) = (1 - F(\tilde{c}))^{\tilde{N}} - (b_A^I(\tilde{c}) - \tilde{c}) \). When \( p > b_A^I(\tilde{c}) \), there exists a unique such \( \tilde{c} \) (see footnote 14). Therefore, in this case the unique symmetric equilibrium is \( b_p^A \). ■

**Proof of Corollary 2.** Suppose first that \( p \leq b_A^I(\tilde{c}) \). Then, \( \text{prob}(\beta_p^A > q | \beta_p^A > p) = \text{prob}(\beta_0^A > q | \beta_0^A > p) \) for all \( q > p \).

Consider next the case in which \( p > b_A^I(\tilde{c}) \). For all \( b \in [b_A^I(\tilde{c}), b_A^I(\pi)] \), let \( c(b) \) be such that \( b_A^I(c(b)) = b \). Since \( \tilde{c} \) is such that \( b_A^I(\tilde{c}) > p \), it follows that \( \tilde{c} > c(p) \). Note then that, for all \( q \geq b_A^I(\tilde{c}) \), \( \text{prob}(\beta_p^A > q | \beta_p^A > p) = \frac{(1 - F(c(q)))^\tilde{N}}{(1 - F(\pi))^\tilde{N}} > \frac{(1 - F(c(\tilde{c})))^\tilde{N}}{(1 - F(\pi))^\tilde{N}} = \text{prob}(\beta_0^A > q | \beta_0^A > p) \).

For \( q \in (p, b_A^I(\tilde{c})) \), \( \text{prob}(\beta_p^A > q | \beta_p^A > p) = 1 > \frac{(1 - F(c(q)))^\tilde{N}}{(1 - F(\pi))^\tilde{N}} = \text{prob}(\beta_0^A > q | \beta_0^A > p) \). ■

**C.2 Proofs for Section 4 and Appendix A.2**

Fix a SPE \( \sigma \) and a history \( h_t \), and suppose that the entry cost of the short-lived firm at time \( t \) is \( k \) and her entry decision is \( E \). Let \( \beta(c) \) and \( T(c, b, x) \) be the bidding profile of cartel members and short-lived firm and the transfer profile of cartel members in this

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equilibrium after history \( h_t \sqcup (k, E) \). For each \( c \), let \( \beta^W(c) \) and \( x(c) \) be winning bid and the allocation induced by bidding profile \( \beta \). For each \( h_{t+1} = h_t \sqcup (k, E, c, b, x, T) \), let \( \{V(h_{t+1})\}_{i \in N} \) be the vector of continuation payoffs of cartel members after history \( h_{t+1} \). We let \( h_{t+1}(c) = h_t \sqcup (k, E, c, \beta(c), x(c), T(k, E, c, \beta(c), x(c))) \) denote the on-path history that follows \( h_t \sqcup (k, E) \) when current costs are \( c \). With this notation, the inequalities (13)-(15) must hold in this setting. Moreover, if \( E = 1 \), it must also be that

\[
x_e(c)[\beta^W(c) - c_e]^+ \geq \rho_e(\beta^W, x)(c)[\beta^W(c) - c_e]^+ \text{ and } x_e(c)[\beta^W(c) - c_e]^- \leq 0. \tag{16}
\]

Conversely, suppose there exists a winning bid profile \( \beta^W(c) \), an allocation \( x(c) \), a transfer profile \( T \) and equilibrium continuation payoffs \( \{V_i(h_{t+1}(c))\}_{i \in N} \) that satisfy inequalities (13)-(15) and (16) if \( E = 1 \). Then, \( (\beta^W, x, T) \) can be supported in a SPE using continuation payoffs \( \{V(h_{t+1}(c))\}_{i \in N} \) whenever all firms follow their equilibrium strategy, and by punishing a deviation by firm \( i \in N \) by reverting to an equilibrium that gives this firm a payoff of \( V_p \).

**Proof of Lemma A.1.** The proof is identical to the proof of Lemma 1, and hence omitted.

\( \blacksquare \)

