Self-Fulfilling Debt Crises, Revisited: The Art of the Desperate Deal

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Harold L. Cole    Zachary Stangebye

May 12, 2016
Some motivation...

- Traditional approach to sovereign debt crises emphasizes negative shocks to output and/or fiscal balance
  - Many examples in the data: Natural disasters, terms-of-trade shocks, wars, banking crises, etc.
  - Consistent with Eaton-Gersovitz approach and the large quantitative literature that has developed subsequently
  - Tomz and Wright document roughly two-thirds of defaults occur when output is below trend
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- Other shocks in data...
  - The Latin American debt crisis of the 1980s driven in part by sharp rise in US interest rates
  - Political transitions (Ecuador 2009, Greece recently)
Debt crises often associated with only small (or no) declines in output or other fundamentals (more on this later)
Crises without fundamental shocks...

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- Quantitative models typically require large falls in output to trigger default (more on this later)
  - More than business cycles needed
Crisis without fundamental shocks...

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- Self-fulfilling debt crises have been the focus of a literature that has arisen primarily in response to the European crisis
  - In the Calvo (1988) tradition: Lorenzoni-Werning, Navarro-Nicolini-Teles
  - In the Cole-Kehoe (2000) tradition: Conesa-Kehoe, Aguiar-Amador-Farhi-Gopinath, this paper
What does a sovereign do when faced with the prospect of a failed auction?
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- In Cole-Kehoe they default
Desperate Deals

- What does a sovereign do when faced with the prospect of a failed auction?
  - In Cole-Kehoe they default
  - In practice, they look for alternative financing
Portugal

- Difficulty in raising funds through auctions starting in 2011
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- Private placement of bonds in January 2011 (reported as purchased by China)
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- Official €78 billion package in May 2011
  - €34.2 billion dispersed in 2011 and €28.5 in 2012
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- Private placement of bonds in January 2011 (reported as purchased by China)
- Official €78 billion package in May 2011
  - €34.2 billion dispersed in 2011 and €28.5 in 2012
- Dual auction in October 2012
  - Bought “September 2013” bonds
  - Sold “October 2015” bonds
- Goal: Clear “space” for auctions in 2013/2014
- Launched new issue in early 2013
What we do...

- Consider an environment in which fundamentals and beliefs both matter and evolve stochastically.

Some findings:
- Crisis issuances important in generating volatile spreads.
- Defaults absent a shift in beliefs (“fundamental defaults”) require large negative declines in output (preceded by a boom).
- Interaction of fundamentals and “beliefs” play an important role in generating patterns similar to the data.
What we do...

- Consider an environment in which fundamentals and beliefs both matter and evolve stochastically
- Add the possibility of issuances during crisis
  - Consistent with conditions for competitive equilibrium
- Quantitatively explore the role of each in generating defaults as well as their interaction

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Why we think beliefs matter...
Italy

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth</th>
<th>Spread</th>
</tr>
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<tbody>
<tr>
<td>2005q1</td>
<td></td>
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<tr>
<td>2008q1</td>
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<td></td>
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<tr>
<td>2011q1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014q1</td>
<td></td>
<td></td>
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</tbody>
</table>
Pooled EMBI Spread Data from EM countries
1993Q4 - 2014Q4

<table>
<thead>
<tr>
<th>Argentina</th>
<th>Brazil</th>
<th>Bulgaria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>Colombia</td>
<td>Estonia</td>
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<tr>
<td>Hungary</td>
<td>India</td>
<td>Indonesia</td>
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<tr>
<td>Latvia</td>
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<td>Russia</td>
</tr>
<tr>
<td>South Africa</td>
<td>Thailand</td>
<td>Turkey</td>
</tr>
<tr>
<td>Ukraine</td>
<td>Venezuela</td>
<td></td>
</tr>
</tbody>
</table>
Distribution of Changes in Spreads

Truncated at $-500$ and $500$
Distribution of Contemporaneous Growth
With and Without Jump in Spreads

