Robust Policy Counterfactuals
with an Application to Fiscal Unions*

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Abstract

Counterfactuals in fully specified structural models are the leading paradigm for analyzing systematic policy changes because they are immune to Lucas Critique. However, such counterfactuals are less credible whenever they lack robustness to variations in primitives across models that are observationally equivalent and reasonable a-priori. In this paper, I propose a methodology to construct policy counterfactuals in a set of linear models of dynamic stochastic economies that are both observationally equivalent under the benchmark policy and have identical counterfactual equilibrium under the alternative policy. These counterfactuals are immune to Lucas critique and robust to variations in primitives across models within the set. Then, I apply the methodology to quantify how fiscal unions contribute to regional stabilization. I focus on models where the federal government redistributes resources via a transfer policy rule in order to smooth local shocks. This rule is a function of local variables. Using US state-level data, I construct a counterfactual US economy without the rule in place. This counterfactual is identical in many fiscal union models with rich features, such as nominal rigidities and asset market incompleteness. I find that, during the Great Recession, fiscal integration significantly reduced cross-state employment differences by redistributing resources from well to poorly performing states. Finally, I discuss how the methodology can further be used to falsify a set of models, provided data before and after a policy change is available.

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1 Introduction

During the Great Recession, many US states were hit with large negative shocks that depressed their economies. Had they not been member of a fiscal union, how would they have fared? Economists commonly follow either structural or reduced-form approaches in order to answer such counterfactual questions. The structural approach relies on specifying primitives of a particular model and identifying its parameters, a daunting task whenever multiple models are both observationally equivalent and reasonable a-priori. If counterfactuals differ under these alternative models, their credibility is undermined because they lack robustness. The reduced-form approach requires less commitment to particular model assumptions and has proven very useful to identify consequences of observed policy changes that are robust across models. However, the structural approach is the leading paradigm for studying unobserved, potential changes in systematic components of policy\(^1\) because structural counterfactuals are not subject to the critique in Lucas (1976), whereas reduced-form counterfactuals typically are.

In this paper, I propose a methodology to conduct counterfactual analysis with respect to unobserved systematic policy changes in a way that is both immune to Lucas critique and robust to specific assumptions about primitives and parameterizations. The methodology applies to widely used models in macroeconomics, i.e., linear models of dynamic stochastic economies. Specifically, I show how to identify a set of linear models that are (i) observationally equivalent under a benchmark policy, and (ii) yield identical counterfactual equilibrium under an alternative policy. Identification is achieved by imposing restrictions directly on structural equations characterizing the equilibrium (e.g., exclusion restrictions). In practice, this counterfactual equilibrium can be constructed using data from the equilibrium under the benchmark policy. Thus, the methodology allows researchers and policy-makers to obtain quantitative predictions with respect to changes in systematic components of policy with minimal a-priori structural assumptions, enhancing credibility of the analysis because it is robust across models within the identified set. This robustness echoes the original motivation in Sims (1980) as well as a more recent literature using sufficient statistics. Moreover, I show that the methodology can further be used to falsify a set of models whenever data before and after a policy change is available.

Then, I apply this methodology to quantify how fiscal unions contribute to regional stabilization by redistributing resources between its members through the federal tax-and-transfer system. The purpose of the application is to transparently illustrate the methodology. As such, I study a set of fiscal union models that are simple enough to achieve this goal. However, I shall argue that this set is rich enough to inform policy discussions\(^2\) about fiscal integration because it encompasses models with realistic features (e.g., nominal rigidities, adjustment costs, asset market incompleteness, and correlated exogenous processes). In particular, I focus on models

\(^1\)As opposed to exogenous components of policy, i.e, policy shocks.

\(^2\)For instance, current discussions about the consequences of European fiscal integration, a question in much need of robust quantitative results that go beyond existing reduced-form calculations and calibration exercises in specific models. See literature review for a number of examples.
where the federal government redistributes resources between the union members via a transfer policy rule that is a function of local variables. These transfers smooth the response to local, temporary shocks. This implies that the presence of the transfer policy rule is the only difference between an economy that is fiscally integrated vis-a-vis one that is not. Using US state-level data, I estimate the current transfer policy rule and construct a counterfactual US economy without it. This counterfactual is identical in a set of observationally-equivalent fiscal union models whose equilibrium equations satisfy a number of simple exclusion restrictions. I find that employment differences across US states in the Great Recession would have been significantly larger in a counterfactual economy without a transfer policy rule because this rule redistributed resources from well to poorly performing states. I conclude that US fiscal integration contributed to stabilization of regional economies.

The paper is divided into two parts. The first part is concerned with the general methodology to construct Robust Policy Counterfactuals and the second part with the application to fiscal unions. Readers only interested in the general methodology could skip the second part of the paper, provided they are comfortable with linear models of dynamic stochastic economies. However, the application to fiscal unions serves a pedagogical purpose and may help readers grasp intuitions that could be lost in the abstract exposition of the general methodology.

In the first part of the paper, I start by describing a class models of dynamic stochastic economies where the equilibrium is unique and is characterized (or approximated) by a system of linear equations. In particular, models with a unique, stable, and finite structural vector autoregression (SVAR) representation of this equilibrium. I note that this linear system is fully described by (i) a vector of variables that are sufficient for computing equilibria and (ii) the coefficients associated with these variables. Then, I emphasize how the coefficients in the system can always be written without loss of generality as a sum of two parts: a part belonging to the policy in place and a part belonging to the structure that is invariant to such policy. The structure is simply a collection of matrices whose elements involve combinations of subsets of parameters derived from model primitives (e.g., preferences and technologies) that typically lack a direct economic interpretation. The policy is also a collection of matrices but whose elements generally have an economic interpretation. For example, the policy would include the elasticities in the Taylor rule in New Keynesian models, and the structure would include non-linear combinations of labor supply and demand elasticities, as well as Calvo price-setting parameters.

An important step towards constructing Robust Policy Counterfactuals is realizing that models with identical structure and policy are observationally equivalent—regardless of their microfoundations or parameterizations—because they have an identical SVAR representation of the equilibrium \( \Gamma(\xi, \Theta) \). Hence, for the purposes of this paper, models can be described by their structure and policy alone. Then, given a particular policy \( \Theta^0 \) and SVAR representation \( \Gamma^0 \), I

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3 As an analogy, a Taylor Rule for the nominal interest rate is a function of aggregate variables and smoothes response to aggregate shocks.