**Proof of Lemma A.2.** Suppose there exists a SPE \( \sigma \) and a history \( h_t \sqcup (k, E) \) in which all participating firms bid according to a bidding profile \( \beta \) that induces winning bid \( \beta^W \) and allocation \( x \). For each \( i \in \hat{N} \), let \( T_i(c, \beta(c), x(c)) \) be firm \( i \)'s transfers at history \( h_t \sqcup (k, E) \) when costs are \( c \) and all firms play according to the SPE \( \sigma \). Let \( h_{t+1}(c) = h_t \sqcup (k, E, c, \beta(c), x(c), T(c, \beta(c), x(c))) \) be the on-path history that follows \( h_t \sqcup (k, E) \) when costs are \( c \), and let \( V_i(h_{t+1}(c)) \) be firm \( i \)'s equilibrium payoff at history \( h_{t+1}(c) \). Since the
equilibrium must satisfy (13)-(15), it follows that for all \( c \),

\[
\sum_{i \in N} \left\{ (\rho_i(\beta^W, x)(c) - x_i(c)) \left[ \beta^W(c) - c_i \right]^+ + x_i(c) \left[ \beta^W(c) - c_i \right]^- \right\}
\leq \sum_{i \in N} T_i(c, \beta(c), x(c)) + \delta \sum_{i \in N} (V_i(h_{t+1})(c) - V_p) \leq \delta(V_p - nV_p),
\]

where we used \( \sum_i T_i(c, \beta(c), x(c)) = 0 \) and \( \sum_i V_i(h_{t+1}(c)) \leq V_p \). Moreover, since (16) must hold whenever \( E = 1 \), \( E \times \{(\rho_e(\beta^W, x)(c) - x_e(c))\left[ \beta^W(c) - c_e \right]^+ + x_e(c)\left[ \beta^W(c) - c_e \right]^- \} \leq 0. \)

Next, consider a winning bid profile \( \beta^W(c) \) and an allocation \( x(c) \) that satisfies (11) and (12) for all \( c \). We now construct a SPE that supports \( \beta^W(c) \) and \( x(c) \) in the first period. Let \( \{V_i\}_{i \in N} \) be an equilibrium payoff vector with \( \sum_i V_i = V_p \). For each \( c = (c_i)_{i \in \hat{N}_e} \) and \( i \in N \), we construct transfers \( T_i(c) \) as follows:

\[
T_i(c) = \begin{cases} 
-\delta(V_i - V_p) + (\rho_i(\beta^W, x)(c) - x_i(c))(\beta^W(c) - c_i) + \epsilon(c) & \text{if } i \in \hat{N}, c_i \leq \beta^W(c), \\
-\delta(V_i - V_p) + x_i(c)(\beta^W(c) - c_i) + \epsilon(c) & \text{if } i \in \hat{N}, c_i > \beta^W(c), \\
-\delta(V_i - V_p) + \epsilon(c) & \text{if } i \notin \hat{N};
\end{cases}
\]

where \( \epsilon(c) \geq 0 \) is a constant to be determined below. Since \( \beta^W(c) \) and \( x(c) \) satisfy (11), it follows that for all \( c \),

\[
\sum_{i \in N} T_i(c) - n\epsilon(c) = -\delta(V_p - nV_p) + \sum_{i \in \hat{N}} \left\{ (\rho_i(\beta^W, x)(c) - x_i(c)) \left[ \beta^W(c) - c_i \right]^+ + x_i \left[ \beta^W(c) - c_i \right]^- \right\} \leq 0.
\]

We set \( \epsilon(c) \geq 0 \) such that transfers are budget balance; i.e., such that \( \sum_{i \in N} T_i(c) = 0 \).

The SPE we construct is as follows. At \( t = 0 \), for each \( c = (c_i)_{i \in \hat{N}_e} \) all firms \( i \in \hat{N}_e \) with \( x_i(c) > 0 \) bid \( \beta^W(c) \) and all firms \( i \in \hat{N}_e \) with \( x_i(c) = 0 \) bid \( \beta_i > \beta^W(c) \). If no firm deviates at the bidding stage, cartel members exchange transfers \( T_i(c) \). If no firm deviates at the transfer stage, from \( t = 1 \) onwards cartel members play a SPE that supports payoff vector
\{V_i\}. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play a SPE that gives firm $i$ a payoff $V_p$ (if more than one firm deviates, then firms punish the lowest indexed firm that deviated). One can check that this strategy profile satisfies (13)-(15). Moreover, since it satisfies (12), it also satisfies (16). Hence, winning bid profile $\beta^W$ and allocation $x$ are implementable. ■

**Proof of Lemma A.3.** Suppose first that $p = 0$, and consider the following bidding profile. If $E = 0$, cartel members bid according to a static equilibrium of the stage game. If $E = 1$, the firm with the lowest cost among participating cartel members bids $\min\{c_{(2)}, c_e + \epsilon\}$, where $c_{(2)}$ is the second lowest cost among participating cartel members and $\epsilon > 0$ is arbitrarily small. All other participating cartel members bid above $c_{(2)}$. Upon entry, the short-lived firm wins the auction if and only if $c_e < c_{(2)}$, and obtains profits that are bounded above by $\epsilon$. This bidding profile and its induced allocation satisfy (11)-(12), and hence it is sustainable in a SPE. It follows that $\pi^e_0 = 0$.