Crisis: Contemporaneous with $\Delta EMBI > 158$bp
Median Growth: $-0.4$ and $1.1$, resp
Distribution of Lagged Growth
With and Without Jump in Spreads

The graph illustrates the distribution of lagged growth with and without a jump in spreads. The density is plotted against the growth rate. Two lines are shown, one for the crisis scenario (dashed line) and one for the no crisis scenario (solid line). The crisis scenario shows a higher density near the zero growth rate, indicating a more concentrated distribution compared to the no crisis scenario.
Distribution of Subsequent Growth

With and Without Jump in Spreads

![Graph showing distribution of subsequent growth with and without jump in spreads. The graph compares the density of growth rates during crisis versus no crisis periods, highlighting a shift in distribution characteristics.]
High and increasing spreads are often associated with subsequent reductions in debt

\[ \text{Corr}(r - r^*, \%\Delta B) = -0.19 \text{ in the pooled sample} \]

\[ \text{Corr}(\Delta(r - r^*), \%\Delta B) = 0.01 \]
Distribution of Changes in Debt
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong> $r - r^*$</td>
<td>4.31</td>
</tr>
<tr>
<td><strong>Std Dev</strong> $r - r^*$</td>
<td>6.76</td>
</tr>
<tr>
<td><strong>Std Def</strong> $\Delta (r - r^*)$</td>
<td>2.29</td>
</tr>
<tr>
<td><strong>95th Pctile</strong> $\Delta (r - r^*)$</td>
<td>1.58</td>
</tr>
<tr>
<td><strong>Annualized Realized Return</strong></td>
<td>9.7</td>
</tr>
<tr>
<td><strong>Mean</strong> $\frac{B}{4Y_Q}$ (percent)</td>
<td>46</td>
</tr>
<tr>
<td><strong>Quarterly Corr</strong>($\Delta (r - r^*), \Delta y$)</td>
<td>-0.27</td>
</tr>
<tr>
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Distribution of European Spreads
Level and Quarterly Change
European Crisis and Growth

Crisis: Contemporaneous with $\Delta r > 136bp$
Taking Stock

- Interest rate spikes much more common than defaults;
- Crises are not tightly connected to poor fundamentals;
- Spreads are highly volatile;
- Rising spreads are associated with de-leveraging by the sovereign;
- Risk premia are an important component of sovereign spreads;
- Coordination failures may be a relevant factor in sovereign bond markets.
Framework

Key Ingredients

▶ Markov process for endowment growth
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- Risk averse lenders
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- Risk averse lenders
- Multiplicity of equilibria
- Sunspot to coordinate beliefs
- Desperate Deals
Environment
Domestic Economy

- Small open economy
- Discrete time $t = 0, 1, ...$
- Single tradable good
- Endowment process:
  \[
y_t \equiv \ln Y_t = \sum_{s=0}^{t} g_s + z_t
  \]
  \[
  = y_{t-1} + g_t + z_t - z_{t-1}
  \]
- $g_t$ follows an AR(1) process
- $z_t$ is iid
Financial Markets

- Sovereign issues non-contingent “random-maturity” bonds
- Bonds mature with Poisson probability $\lambda$
- Assume that in a non-degenerate portfolio of bonds, a fraction $\lambda$ matures with probability 1
- Perpetual-youth bonds allow for tractably incorporating maturity without adding separate state variables for each cohort of bond issuances
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- Perpetual-youth bonds allow for tractably incorporating maturity without adding separate state variables for each cohort of bond issuances
- Bonds pay coupon $r^*$ each period up to and including maturity
- Payments due in period $t$: $(r^* + \lambda)B_t$
- New issuances: $B_{t+1} - (1 - \lambda)B_t$
Some Useful Notation

\[ b_t \equiv \frac{B_t}{Y_t} \]
\[ b'_t \equiv \frac{B_{t+1}}{Y_t} \]