4 This class rules out, for example, models with multiple equilibria or with non-fundamental shocks.

5 In other words, there exist an economy without a policy \( \Theta \) in place that is well defined by the structure \( \xi \).
define a set of models that are consistent with \( \Gamma^0 \) being the SVAR representation of equilibrium under policy \( \Theta^0 \). Moreover, I define a subset of these observationally-equivalent models that also generate identical SVAR representation \( \Gamma(\xi, \Theta^1) \) under the alternative policy \( \Theta^1 \). Next, I show that imposing a number of linear restrictions on the elements of \( \xi \) is sufficient to identify this subset. Because this proof is constructive, it provides a methodology for constructing a counterfactual equilibrium under the alternative policy, provided data under the benchmark policy is available. Hence, a Robust Policy Counterfactual is a mapping from (1) the SVAR representation of the equilibrium under the benchmark policy, (2) the benchmark and alternative policies, and (3) the imposed linear restrictions, onto the counterfactual SVAR representation under the alternative policy. This counterfactual is identical in all models that are observationally equivalent under the benchmark policy and satisfy the linear restrictions.

The key intuition behind the construction of Robust Policy Counterfactuals is the following. In order to find the equilibrium in a particular fully specified model, we typically solve a non-linear system of equations for the SVAR representation, given values for the structure and policy that result from describing primitives and picking parameters. However, the system of equations is linear in the structure, given values for the SVAR representation and policy. Because of this linearity, we can identify the set of observationally equivalent models under a benchmark policy that also yield identical counterfactual equilibrium under an alternative policy, as long as we impose enough linear restrictions on the structure in order to make the linear system exactly determined. Lastly, given knowledge of both policies and the benchmark SVAR, we can construct this counterfactual equilibrium because the identified structure is invariant to policy changes for the set of models that satisfy the imposed linear restrictions.

I conclude the first part of the paper by discussing an alternative application of some ideas used in the construction of Robust Policy Counterfactuals. Although I do not pursue it here, I believe it may be relevant to readers interested in model testing or provide useful information for picking linear restrictions when constructing Robust Policy Counterfactuals in future applications. In particular, I show how to use the methodology to falsify a set of models, provided data before and after a policy change is available. In such case, I argue that we can use a simple Wald \( \chi^2 \) test in order to reject that a set of models is consistent with the observed reduced-form VAR representation after the policy change. The set of models is rejected if these are “too far apart” in a statistical sense.

In the second part of the paper, I start by describing several models of an economy with many islands (or states) that are hit by idiosyncratic local shocks, where a federal transfer policy rule redistributes resources across islands, and where asset markets are incomplete. For instance, I present models with and without nominal rigidities, or where leisure is a durable good. The goal is to show examples where the federal transfer policy rule can be evaluated, emphasizing their differences and commonalities. Moreover, I discuss the economic intuition behind Robust

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6For example, by guessing a recursive equilibrium exists and then using the method of undetermined coefficients to verify the guess by solving a quadratic matrix equation.
Policy Counterfactuals using two of these models. The two models have very different economic mechanisms (i.e., leisure durability v. wage rigidities) but are observationally equivalent and produce identical counterfactual equilibria for alternative transfer policy rules.

Next, I show how to construct a Robust Policy Counterfactual with respect to the transfer policy rule in a more general class of fiscal union models. In particular, I consider fiscal union models that satisfy five properties. The first is that state-level economies in log-deviations from the aggregate union behave to a first-order approximation as if they were small open economies—economies that are independent of others in the union and differ only in the realizations of purely idiosyncratic shocks that hit them. The second property is that a sufficient set of variables for characterizing the state-level equilibrium in log-deviation from aggregates includes employment, nominal wages, and assets, in addition to exogenous processes for the discount rate of households, wealth of households, and productivity of firms. The third is that the federal government gives lump-sum transfers that are a function of state-level employment, wages, and assets. This function is the transfer policy rule. These properties imply that a linear dynamic system of equations characterizes equilibria in these models. This system involves three equations: a log-linearized Euler equation, a sequential budget constraint, and an equation describing the labor market equilibrium. The fourth property is that a unique, stable equilibrium solution to this system of equations exist and that this equilibrium has a SVAR representation. The fifth and final property is that the three equilibrium equations satisfy a number of exclusion restrictions. For example, I impose that assets do not enter in the Euler equation or the discount rate process does not enter in the sequential budget constraint, which is true in many models of fiscal unions. Moreover, because of the lump-sum-transfers property, I impose that the transfer policy rule only appears in the sequential budget constraint. This is a key property of the set of fiscal union models that I analyze because it implies that both the Euler and labor market equations are invariant to policy changes, and that the policy enters in the sequential budget constraint in an additive fashion. This makes it possible to identify the policy-invariant structure of a fairly large and interesting set of fiscal union models that are both observationally equivalent under the benchmark transfer policy rule and yield identical counterfactual equilibrium without it. Intuitively, the primitives of the models in this set might be very different but they express themselves as differences in Euler and labor-market equations, not budget constraints—for instance, as in models with leisure durability vis-a-vis wage rigidities.

I follow by using US state-level data to construct a counterfactual US economy without a federal transfer policy rule redistributing resources between states. I use annual state-level data on employment, wages, and assets between 2006 and 2011 to estimate all inputs required to implement the methodology. I estimate a regional SVAR where I identify the structural shocks and impulse response matrix by using a set of theoretical restrictions on the joint response of

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7This is an assumption about aggregation of linear economies with idiosyncratic shocks. The key conditions are that identical parameters associate with the variables in each system characterizing the equilibrium, and the idiosyncratic shocks “wash out” when aggregated. The special cases I described in the previous section satisfy these.

8In Section 7.7 I consider the case where taxes and transfers distort decisions at the margin.
variables in the SVAR to each type of shock. These restrictions are derived from the parameterized sequential budget constraint. This identification scheme extends results in Beraja et al. (2015) to the larger class of models considered here. This is an ancillary contribution of this paper. Furthermore, I estimate the federal transfer policy rule elasticities with respect to state-level employment, wages, and assets. I use both OLS and, to deal with potential bias due to reverse causality, two-stage least squares. I instrument potentially endogenous variables using local house prices and the SVAR shocks that are orthogonal to the innovation in the policy rule but correlated with the endogenous variables. Results are similar in all specifications. I find that net transfers increase between 20 and 30 percent for every 1 percent decrease in nominal total wage income, and asset variation is not significant in explaining transfers variation once wages and employment are considered.