Consider next $p > c$. Note that $\pi^e_p \geq \mathbb{E}\left[\frac{1}{N+1}c_e \leq p(p - c_e)\right] > 0$, which is the short-lived firm’s expected payoff from entering when all participating cartel members submit a bid equal to $p$. ■

**Proof of Proposition A.1.** The proof of part (i) is identical to the proof of Proposition 1, and hence omitted.

We now turn to part (ii). Note first that, by Lemma A.2, entry by the short-lived firm reduces the set of sustainable bidding profiles and thus the profits that the cartel can obtain in an auction. Therefore, in an optimal equilibrium the cartel uses the most deterrent strategy: if $k > \pi^e_p$, upon entry by the short-lived firm the cartel members play a bidding profile that gives firm $e$ an expected payoff of $\pi^e_p$. This way, the short-lived firm enters the auction if and only if $k \leq \pi^e_p$.

Suppose $E = 1$ (by the previous paragraph, this occurs on the equilibrium path if and only
if $k \leq \pi_p$). For any $c$, let $\beta^W(c)$ and $x(c)$ be, respectively, the winning bid and allocation in an optimal equilibrium. Consider first cost realizations $c$ such that $c_{(1)} \geq \max\{p, c_e\}$. In this case, $x_e(c) = 1$ in an optimal bidding profile. Indeed, by equation (12), $\beta^W(c) \leq \max\{c_e, p\}$ if $x_e(c) < 1$. Hence, the cartel is better-off letting the short-lived firm win whenever $c_{(1)} \geq \max\{p, c_e\}$. In this case, $\beta^W(c) = c_{(1)}$.

Consider next $c$ such that $c_{(1)} \in [p, c_e)$. Clearly, an optimal bidding profile for the cartel must be such that $x_e(c) = 0$. Equation (12) then implies that $\beta^W(c) \leq c_e$. We now show that, in this case, $\beta^W(c) = \max\{p, \min\{c_e, b^*_p(c)\}\}$. There are two cases to consider: (a) $b^*_p(c) > c_e$, and (b) $b^*_p(c) \leq c_e$. Consider case (a), so $b^*_p(c) \geq c_e \geq p$. It follows that

$$\sum_{i \in \hat{N}} (1 - x^*_i(c))[c_e - c_i]^+ < \sum_{i \in \hat{N}} (1 - x^*_i(c))[\beta^*_p(c) - c_i]^+ \leq \delta(\overline{V}_p - n\overline{V}_p).$$

Therefore, a bidding profile that induces winning bid $c_e$ and allocation $x^*(c)$ satisfies (11) and (12). Since such a bidding profile is optimal for the cartel among all bidding profiles with winning bid lower than $c_e$, it must be that $\beta^W(c) = c_e$.

Consider next case (b). Note that for all $b > \max\{b^*_p(c), p\}$ and any allocation $x(c)$,

$$\sum_{i \in \hat{N}} \left\{(1 - x_i(c))[b - c_i]^+ + x_i(c)[b - c_i]^-\right\} \geq \sum_{i \in \hat{N}} (1 - x^*_i(c))[b - c_i]^+ > \delta(\overline{V}_p - n\overline{V}_p),$$

so $\max\{b^*_p(c), p\}$ is the largest winning a bidding that can be supported in an equilibrium. Therefore, in an optimal equilibrium cartel members must use a bidding profile inducing winning bid $\max\{b^*_p(c), p\}$.

Finally, consider $c$ such that $c_{(1)} < p$ and $c_e < p$. We now show that, in an optimal equilibrium, $\beta^W(c) = p$. Indeed, by (12), a winning $\beta^W(c) > p > c_e$ can only be implemented if $x_e(c) = 1$. But this is clearly suboptimal for the cartel, since it can make strictly positive profits by having a firm with cost $c_{(1)}$ bidding $p$. Therefore, in an optimal equilibrium it must be that $\beta^W(c) = p$. ■
Proof of Lemma A.4. By Lemma A.3, when $p = 0$ the cartel can completely deter entry by short-lived bidders. Therefore, when Assumption 1 holds the strategy profile used in the proof of Lemma 3 can be sustained as an equilibrium at all periods at which $E = 0$, and so again have $V_0 = 0$.