- **Evolution:**

\[ b_{t+1} = b'_t \frac{Y_t}{Y_{t+1}} \]
\[ = b'_t e^{-g_{t+1}-z_{t+1}+z_t} \]
Timing

Initial State: \( s = (g, z, b, \rho) \)

Auction:
\[ b' - (1 - \lambda)b \]
at price \( q(s, b') \)

Settlement

No Default \( V^R(s, b') \)

Default \( V^D(s) \)

Next Period: \( s' \)
The Government’s Problem

Preferences

- Sovereign government makes all consumption-savings-default decisions

- Sovereign’s preferences over sequence of aggregate consumption \( \{ C_t \}_{t=0}^{\infty} \):

\[
\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(C_t)
\]

with

\[
u(C) = \frac{C^{1-\sigma}}{1-\sigma}
\]
Value Functions

- $V(s)$ denotes start-of-period value of government
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- \( V^R(s, b') \) denotes value if having auctioned \( b' - (1 - \lambda)b \) the government decides to repay \( (r^* + \lambda)b \) at settlement
- \( V^D(s) \) denotes the value of defaulting at settlement (independent of amount auctioned) \( \Rightarrow \) lose fraction \( \phi(s) \) of endowment until “redemption” from default status
- Strategic default implies:

\[
V(s) = \max \left\{ \max_{b' \leq \bar{b}} V^R(s, b'), V^D(s) \right\}
\]
Bellman Equations

- If repay...

\[ V^R(s, b') = u(C) + \beta \mathbb{E} \left[ V(s') \middle| s, b' \right], \]

with

\[ C = Y + q(s, b')(B' - (1 - \lambda)B) - (r^* + \lambda)B \]
\[ = Y \left[ 1 + q(s, b')(b' - (1 - \lambda)b) - (r^* + \lambda)b \right]. \]
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- If default...

\[ V^D(s) = u(C) + \beta (1 - \xi) \mathbb{E} \left[ V^D(s') \middle| s \right] + \beta \xi \mathbb{E} \left[ V(s') \middle| s, b' = 0 \right] , \]

with

\[ C = (1 - \phi(s))Y \]
Marginal Bond Issuance

Recall budget constraint if repay:

\[ C = Y \left[ 1 + q(s, b')(b' - (1 - \lambda)b) - (r^* + \lambda)b \right]. \]

Normalized marginal revenue raised from a new issuance:

\[ q(s, b') + \frac{\partial q(s, b')}{\partial b'}(b' - (1 - \lambda)b) \]

Second term quantitatively important

Increases with marginal cost of debt

Increases with \( \lambda \) (shorter maturity)
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- Normalized marginal revenue raised from a new issuance:

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- Second term quantitatively important

- Incentive to reduce debt:
  - Increases with marginal cost of debt
  - Increases with \( \lambda \) (shorter maturity)
Policy Functions

- Debt issuance:

\[ \mathcal{B} : S \rightarrow (-\infty, \bar{b}] \]

- Default (allow mixed strategies when indifferent):

\[ \mathcal{D} : S \times (-\infty, \bar{b}] \rightarrow [0, 1] \]
Lenders

- Risk averse lenders

- Financial markets are segmented: Only a fraction of potential investors participate in bond market at a point in time

- Tractability: Period $t$’s set of investors hold bonds for one period and then sell them to a new cohort of investors at start of $t + 1$
Timing

- At start of current period:
  - New lenders purchase non-maturing bonds from old lenders at auction
  - New lenders purchase new bonds from government at same auction

- At end of auction:
  - Old lenders have received \( q(s, b') \left( 1 - \lambda \right) b \) and hold current claims on government \( r + \lambda b \)
  - New lenders have paid out \( q(s, b') \) and hold claims with face value \( b' \)
  - Government has net receipts \( q(s', b') (b' - (1 - \lambda) b) \)
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Settlement

- Government’s net receipts (if positive) from auction held in settlement fund until default decision:

\[ x(s, b') \equiv \max \{ q(s, b')(b' - (1 - \lambda)b), 0 \} . \]