I conclude the second part of the paper by presenting primary findings of the counterfactual exercise. The employment rate’s cross-state standard deviation in the US in 2010 was 2.6 percent. I find that the standard deviation would have been 3.5 percent without the transfer policy rule redistributing resources from well performing to poorly performing states. The results are similar for 2009 and 2011, and in the stationary distribution (which I construct using Monte Carlo simulations). To put these magnitudes in context, aggregate output volatility during the pre-Volcker period (1960:1 to 1979:2) was 1.8 percent whereas during the post-Volcker-disinflation period (1982:4 to 1996:4) it was 1.3 percent. The stabilization role of fiscal integration seems to be in the same order of magnitude than this "Great Moderation". Moreover, I assess how nominal and real rigidities interact with fiscal integration—the rigidities channel—by constructing a robust transfer policy counterfactual in an economy without nominal or real rigidities, i.e, an economy where labor supply and demand are static. This allows me to conduct a theoretical difference-in-differences experiment to quantify the interaction between rigidities and the transfer policy rule. I find that rigidities amplified the gains from fiscal integration in poorly performing states during the Great Recession, accounting for one-third of their counterfactual employment gains, whereas it accounted for none of the counterfactual losses in the well performing states. Finally, I show that main quantitative findings are insensitive to alternative parameterizations and specifications of the policy rule. In particular, I allow for the possibility that taxes and transfers distort labor supply decisions.

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9 Identification of the impulse response matrix relies on a series of theoretical linear restrictions that are implied by these equations, and link the reduced form errors to the structural shocks.

10 OLS estimates are consistent if the error term in the policy rule has no regional component (or if the error term is measurement error). However, the issue of endogeneity might arise because of reverse causality. When the innovation in the transfer policy rule causes regional variation in employment, wages, and assets, they both cause and are caused by transfers.

11 The estimates are similar to those in Feyrer and Sacerdote (2013), who find a 0.25 decrease, and Bayoumi and Masson (1995), who find a 0.31 decrease.

12 See Clarida et al. (2000) for details of this calculation and analysis of interest rate policy stabilization.

13 Findings under this alternative policy are essentially unchanged because the estimated transfer policy rule elasticities with respect to wages and employment are close to 1. What is important for determining the agents marginal decisions is the marginal effect of employment (or wages) in taxes and transfers. When coefficients are close to 1, the marginal effect is close to zero.
The rest of this paper is organized as follows. Section 2 discusses related literature. Section 3 lays out the general methodology. Section 4 shows how to falsify a set of models via the methodology. Section 5 illustrates the concepts observational equivalence and robustness in simple fiscal union models. Section 6 shows how to apply the methodology to a larger set of fiscal union models. Section 7 constructs a counterfactual US economy without fiscal integration and presents primary findings. Section 8 offers concluding remarks.

2 Literature review

This paper relates to much literature. The original motivation of Sims (1980) seminal contribution on structural vector autoregressions (SVARs) was to do policy analysis in a way that was robust across many models. SVARs have been very successful for doing analysis with respect to the exogenous component of policy (i.e., policy shocks) in a relatively model-free way. However, because of Lucas critique, it has not been possible to use SVARs to do analysis with respect to unobserved systematic components of policy (i.e., policy rules). The methodology in this paper shows how to restrict the set of structural models that generate SVARs in order to conduct this type of analysis in a way that is immune to Lucas critique and retains robustness to model selection within the restricted set.

Furthermore, since seminal research by Harberger (1964), sufficient statistics have been used to evaluate the consequences of policy changes, without requiring full knowledge of the underlying structural model. Chetty (2008) provides an excellent survey. More recent examples in trade literature include Arkolakis et al. (2012) and Blaum et al. (2015), and in macroeconomics, Alvarez et al. (2014), Auclert (2014), and Atkeson and Burstein (2015). The non-parametric counterfactuals in Adao et al. (2015) are in the same spirit. The methodology for conducting Robust Policy Counterfactuals shares their motivation and philosophy, using reduced-form statistics to construct counterfactuals that are identical in a given set of models.

Economic literature on robust control and estimation, pioneered by Hansen and Sargent (see Hansen and Sargent (2008) for a summary), is also concerned with model misspecification and its consequences for policy decision-making. While their analysis is rather different, we both emphasize the study of these issues in dynamic, stochastic economies. Consequently, we use similar techniques from time series econometrics (e.g. Hamilton (1994), Stock and Watson (2001), DeJong and Dave (2011)) and for solving such models (e.g. Blanchard and Kahn (1980), Uhlig (1995)).

I extend the scheme proposed by Beraja et al. (2015) for identification of structural shocks in SVARs, which is similar to a recent paper by Baumeister and Hamilton (2015). This identification scheme fits nicely with the philosophy in this paper by using certain parts of the theoretical model about which we feel more confident. It is part of growing literature developing “hybrid”

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14 In the application to fiscal unions, policy shocks would be exogenous, unanticipated transfers to a state. On the other hand, I am concerned with the systematic, anticipated component of tax-and-transfer policy as a known function of state-level variables—a transfer policy rule.
methods that, for example, construct optimal combinations of econometric and theoretical models (Carriero and Giacomini (2011), Del Negro and Schorfheide (2004)), or use the theoretical model to inform the econometric model’s parameter (An and Schorfheide (2007), Schorfheide (2000)). Other identification schemes that use restrictions derived from theory include sign restrictions as in Uhlig (2005), long run restrictions as in Blanchard and Quah (1989), and short-run restrictions as in Clarida and Gertler (1997), Christiano et al. (1999).