We now turn to part (ii). Consider an auction with minimum price $p > c$, and note that

$$V_p \geq u_p = \frac{1}{1-\delta} \mathbb{E}\left[\frac{1}{N+1}1_{c_i \leq p}(p - c_i)\right] > V_0 = 0,$$

where the inequality follows since firm $i$ can always guarantee a payoff at least as large as $u_p$ by bidding $p$ whenever $c_i \leq p$ and bidding $b \geq c_i$ otherwise. Note further that

$$b_0 = \inf_c \beta_0^*(c) = c + \frac{nV_0}{n-1} > c.$$  

We show that $V_p - nV_p < V_0$ for any minimum price $p \in [c, b_0)$. Suppose by contradiction that $V_p \geq V_0 + nV_p > V_0$. Let $\beta$ be the bidding profile that the cartel uses at periods in which $E = 0$ under an optimal equilibrium that attains $V_p$. By Proposition A.1, $\beta$ induces winning bid $\beta_p^*(c) \geq \beta_0^*(c) > p$, where the inequality follows since $V_p - nV_p \geq V_0$. Moreover, since $\beta_p^*(c) > p$ for all $c$, $\beta$ induces the efficient allocation $x^*(c)$ for all $c$.

Suppose $p = 0$ and consider the following strategy profile for cartel members. For all periods in which $E = 0$, the cartel uses bidding profile $\beta(c)$. If firm $i$ deviates at the bidding stage, there are no transfers and in the next period cartel members play an equilibrium that gives firm $i$ a payoff of $V_0 = 0$ (if more than one firm deviates, cartel members punish the lowest indexed firm that deviated). If no firm deviates at the bidding stage, each firm $i$ makes transfer $T_i(c)$ to be determined below. If a firm $i$ deviates at the transfer stage, in the next period firms play an equilibrium that gives firm $i$ a payoff of $V_0 = 0$ (if more than one firm deviates, cartel members again punish the lowest indexed firm that deviated). Otherwise, if no firm deviates at the bidding and transfer stages, in the next period firms continue playing the same strategies as above. If $E = 1$, the cartel uses a bidding profile
satisfying (11)-(12) that leaves the short-lived firm with zero expected payoff (see proof of Lemma A.3). Note that the short-lived firm never enters when the cartel uses this strategy profile.

Let $\hat{V}$ be the total surplus that the cartel obtains from this strategy profile. Note that $\hat{V} \geq V_p$, since in an optimal equilibrium with minimum price $p$ the short-lived firm enters with weakly positive probability, and since the cartel’s per period profits conditional on $E = 1$ are lower than its per period profits conditional on $E = 0$.

Let $\beta^*_p(c)$ and $x^*_i(c)$ be the winning bid and allocation induced by bidding profile $\beta(c)$, and let $V = \hat{V}/n$. The transfers $T_i(c)$ are determined as follows:

$$T_i(c) = \begin{cases} 
-\delta V + (1 - x^*_i(c))(\beta^*_p(c) - c_i) + \epsilon(c) & \text{if } i \in \hat{N}, c_i \leq \beta^*_p(c), \\
-\delta V + \epsilon(c) & \text{otherwise}, 
\end{cases}$$

where $\epsilon(c) \geq 0$ is a constant to be determined. Note that

$$\sum_i T_i(c) - n\epsilon(c) = -\delta \hat{V} + \sum_i (1 - x^*_i(c))[\beta^*_p(c) - c_i]^+ \leq 0,$$

where the inequality follows since $\beta^*_p(c)$ is implementable with minimum price $p$, and since $\delta \hat{V} > \delta V_p - nV_p$. We set $\epsilon(c) \geq 0$ such that $\sum_i T_i(c) = 0$. This strategy profile generates total surplus $\hat{V} \geq V_p > V_0$ for the cartel. Since firms play symmetric strategies, it gives a payoff $V = \hat{V}/n$ to each cartel member. One can check that no firm has an incentive to deviate at any stage, and so this strategy profile constitutes an equilibrium. Hence, it must be that $V_0 > V_p - nV_p$.

**Proof of Proposition 5.** Consider first a collusive environment and suppose $E = 0$. By Proposition A.1 and Lemma A.4, $\beta^*_p(c) \leq \beta^*_0(c)$ for all $c$ such that $\beta^*_0(c) > p$, with strict inequality if $\beta^*_0(c) < r$. Therefore, $\text{prob}(\beta^*_p > q|\beta^*_p > p, E = 0) \leq \text{prob}(\beta^*_0 > q|\beta^*_0 > p, E = 0)$, and the inequality is strict for some $q > p$ whenever $\text{prob}(\beta^*_0(c) < r|E = 0) > 0$. This
proves part (i).

Consider next part (ii). By Corollary A.1, under competition \( \text{prob}(\beta_p^* > q | \beta_p^* > p, E = 0) = \text{prob}(\hat{c}(2) > q | \hat{c}(2) > p, E = 0) \) for all \( q > p \) and all \( \tilde{p} \leq p \).

References