- Can be used to pay current liabilities if decide to repay

- Distributed to bondholders pro rata if default:

\[ R^D(s, b') = \frac{x(s, b')}{b' + (r^* + \lambda)b} . \]
Return to Sovereign Bond
Per Unit Face Value

▶ Let $\delta$ and $\delta'$ denote indicator for default realization today and tomorrow
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$$R = \frac{1}{q(s, b')} \left[ (1 - \delta)(1 - \lambda)q(s', b'') \leftarrow \text{auction tomorrow} \right] + \ldots$$
Return to Sovereign Bond
Per Unit Face Value

Let $\delta$ and $\delta'$ denote indicator for default realization today and tomorrow

$$ R = \frac{1}{q(s, b')} \left[ (1 - \delta)(1 - \lambda)q(s', b'') + \delta R^D(s, b')(1 + r^*) \right] \leftarrow \text{default today} $$

+ 

+ 

+ 

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R = \frac{1}{q(s, b')} \left[ (1 - \delta)(1 - \lambda)q(s', b'') \right. \\
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Representative lender’s problem

- Preferences over wealth when old:

\[ \nu(W_0) = \frac{W_0^{1-\gamma}}{1-\gamma} \]
Representative lender’s problem

- Preferences over wealth when old:

\[ \nu(W_o) = \frac{W_o^{1-\gamma}}{1-\gamma} \]

- Portfolio problem:

\[ \mu^*(s, b') = \arg\max_{\mu} \mathbb{E} \left[ \nu((1-\mu)(1+r^*) + \mu R) \right| s, b' \], \]

- In forming expectations over \( R \), the lender uses the equilibrium policy functions of the government:

\[ \delta = 1 \text{ with probability } \mathcal{D}(s, b') ; \]
\[ \delta' = 1 \text{ with probability } \mathcal{D}(s', b'') \text{ in state } s' ; \text{ and} \]
\[ b'' = \mathcal{B}(s'). \]
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- FOC: \( \mathbb{E}\nu'(W_o)(R - (1 + r^*)) = 0. \)
Equilibrium

- States \( s \in S \) elements of \( s \) are:
  - Endowment: \((Y, g, z)\)
  - Bonds: \( b \)
  - Normalized wealth of lenders: \( w = \frac{W}{Y} \)
  - Beliefs: \( \rho \)
Equilibrium

- States $s \in S$ elements of $s$ are:
  - Endowment: $(Y, g, z)$
  - Bonds: $b$
  - Normalized wealth of lenders: $w = \frac{W}{Y}$
  - Beliefs: $\rho$

- Policy Functions:
  - Bond-issuance: $B(s) \in [0, \bar{b}]$
  - Default: $\mathcal{D}(s, b') \in [0, 1]$
  - Bond-demand: $\mu^*(s, b') \in \mathbb{R}$
Equilibrium

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- Policy Functions:
  - Bond-issuance: $B(s) \in [0, \bar{b}]$
  - Default: $D(s, b') \in [0, 1]$
  - Bond-demand: $\mu^*(s, b') \in \mathbb{R}$

- Price function: $q(s, b') \in [0, 1]$

- Market clearing: $\mu^*(s, b')W = q(s, b')b'$. 
Multiplicity of Equilibria

- There is a “static” multiplicity in a given period
- Arises because of timing convention: Failed auction even for small levels of bond issuances can be supported in equilibrium
- Suppose the continuation equilibrium is held constant and we consider alternative price schedules for the current period’s auction
- Consider two scenarios for today’s auction
Rollover Crisis

- Zero price for any $b' \geq (1 - \lambda)b$:

$$V^R(s, (1 - \lambda)b)$$

$$= u(Y[1 - (r^* + \lambda)b]) + \beta \mathbb{E}[V(s')|s, b' = (1 - \lambda)b]$$

$$< V^D(s).$$
Rollover Crisis

- Zero price for any $b' \geq (1 - \lambda)b$:

$$V^R(s, (1 - \lambda)b) = u(Y[1 - (r^* + \lambda)b]) + \beta\mathbb{E}[V(s')|s, b' = (1 - \lambda)b] < V^D(s).$$