Del Negro and Schorfheide (2009) are also concerned with doing policy analysis in misspecified models. In particular, they study changes in the nominal interest rate policy rule. They consider a benchmark DSGE model with sticky prices and a SVAR representation of the equilibrium in this model. Then, they consider an alternative “true” model that might be different from the benchmark and is unknown. They write the SVAR representation of the equilibrium in this “true” model as a sum of the benchmark SVAR and perturbation terms. They consider various assumptions about the perturbation terms concerning how they are affected by changes in policy. Taking a stance on this is necessary to do counterfactuals that are immune to Lucas critique within their framework. For example, they conduct the analysis assuming that the perturbation terms are invariant to changes in the policy rule and that they can be estimated jointly with the benchmark DSGE model parameters from past data. This approach is useful for considering the sensitivity of results to model misspecification around the benchmark model. The methodology in this paper is rather different. While I do not describe alternative models in terms of deviations from a benchmark model in the paper, it is possible to do so and useful for comparison with their paper. In this case, we would write the structural equations characterizing the equilibrium in the “true” model as the sum of a benchmark model and a perturbation term. Then, we would state which perturbation terms are policy invariant and which ones are not. The key difference is that my perturbation would be done around a benchmark structure of the model (e.g. Euler equations, labor demand, etc.) whereas Del Negro and Schorfheide (2009) do perturbations around the SVAR representation of the equilibrium of such benchmark model. Perturbing the structure of the model makes it easy to see which models are consistent with the perturbation terms being policy-invariant. However, perturbing the equilibrium SVAR representation implies that the perturbation terms are not policy invariant (except in very particular models), making the analysis subject to the Lucas Critique.

The theory behind the stabilization benefits of fiscal integration, or more generally aggregate risk sharing arrangements, is well developed. Examples include Farhi and Werning (2014), Bucovetsky (1998), Persson and Tabellini (1996a,b), Lockwood (1999). However, there is surprisingly little quantitative and empirical research on regional stabilization in fiscal unions. On the purely empirical side, Sala-i Martin and Sachs (1991), Feyrer and Sacerdote (2013), and Bayoumi and Masson (1995) focus on reduced form estimates of properties of the tax-and-transfer system in fiscal unions, and in some cases, present back-of-the-envelope counterfactuals. Asdrubali et al. (1996) quantify the amount of risk sharing among states in the United States by decomposing cross-sectional variance in output. Atkeson and Bayoumi (1993) examine evidence for private
insurance of regional risk in the United States and Europe.

Regarding quantitative research, Evers (2015) is closest to the application in this paper. He provides a quantitative analysis of federal fiscal rules in monetary unions by using a fully structural DSGE model, which builds on Chari and Kehoe (2007) and Kollmann (2001). The model has two countries, nominal wage, and price rigidities, and incomplete markets. It includes productivity and government spending shocks alone. He calibrates the model and finds very small differences in nearly every relevant business cycle statistic when comparing a monetary union with and without a central fiscal authority. I hypothesize that there are primarily two reasons he does not find quantitatively significant benefits from fiscal integration, while the opposite is true for this paper. First, his model does not include shifters in the Euler equation (e.g., discount rate shocks or borrowing constraints). In contrast, I find that most of the regional employment stabilization because of fiscal integration is because the tax-and-transfer system stabilizes "discount rate" shocks, and not "productivity" shocks. Second, his is a calibration exercise in a fully specified model, and so it fails to account for several statistical properties of the joint distribution of the variables in the data (e.g., the autocorrelation of consumption or cross correlation between employment and output). Oh and Reis (2012) and McKay and Reis (2013) quantify the impact of targeted transfers and automatic stabilizers in an incomplete markets model, with heterogeneous agents and nominal rigidities. This paper shares their interest in the study of real-world tax-and-transfer systems and their macroeconomic consequences. However, they are largely concerned with implications for aggregate business-cycle stabilization, whereas I am concerned with implications for regional stabilization.

Moreover, the paper contributes to the recent surge in papers that exploit regional variation to highlight mechanisms of importance for economic fluctuations. Nakamura and Steinsson (2014) use sub-national US variation to inform the size of local government spending multipliers. Hurst et al. (2015) study regional redistribution through the US mortgage market. Mian and Sufi (2014), Mian et al. (2013), and Midrigan and Philippon (2011) exploit regional variation in the United States to explore the extent to which household leverage contributed to the Great Recession. Blanchard et al. (1992), Autor et al. (2013), and Charles et al. (2014) use regional variation to measure the responsiveness of labor markets to labor demand shocks. Beraja et al. (2016) study how the regional distribution of housing equity shapes aggregate and distributional consequences of monetary policy.

Finally, the paper relates to econometrics literature on "partial identification" of structural models and how to perform statistical tests in these cases. Canay and Shaikh (2015) provides an extensive summary.

3 Robust Policy Counterfactuals: A General Methodology

This section describes how to construct Robust Policy Counterfactuals in a class of linear models of dynamic stochastic economies. First, I define this class of models, and develop notation and
language that I use throughout the section. Second, I offer a formal definition of Robust Policy Counterfactuals. I conclude by showing how to construct them, given data from an economy under a benchmark policy rule and restrictions on the set of models that could have generated this data.

I consider a class of models whose equilibrium is characterized (or approximated) by the linear system of equations,

\[ 0 = (F + \Theta_f)E_t[x_{t+1}] + (G + \Theta_c)x_t + (H + \Theta_p)x_{t-1} + LE_t[z_{t+1}] + Mz_t \]

\[ 0 = -z_t + Nz_{t-1} + \Sigma u_t; \quad \text{iid } u_t \text{ with } E[u_t] = 0, \text{Var}(u_t) = 1 \]  \hspace{1cm} (SME1)

\[ 0 = y_t + R_1x_t + R_2x_{t-1} + R_3z_t \]

\[ (H + \Theta_p)x_0; Mz_0 \text{ given} \]

where \(x_t\) is a column vector that includes all endogenous state variables, and could include endogenous control variables too; \(z_t\) is a column vector of exogenous state variables; and \(y_t\) a column vector of other endogenous variables not included in \(x_t\). For example, in the context of Hansen (1985) real business cycles model, \(x_t\) includes log-deviations from the steady state of capital and potentially employment, \(z_t\) includes productivity, and \(y_t\) consumption. Alternatively, in the application to fiscal unions, \(x_t\) will include employment, wages and assets at the state-level in log-deviations from the aggregate union equilibrium, and \(z_t\) will include the exogenous household discount rate, productivity and wealth processes.