Non-Crisis

- A pair $(\tilde{q}, \tilde{b})$ such that:

$$V^R(s, b') = u\left( Y\left[1 - (r^* + \lambda)b + \tilde{q}(\tilde{b} - (1 - \lambda)b)\right]\right) + \beta\mathbb{E}\left[ V(s')|s, \tilde{b}\right] > V^D(s).$$
Rollover Crises

Note that whether a rollover crisis can be supported in equilibrium depends on endowment and inherited debt.
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Note that whether a rollover crisis can be supported in equilibrium depends on endowment and inherited debt.

Define the states in which a crisis is possible by $\mathcal{C}$:

$$\mathcal{C} \equiv \left\{ s \in S \mid u(Y [1 - (r^* + \lambda)b]) + \beta \mathbb{E} [V(s') | s, b' = (1 - \lambda)b] \leq V^D(s) \right\}.$$
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  $$

- Crisis zone combination of high $b$ and low $(Y, g, z)$.
In canonical Cole-Kehoe equilibrium, a rollover crisis is an equilibrium in which prices are zero for any positive amount of debt issuance.
Desperate Deals

- In canonical Cole-Kehoe equilibrium, a rollover crisis is an equilibrium in which prices are zero for any positive amount of debt issuance.

- We relax this and consider a broader set of crisis equilibria.
  - Build on the mixed strategy equilibria of Aguiar and Amador (2014).
  - That model had potential buybacks and randomization off the equilibrium path.
  - We now bring this onto the equilibrium path and consider crisis issuances.
Define a price schedule along which the government is indifferent to default:

\[ q^D(s, b') = \{ \tilde{q} \, \mid \, u \left( Y \left[ 1 - (r^* + \lambda)b + \tilde{q}(b' - (1 - \lambda)b) \right] \right) + \beta \mathbb{E} \left[ V(s') \mid s, b' \right] \]

\[ = V^D(s) \}. \]
Define a price schedule along which the government is indifferent to default:

\[
q^D(s, b') = \left\{ \tilde{q} \right\}
\]

\[
u(Y [1 - (r^* + \lambda)b + \tilde{q}(b' - (1 - \lambda)b)] ) + \beta \mathbb{E} [V(s')|s, b']
\]

\[
= V^D(s)
\]

Construct crisis price schedule as (admissible part of) this locus

Support as equilibrium with appropriate choice of \( D(s, b') \)

Government indifferent regarding default/repayment and across possible issuances at this price
Desperate Deals

Noteworthy Features

- Part of a *competitive* equilibria

- Not a bargaining outcome: No role for “walking away” or “holding out”
Desperate Deals

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Desperate Deals

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▶ Not a bargaining outcome: No role for “walking away” or “holding out”

▶ New lenders are indifferent at margin given equilibrium price and mixed strategy probability

▶ However: Legacy lenders are not indifferent

▶ No market mechanism to have the government select issuance policy (from policy correspondence) that maximizes market value of outstanding debt

▶ Possibility of positive price in a crisis raises ex ante price of bonds
  ▶ Increases efficiency of bond market conditional on crisis probability
Evolution of Beliefs

- Let $\rho$ index beliefs and assume it follows Markov process:
  - $\rho \in \{r_V, r_C\}$
  - If $\rho = r_C$, coordinate on crisis price schedule if $s \in C$
  - In crisis, $B(s) = b$
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  - Same as benchmark, but assume $B(s) = (1 - \lambda)b$ in a crisis
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Alternative Belief Regimes:

$\rho \in \{r_T, r_V, r_C\}$

- $r_T$ differs from $r_V$ in probability of crisis next period
Equilibrium Price Schedule

\[ q(s, b') = \begin{cases} 
(1 - D(s, b'))q^{EG}(s, b') & \text{if } \rho = r_V \text{ or } s \notin C \\
q^D(s, b') & \text{if } \rho = r_C \text{ and } s \in C \text{ and } q^D(s, b') \leq q^{EG}(s, b') \\
0 & \text{otherwise.} 
\end{cases} \]

- \( q^{EG} \) is “Eaton-Gersovitz” price schedule which “assumes” repayment this period

- By backward induction, non-crisis price is
  \( (1 - D(s, b'))q^{EG}(s, b') \)
Equilibrium Price Schedule

\[ q(s, b') = \begin{cases} 
(1 - \mathcal{D}(s, b'))q^{EG}(s, b') & \text{if } \rho = r_V \text{ or } s \notin \mathcal{C} \\
q^D(s, b') & \text{if } \rho = r_C \text{ and } s \in \mathcal{C} \text{ and } q^D(s, b') \leq q^{EG}(s, b') \\
0 & \text{otherwise.}
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- By backward induction, non-crisis price is \( (1 - \mathcal{D}(s, b'))q^{EG}(s, b') \).

- Implied spread: \( r(s, b') = \frac{r^*(1-\lambda)+\lambda}{q(s,b')} - \lambda. \)
Calibration
Pre-Set Parameters

- Calibrate endowment to Mexico 1980Q1-2001Q4
- Set risk aversion coefficient for sovereign and lenders at 2
- Set quarterly risk free rate to 1%
- Set average maturity to 8 quarters
- Set average exclusion to 8 quarters
## Endowment: Mexico 1980Q1-2001Q4

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - \rho_g)\bar{g}$</td>
<td>0.0034</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.445</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.012</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.003</td>
</tr>
</tbody>
</table>
## Matched Moments

<table>
<thead>
<tr>
<th>Target Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt-to-Income (Quarterly)</td>
<td>65.6%</td>
<td>65.6%</td>
</tr>
<tr>
<td>Mean Spread (Annualized)</td>
<td>3.4%</td>
<td>3.4%</td>
</tr>
<tr>
<td>Quarterly Std Dev of Annualized Spread</td>
<td>2.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Default Frequency (Annually)</td>
<td>2.0%</td>
<td>2.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crisis Probability $\Pr(\rho = r_C)$</td>
<td>50%</td>
</tr>
<tr>
<td>Discount factor $(\beta)$</td>
<td>0.82</td>
</tr>
<tr>
<td>Default Cost $(d)$</td>
<td>0.068</td>
</tr>
<tr>
<td>Creditor Wealth Relative to $Y$ $(w)$</td>
<td>3.03</td>
</tr>
</tbody>
</table>
Equilibrium Price Schedule

\[ q(b', \cdot) \]

\[ b' \]
Ergodic Distribution of Debt

![Graph showing the density distribution of debt to GDP ratio ranging from 0.62 to 0.7. The peak density is around 0.66.]
Equilibrium Price Schedule

Shocks to $g$

\[ g = \bar{g} - 3 \frac{\sigma_g}{\sqrt{1 - \rho_g^2}} \]

\[ g = \bar{g} \]

\[ g = \bar{g} + 3 \frac{\sigma_g}{\sqrt{1 - \rho_g^2}} \]
Crisis Price Schedule
Policy Functions

Shocks to $g$
Distribution of $r - r^*$

Unconditional
Distribution of $r - r^*$
Conditional on No Rollover Crisis
Distribution of $r - r^*$

Conditional on Crisis
Randomization Probabilities

Conditional on Rollover Crisis

Density

Default Probability
Growth and Interest Rate Crises

Threshold: $\Delta r \geq 0.3$
Crises and Default

- Probability $\rho = r_C$ is $\frac{1}{2}$
- Fraction of quarters in crisis: 2.6%
  - Only a small fraction of the time spent in “crisis zone”
  - Incentive to deleverage to avoid rollover crises
- Conditional on rollover crisis, default on average 15% of time
Types of Interest Rate Crises