Given initial conditions, I say that a particular economy is characterized by the structure \(\{\xi, \xi^a\}\) and the policy \(\Theta\), where \(\xi \equiv \{F, G, H, L, M, N, \Sigma\}\), and \(\xi^a \equiv \{R_1, R_2, R_3\}\) are policy-invariant matrices, and \(\Theta \equiv \{\Theta_f, \Theta_c, \Theta_p\}\) are matrices that characterize endogenous policy rules. \(\Theta_f\) contains the policy parameters associated with future expected variables, \(\Theta_c\) with contemporaneous variables, and \(\Theta_p\) past variables. Elements in \(\{\xi, \xi^a\}\) typically involve combinations of subsets of parameters derived from a fully specified model. They typically lack direct economic interpretation in terms of the primitives of such a model. Elements in \(\Theta\) generally have a direct economic interpretation. For example, in the application to fiscal unions, the structure would include non-linear combinations of labor supply and demand elasticities, as well as parameters related to the degree of nominal wage rigidity. The policy, on the other hand, would include the transfer policy rule elasticities.

The system (SME1) evaluated at \(\Theta = 0\) corresponds to a benchmark economy without a policy in place.\(^\text{15}\) Moreover, because of the linearity of the system, we can always write the policy as the difference between two economies. For example, if there is a benchmark economy with \(F_0 \in \xi_0\) and and alternative economy with \(F_1 \in \xi_1\), then we can define \(\Theta_f = F_1 - F_0\) without loss of generality.

Next, I restrict the class of linear models to those that satisfy three assumptions.

\(^\text{15}\)Uhlig (1995), from whom I borrow some notation, studies a very similar system of equations, and provides a computational toolkit for finding recursive solutions.
Assumption 1 (Stability)  $\xi, \Theta$ are such that the system $(\text{SME1})$ is stabilizable.

Under Assumption 1, we have a stable recursive solution to $(\text{SME1})$ that can be written as:

$$x_t = P(\xi, \Theta)x_{t-1} + Q(\xi, \Theta)z_t \quad \text{(RR)}$$

$$z_t = Nz_{t-1} + \Sigma u_t$$

where $P(\xi, \Theta)$ and $N$ have all eigenvalues inside the unit circle.

Assumption 2 (Invertibility)  $\xi, \Theta$ are such that $Q(\xi, \Theta)$ is a non-singular square matrix.

Claim 1 If Assumptions 1 and 2 hold, there is a structural vector autoregression (SVAR) representation to the solution (RR) of the form:

$$x_t = \rho_1(\xi, \Theta)x_{t-1} + \rho_2(\xi, \Theta)x_{t-2} + \Lambda(\xi, \Theta)u_t \quad \text{(SVAR)}$$

where $\rho_1(\xi, \Theta) \equiv P(\xi, \Theta) + Q(\xi, \Theta)NQ(\xi, \Theta)^{-1}$; $\rho_2(\xi, \Theta) \equiv (P(\xi, \Theta) - \rho_1(\xi, \Theta))P(\xi, \Theta)$ and $V(\xi, \Theta) \equiv \text{Var}(\Lambda(\xi, \Theta)u_t) = \Lambda(\xi, \Theta)\Lambda(\xi, \Theta)'$, where $\Lambda(\xi, \Theta) \equiv Q(\xi, \Theta)\Sigma$.

Proof. We can write $z_{t-1} = Q(\xi, \Theta)^{-1}(x_{t-1} - P(\xi, \Theta)x_{t-2})$ and replace it and the law of motion for the exogenous states into the law of motion for the endogenous variables to obtain the VAR(2) representation □.

Assumption 3 (Uniqueness)  $\xi, \Theta$ are such that, given $\{\rho_1(\xi, \Theta), \rho_2(\xi, \Theta)\}$, the equation $\rho_2(\xi, \Theta) = (P - \rho_1(\xi, \Theta))P$ has a unique solution $P$ with all eigenvalues inside the unit circle.

In what follows, I ask: how does the structural vector autoregression representation of the equilibrium change when the policy changes from a benchmark policy $\Theta^0$ to an alternative policy $\Theta^1$? A Robust Policy Counterfactual is an answer to this question in models whose equilibrium is characterized (or approximated) by a system like $(\text{SME1})$, and whose structure and policy $\xi, \Theta$ satisfy Assumptions 1, 2 and 3. In particular, I show that the equilibrium after the policy change is identical in a set of observationally equivalent models whose structure satisfies a number of linear restrictions. Then, I provide a methodology for constructing such counterfactual equilibrium in practice. Implementing this methodology only requires knowledge of the benchmark and alternative policies, the SVAR representation of the equilibrium under the benchmark policy, and imposing a number a linear restrictions on the structure of models that generated that benchmark equilibrium.

The first step is realizing that models with an identical structure $\xi$ are observationally equivalent under policy $\Theta$ because they have an identical SVAR representation of the equilibrium $\Gamma(\xi, \Theta) \equiv \{\rho_1(\xi, \Theta), \rho_2(\xi, \Theta), \Lambda(\xi, \Theta)\}$. Then, for a given benchmark policy $\Theta^0$ and SVAR representation $\Gamma^0$, we have the following definition,
Definition 1 (Observational Equivalence)  Given \( \{ \Theta^0, \Gamma^0 \} \), define \( O(\Theta^0, \Gamma^0) \) as the set of structures \( \xi \) that generate equilibrium SVAR representation \( \Gamma^0 \) under benchmark policy \( \Theta^0 \).

By definition, all models which have structures in \( O(\Theta^0, \Gamma^0) \) are observationally equivalent. From an econometrician’s perspective, this definition of observationally-equivalent models is stronger than the usual one because models with structures in \( O(\Theta^0, \Gamma^0) \) are not identified even if the structural VAR representation of the equilibrium was known, whereas the usual definition would involve knowledge of the reduced-form VAR representation alone.

Next, I define a subset of these observationally-equivalent models that yield identical counterfactual equilibrium representation \( \Gamma^1 \) under an alternative policy \( \Theta^1 \). Moreover, I give a formal definition of Robust Policy Counterfactuals and show how to identify them.

Definition 2 (Counterfactual Equivalence)  Given \( \{ \Theta^0, \Gamma^0 \} \) and \( \{ \Theta^1, \Gamma^1 \} \), define \( C(\Theta^0, \Gamma^0, \Theta^1, \Gamma^1) \) as the subset of structures \( \xi \) in \( O(\Theta^0, \Gamma^0) \) that generate identical counterfactual equilibrium SVAR representation \( \Gamma^1 \) under alternative policy \( \Theta^1 \). Formally, \( C(\Theta^0, \Gamma^0, \Theta^1, \Gamma^1) = O(\Theta^0, \Gamma^0) \cap O(\Theta^1, \Gamma^1) \).