- Interest rate crises happen 5% of quarters (by definition)
- 86% of interest rate crises coincide with negative growth
- 50% of interest rate crises coincide with $\rho = r_C$
Types of Defaults

- Default rate 2% per annum (targeted)
- 97% of defaults coincide with negative growth
- 85% of defaults coincide with $\rho = r_C$
Anatomy of a Default

Growth: Fundamental and Self-Fulfilling

![Graph showing the growth of Fundamental and Self-Fulfilling over time](image-url)
Anatomy of a Default

Debt: Fundamental and Self-Fulfilling

[Diagram showing the relationship between time to default and debt/income for fundamental and self-fulfilling debts.]
Anatomy of a Default

Spreads: Fundamental and Self-Fulfilling

![Graph showing the relationship between time to default and spread for Fundamental and Self-Fulfilling spreads.]

- **Fundamental** spread remains relatively flat until close to default, then increases sharply.
- **Self-Fulfilling** spread shows a gradual increase as time to default decreases, becoming very high near default.

Key points:
- Time to Default: -5, -4, -3, -2, -1, 0, 1
- Spread values: 0.05, 0.1, 0.15, 0.2

Legend:
- **Solid line**: Fundamental
- **Dashed line**: Self-Fulfilling
The Role of Desperate Deals

<table>
<thead>
<tr>
<th>Target Moment</th>
<th>Data</th>
<th>Benchmark</th>
<th>No Deals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{B}{Y}$</td>
<td>65.6%</td>
<td>65.6%</td>
<td>63.7%</td>
</tr>
<tr>
<td>$r - r^*$</td>
<td>3.4%</td>
<td>3.4%</td>
<td>3.1%</td>
</tr>
<tr>
<td>$\sigma(r - r^*)$</td>
<td>2.5%</td>
<td>2.5%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Default Freq</td>
<td>2.0%</td>
<td>2.0%</td>
<td>1.9%</td>
</tr>
</tbody>
</table>
Equilibrium Price Schedule
With and Without Crisis Issuances
Equilibrium Price Schedule: Relevant Domain
With and Without Crisis Issuances

![Graph showing the equilibrium price schedule with and without crisis issuances. The graph compares the benchmark case with and without crisis deals.]

- **Benchmark**: Black line
- **No Deals**: Red dashed line

The graph illustrates the impact of crisis issuances on equilibrium prices, highlighting the differences in price schedules between the benchmark and no deals scenarios.
Policy Functions
With and Without Crisis Issuances

![Graph showing policy functions with and without crisis issuances. The x-axis represents the benchmark (b) and the y-axis represents b'. The graph compares the outcomes with and without desperate deals.]
Policy Functions: Relevant Domain
With and Without Crisis Issuances
Distribution of \( b \)
Distribution of $r - r^*$
Conditional on No Rollover Crisis

Spread

Density

Benchmark
No Desperate Deals
## Interest Rate Crises

Defined as $\Delta (r - r^*) \geq 95^{th}$ pctile

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>No Deals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $r - r^*$</td>
<td>11.5</td>
<td>3.3</td>
</tr>
<tr>
<td>Mean $\Delta (r - r^*)$</td>
<td>8.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Mean $\Delta y$</td>
<td>-1.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>Fraction $\Delta y &lt; 0$</td>
<td>0.859</td>
<td>0.627</td>
</tr>
<tr>
<td>Mean $b$</td>
<td>66.4</td>
<td>63.8</td>
</tr>
<tr>
<td>Fraction $\rho = r_C$</td>
<td>0.497</td>
<td>NA</td>
</tr>
</tbody>
</table>