Definition 3 (Semi-structure)  Let \( R_{n_l} \) be a set of \( n_l \) independent linear restrictions on the coefficients in line \( 'l' \) of structure \( \xi \). Then, define a semi-structure \( \bar{\xi} \) as a structure \( \xi \) that satisfies \( \bar{\xi} \equiv \{ R_{n_l} \}_{l=1}^k \) for all lines \( l = \{1, ..., k\} \) in \( \xi \).

Definition 4 (Robust Policy Counterfactual)  Let \( \bar{\xi}^* \) be a semi-structure satisfying linear restrictions \( R \) and \( \bar{\xi}^* \in C(\Theta^0, \Gamma^0, \Theta^1, \Gamma(\bar{\xi}^*, \Theta^1)) \). Then, a Robust Policy Counterfactual is a mapping from \( \{ \Gamma^0, \Theta^0, \Theta^1, R \} \) onto \( \Gamma(\bar{\xi}^*, \Theta^1) \).

Proposition 1 (Robust Policy Counterfactual Identification)
Given \( \{ \Theta^0, \Theta^1, \Gamma^0 \} \), a structure \( \bar{\xi}^* \) is the unique solution to \( \bar{\xi}^* = C(\Theta^0, \Gamma^0, \Theta^1, \Gamma(\bar{\xi}^*, \Theta^1)) \) if and only if:

(i). \( \bar{\xi}^* \) is such that \( \Gamma(\bar{\xi}^*, \Theta^0) = \Gamma^0 \)

(ii). \( \bar{\xi}^* \) is a semi-structure satisfying \( R^* \) with \( n_l = 2k - (1 - \text{rank}(\Theta^0)) \) for all \( 'l' \)

Then, a Robust Policy Counterfactual exists and is unique, given \( \{ \Theta^0, \Theta^1, \Gamma^0, R^* \} \)

Proof.  See Appendix A

Definitions [4] and [1] state that we can partition the set of observationally-equivalent models under a benchmark policy into many smaller subsets, where models within each subset yield identical counterfactual equilibrium under the alternative policy. In other words, these subsets include models that are observationally equivalent under both the benchmark and alternative policies.

The key to proving Proposition 1 is noticing that in order to find the equilibrium SVAR representation \( \Gamma^0 \equiv \Gamma(\bar{\xi}^*, \Theta^0) \), we typically solve a non-linear system of equations, given structure and policy \( \{ \bar{\xi}^0, \Theta^0 \} \). However, the system of equations is linear in structure \( \bar{\xi}^0 \), given values for the
SVAR representation and policy $\Gamma^0, \Theta^0$. Because of this linearity, we can identify the structure $\xi^*$ that is consistent with a set of observationally equivalent models under a benchmark policy that also yield identical counterfactual equilibrium under an alternative policy, as long as we impose enough linear restrictions on the structure in order to make the linear system exactly determined. Then, we can construct this counterfactual equilibrium by solving the non-linear system of equations using the alternative policy and identified structure $\xi^*$. Lastly, this methodology for constructing Robust Policy Counterfactuals is immune to Lucas Critique because the identified structure is invariant to policy changes for the set of models that satisfy the imposed linear restrictions.

Why is this methodology useful? There are mainly three reasons. First, conducting counterfactual analysis in a particular structural model requires specifying its primitives as well as identifying its parameters. Both of these are complicated issues, in particular because linear models are many times not identified. The methodology uses this lack of identification "in our favor", showing that the flip-side of lack of identification is robustness. As such, it allows us to derive quantitative predictions for an unobserved policy change without much a-priori knowledge of the "true" economy’s structure, predictions that are immune to Lucas Critique and robust across models within the identified set. Second, if we started with a particular structural model, it is typically hard to know whether its counterfactual predictions are robust to variations in model primitives, undermining the analysis’ credibility. It is hard because the mapping from primitives or parameters to the recursive equilibrium representation is highly non-linear. However, as shown in Proposition 1, the mapping is linear from such equilibrium to the structure $\xi$ that involves combinations of these parameters. Hence, the methodology provides an easy way to check for robustness: simply check whether the alternative model being consider both satisfies the same linear restrictions and can match the same SVAR representation under the benchmark policy—i.e., whether it is in the same subset $C(.)$ than the benchmark structural model. If the answer is yes, then we can be confident that our predictions for a policy change are robust to such variation in model primitives. Third, it provides a way of falsifying a set of models, as long as we have data before and after a policy change. I discuss this application of the methodology in Section 4.

3.1 A quick note on implementation and SVAR identification

As implied by Proposition 1, implementing the methodology to construct Robust Policy Counterfactuals requires estimating the SVAR representation of the equilibrium under the benchmark policy. Estimating the reduced-form VAR $\{\rho_1, \rho_2, V\}$ is typically straightforward, e.g., via OLS equation-by-equation. However, estimating the impulse response matrix $\Lambda$ requires structural identification assumptions. By now, there is a plethora of identification schemes for achieving this: ordering of shocks, sign restrictions, long-run restrictions, etc. These represent several routes that could be followed to identify $\Lambda$ and use it as an input when constructing Robust Pol-

\[ \text{See Section 5 for an example regarding fiscal union models.} \]
icy Counterfactuals, as long as their implied linear restrictions on the structure $\xi$ are consistent with the restrictions imposed on semi-structure $\xi^*$. Alternatively, in this section I show how to use those equilibrium equations of structural models that we feel more confident about in order to derive linear restrictions on structure $\xi$ that are sufficient to identify $\Lambda$. Hence, this identification scheme fits nicely with the previous section and the philosophy in this paper. Moreover, the scheme generalizes the special case in [Beraja et al. (2015)] and is similar to that in a recent paper by [Baumeister and Hamilton (2015)].

The following lemma describes the identification scheme,

**Lemma 1** Given the reduced-form VAR $\{\rho_1, \rho_2, V\}$ and all the elements in 's' lines of $\{F, G, H, \Theta\}$, where $s < k$. Let $I_s$ be a selection matrix for the 's' lines, and $A(s) \equiv \begin{bmatrix} I_s(LN + M)\Sigma \\ I_s(LN + M)N \Sigma \end{bmatrix}$ and $B(s) \equiv \begin{bmatrix} I_s(F\rho_1 + G + \Theta_c) \\ I_s((F\rho_1 + G + \Theta_c)\rho_1 + F\rho_2 + H + \Theta_p) \end{bmatrix}$. The impulse response matrix $\Lambda(\xi, \Theta)$ is identified if the following are satisfied:

1. $B(s)$ has full rank.
2. There are $r(k) = k(k - 1)/2$ elements specified in $A(s)$.
3. There is at least one specified element in $r_k$ in each of 'k - 1' columns of $A(s)$.
4. $\Lambda(\xi, \Theta)\Lambda(\xi, \Theta)' = V$.

**Proof.** See Appendix B.

Conditions 1 through 3 in the lemma imply a set of linear restrictions linking the reduced form errors to the structural shocks. They impose a linear co-movement that the unexpected components in the variables in (SVAR) (i.e., the columns in the impulse response matrix) must satisfy. These restrictions, combined with orthogonalization conditions in 4, are sufficient to identify all elements in the matrix $\Lambda(\xi, \Theta)$. Section 7.2 describes an application of this scheme to a special case, i.e., fiscal union models with three endogenous variables and one specified structural equation: the sequential budget constraint. [Beraja et al. (2015)] is another example.

## 4 Falsifying a Set of Models

So far I have discussed how to construct a counterfactual economy after a policy change given observations of the economy before the policy change. There are many instances, however, where observations before and after the policy change are available. For example, in the context of the application to stabilization in fiscal unions, we could entertain the possibility that in the future Catalunya decided to secede from the fiscal union that is Spain or that Europe would fiscally integrate. Alternatively, when the European Monetary Union was formed, country-level monetary policy rules went from being country-specific to union-wide determined. Or, when Paul Volcker
started his tenure as the Federal Reserve president, the nominal interest rate policy rule changed. In cases like this, I argue that we could exploit this policy change "experiments" in order to falsify a set of models using the Robust Policy Counterfactual methodology. Specifically, by constructing the reduced-form VAR representation predicted by a set of models after a policy change and comparing it to the observed reduced-form VAR representation after the policy change. If these are "too far apart" in a statistical sense, we can reject the null hypothesis that this set of models generated the observed data, under the assumption that only the policy changed and not the structure of the economy.

Although I do not pursue this application of the methodology in this paper, I conjecture that it could be useful in future applications, mainly in two ways. First, it could be used as an input for constructing Robust Policy Counterfactuals. For example, if we had several potential linear restrictions that are reasonable a-priori, we could discard some of them by falsifying the set of models that satisfies. Then, we would using an observed policy change in order to construct a Robust Policy Counterfactuals for an unobserved policy change. Second, it could be used as a guidance for future structural modeling. For example, if a particular policy change "experiment" proved to be inconsistent with restrictions implied by certain models, then researchers building new structural models should take this evidence into consideration.

Formally, suppose we wished to test the null hypothesis

$$H_0 : h (\xi_s, \Theta', \rho'_1, \rho'_2, V') \equiv \{\rho_1(\xi_s, \Theta'), \rho_2(\xi_s, \Theta'), V(\xi_s, \Theta')\} - \{\rho'_1, \rho'_2, V'\} = 0$$

where \(\{\rho_1(\xi_s, \Theta'), \rho_2(\xi_s, \Theta'), V(\xi_s, \Theta')\}\) is the predicted, reduced-form VAR representation under the new policy \(\Theta'\) that results from the Robust Policy Counterfactual of a set of models with identified semi-structure \(\xi_s\) that have identical SVAR representation \(\rho_1, \rho_2, \Lambda\) under the benchmark policy \(\Theta\), and \(\{\rho'_1, \rho'_2, V'\}\) is the observed reduced-form VAR representation after the policy change. Proposition 2 below shows that a Wald \(\chi^2\) test can be used to reject this null hypothesis, as long as we have normally distributed estimators of \(\{\xi_s, \Theta', \rho'_1, \rho'_2, V'\}\). In particular, the test is about the null hypothesis that semi-structure \(\xi_s\)—i.e., its associated linear restrictions—is consistent with the observed equilibrium after the change in policy.

**Proposition 2 (Model Falsification via Wald Test)**

Let \(\{\tilde{\xi}_s, \tilde{\Theta}', \tilde{\rho}'_1, \tilde{\rho}'_2, \tilde{V}'\}\) be normally distributed, consistent estimators. If the function \(h (\xi_s, \Theta', \rho'_1, \rho'_2, V')\):

1. is continuously differentiable in a neighborhood of \(\{\tilde{\xi}_s, \tilde{\Theta}', \tilde{\rho}'_1, \tilde{\rho}'_2, \tilde{V}'\}\).

2. has full rank first derivative matrix.

3. has positive definite second derivative matrix.

then, \(h (\xi_s, \Theta', \rho'_1, \rho'_2, V')\) has asymptotic \(\chi^2\) distribution under the null hypothesis \(H_0\).

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\(^{17}\)For instance, under the assumption of normality of the structural shocks, this is true when estimating the SVAR under the benchmark policy via OLS equation-by-equation together with an identification procedure for the impulse response matrix \(\Lambda\).
5 Illustrating Observational and Counterfactual Equivalence

In this section, I begin by describing a simple fiscal union model. Then, I present a number of alternative models that are both observationally equivalent under fiscal integration and yield identical counterfactual equilibrium in an economy without fiscal integration. The goal is to illustrate the concepts of observational and counterfactual equivalence from previous sections, providing economic intuition about Robust Policy Counterfactuals in particular examples. Moreover, these examples are in the larger set of fiscal union models that I analyze in the following sections when evaluating the contribution of fiscal integration to regional stabilization in the US.

5.1 A Simple Fiscal Union Model

Consider an economy comprised of many islands, inhabited by a representative household and firm. The only other agent in the economy is a federal government. Households consume, work, and save/borrow in a non-state-contingent asset—a nominal bond in zero net supply. Firms produce final consumption goods using labor and intermediate goods. By assumption, the final consumption good is non-tradable, intermediate goods are tradable, and labor is not mobile across islands. Finally, each island has an exogenous endowment of intermediate goods. The federal government sets the nominal interest rate on the nominal bond, and gives lump-sum transfers to the islands. Assume that the nominal interest rate follows an endogenous rule that is a function of only aggregate variables (together with a fixed nominal exchange rate, this implies that the islands are part of a monetary union). Also, assume that federal transfers are a function of island-level variables alone. Throughout, I assume that parameters governing preferences and production are identical across islands and the islands only differ, potentially, in the shocks that hit them—these shocks include a shifter of the households discount rate, a productivity shifter in the production function of final goods, and the exogenous endowment of tradable intermediate goods. Finally, I assume that all labor, goods and asset markets are competitive.