- Recall unconditional mean $b$ is 65.6 and 63.7, respectively
## Defaults

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>No Deals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $r - r$</td>
<td>18.4</td>
<td>NA</td>
</tr>
<tr>
<td>Mean $\Delta(r - r^*)$</td>
<td>13.9</td>
<td>NA</td>
</tr>
<tr>
<td>Mean $\Delta y$</td>
<td>-1.7</td>
<td>-2.2</td>
</tr>
<tr>
<td>Fraction $\Delta y &lt; 0$</td>
<td>0.973</td>
<td>0.999</td>
</tr>
<tr>
<td>Mean $b$</td>
<td>66.7</td>
<td>65.1</td>
</tr>
<tr>
<td>Fraction $\rho = r_C$</td>
<td>0.845</td>
<td>0.994</td>
</tr>
</tbody>
</table>
Getting Volatile Spreads

- Non-contingent debt very poor – and costly – form of insurance
  - Default involves deadweight costs to output and future risk-sharing

- Standard calibration: Avoid borrowing into default regions
  - Even with impatience, marginal cost of debt may be so high as to limit debt

- Lower efficiency loss of default in EG models:
  - Arellano: Make deadweight loss nonlinear function of endowment
  - Yue: Allow for renegotiation to eliminate ex post losses
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    - Yue: Allow for renegotiation to eliminate ex post losses

- ACCS: Use limited market transactions to lessen inefficiency of self-fulfilling crises ⇒ Improves ex ante price schedule
Other Approaches/Shocks

Shocks to Lender Wealth

- Calibrated AR(1) for lender wealth to match persistence of S&P P/E ratio

- Targeted $R^2$ of regression of spreads on P/E ratio with model counterpart of regression of spreads on lender wealth

- Generated variable risk premium

- However, adding wealth shocks did not generate variable spreads:
  - As risk premium increased, probability of default decreased
  - Disciplining effect of risk premia: Sovereigns reduced borrowing in response
Other Approaches/Shocks
Belief Regimes

- Introduced a “Tranquil” regime such that

$$\Pr(\rho' = r_C | \rho = r_T) \ll \Pr(\rho' = r_C | \rho = r_V)$$

- Tranquil regime fairly persistent

- While $\rho = r_T$ borrow due to impatience and relatively low risk of crisis

- Shift to $\rho = r_V$: Either default (if $b$ high) or deleverage quickly

- Generated shifts in price schedules, but limited variance in spreads
  - Incentive to delever too strong
Other Approaches/Shocks

Subsistence Consumption

- Introduced minimum consumption level: \( \tilde{u}(c) = u(c - \bar{c}) \)
- Reduces desire to delever, but also reduces borrowing ex ante
- Ongoing work…
Conclusion

- Fundamentals important but business cycles incomplete description of risk
- Self-Fulfilling Crises generate a mixture of fundamental and belief-driven defaults
  - Interaction of fundamentals and potential for belief change is important
  - Sovereign can influence spreads by adjusting debt issuances
  - Crisis deals generate sharp spikes in spreads
- Risk premia generate strong incentive to reduce debt
Calibration

Beliefs

\[ \rho' = \begin{array}{ccc} 
\rho_T & \rho_V & \rho_C \\
0.97 & 0.028 & 0.006 \\
0.12 & 0.68 & 0.20 \\
0.12 & 0.68 & 0.20 
\end{array} \]
Equilibrium Price Schedule

Shocks to \( \rho \)
Policy Functions
Distribution of $b$ by Belief Regime

Frequency

Debt/Y

Tranquil
Vulnerable
Distribution of $r - r^\ast$ by Belief Regime

![Histogram showing the distribution of $r - r^\ast$ by Belief Regime. The x-axis represents the spread, and the y-axis represents frequency. Two regimes are shown: Tranquil (black) and Vulnerable (red). The majority of data points fall in the Tranquil regime, with a smaller proportion in the Vulnerable regime.](image-url)
Shocks to Lender Wealth

\( q(b', \cdot) \)

- \( w = \bar{w} - 3 \frac{\sigma_w}{\sqrt{1 - \rho_w^2}} \)
- \( w = \bar{w} \)
- \( w = \bar{w} + 3 \frac{\sigma_w}{\sqrt{1 - \rho_w^2}} \)