5.1.1 Firms and Households

Final goods producers use labor $N^y_{kt}$ and intermediates $X_{kt}$ in island $k$ at time $t$ and face prices $P_{kt}$, wages $W_{kt}$, and intermediate prices $Q_t$ (equalized across all islands because of assumed tradability). Their profits are

$$
\max_{N^y_{kt}, X_{kt}} P_{kt} e^{\gamma t} (N^y_{kt})^a (X_{kt})^b - W_{kt} N^y_{kt} - Q_t X_{kt}
$$
where $z_{kt}$ is a productivity shock and $(\alpha, \beta) : \alpha + \beta < 1$ are the labor and intermediates shares. Unlike the tradable goods prices, final good prices ($P_{kt}$) vary across islands.

Households preferences are given by

$$
\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} e^{-\rho_{kt} - \delta_{kt}} \left( C_{kt} - \frac{\phi}{1+\phi} N_{kt}^{\frac{1+\phi}{\phi}} \right)^{1-\sigma} \right]
$$

where $C_{kt}$ is consumption of the final good, $N_{kt}$ is labor, $\delta_{kt}$ is an exogenous processes driving the household’s discount rate.

Households are able to spend their labor income $W_{kt}N_{kt}$ plus profits accruing from firms $\Pi_{kt}$ and exogenous endowment of tradable goods $Q_{tekt}$, financial income $B_{kt}i_t$ and transfers from the government $S_{kt}$, where $B_{kt}$ are nominal bond holdings at the beginning of the period and $i_t$ is the nominal interest (equalized across islands given our assumption of a monetary union where the bonds are freely traded) on consumption goods ($C_{kt}$) and savings ($B_{kt+1} - B_{kt}$). Thus, they face the period-by-period budget constraint

$$
P_{kt}C_{kt} + B_{kt+1} \leq B_{kt}(1 + i_t) + W_{kt}N_{kt} + \Pi_{kt} + S_{kt} + Q_{tekt}
$$

A well known issue in the international macroeconomics literature is that under market incompleteness of the type we just described there is no stationary distribution for bond holdings across islands in the log-linearized economy, and all other island variables in the model have unit roots. This is problematic for reasons both theoretical (I will like to study log-deviations from a deterministic steady state) and empirical (regional data for the US does not suggest the presence of such unit roots). I follow Schmitt-Grohe and Uribe (2003) and let $\rho_{kt}$ be the endogenous component of the discount factor that satisfies $\rho_{kt+1} = \rho_{kt} + \Phi(.)$ for some function $\Phi(.)$ of the average per capita variables in an island. As such, agents do not internalize this dependence when making their choices. This modification induces stationarity for an appropriately chosen function $\Phi(.)$. Schmitt-Grohe and Uribe (2003) show that alternative stationary inducing modifications (a specification with internalization, a debt-elastic interest rate or convex portfolio adjustment costs) all deliver similar quantitative results in the context of a small open economy real business cycle model.

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18It is worth noting that all model shocks will generate endogenous variation in markups given assumed decreasing returns to scale. Additionally, what I call a “productivity shock” is isomorphic to any shifter of unit labor costs and, hence, labor demand schedules. I will not attempt to distinguish between the different interpretations of this shock in this paper.
5.1.2 Federal government

The federal government budget constraint is

\[ B_t^g + \sum_k S_{kt} + Q_t G = B_{t-1}^g (1 + i_t) \]

where \( G \) is some exogenous level of government spending in intermediate goods. The key feature of a fiscally integrated economy is that the federal government has the ability to redistribute resources across islands via transfers \( S_{kt} \). If the islands where fiscally independent such transfers would not be possible.

I assume that the federal government announces a nominal interest rate rule \( i_t = i(\cdot) \) as a function of aggregate variables in the economy alone. Moreover, it announces a transfer policy rule as a function of per-capita employment, wages and assets in an island

\[ S_{kt} = \tilde{S}(\tilde{W}_{kt})^{\delta_w} (\tilde{N}_{kt})^{\delta_n} (\tilde{B}_{kt-1})^{\delta_b} \]

Again, agents do not internalize this dependence when making their choices.

5.1.3 Exogenous shocks and processes

I assume the exogenous processes are AR(1) processes, with an identical autoregressive coefficient across islands, and that the innovations are iid, mean zero, random variables with an aggregate and island specific component. First, define \( \gamma_{kt} \equiv \delta_{kt} - \delta_{kt-1} \). Then,

\[ z_{kt} = \rho z_{kt-1} + \sigma_z v_{kt} + \sigma_z u_{kt}^{\gamma} \]
\[ \gamma_{kt} = \rho_{\gamma} \gamma_{kt-1} + \sigma_{\gamma} v_{kt}^{\gamma} + \sigma_{\gamma} u_{kt}^{\gamma} \]
\[ \eta_{kt} = \rho_{\eta} \eta_{kt-1} + \sigma_{\eta} v_{kt}^{\eta} + \sigma_{\eta} u_{kt}^{\eta} \]

with \( \sum_k u_{kt}^{\gamma} = \sum_k u_{kt}^{\eta} = \sum_k u_{kt} = 0 \). By assumption, I assume the average of the regional shocks sum to zero in all periods.

The "discount rate" process \( \gamma_{kt} \) is a shifter of a household’s discount rate, but it can be viewed as a proxy for the tightening of household borrowing limits. Such shocks have been proposed by Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2011), and Mian and Sufi (2014) as an explanation of the 2008 recession. Beraja et al. (2015) find that these broad types of shocks explain most regional employment variation across states in the United States during the Great Recession and its aftermath. The "productivity" process \( z_{kt} \) can be interpreted as actual productivity, or a shifter of firm’s demand for labor or firm’s mark-ups. As an example, credit supply shocks to firms such as those proposed by Gilchrist et al. (2014) fall in this category. Finally, "wealth" process \( \eta_{kt} \) is modeled as an endowment of intermediate goods but can be interpreted as shifters of the budget constraint that agents face such as exogenous changes in household wealth.
### 5.1.4 Equilibrium

An equilibrium is a collection of prices \( \{P_{kt}, W_{kt}, Q_t\} \) and quantities \( \{C_{kt}, N_{kt}, B_{kt}, N^y_{kt}, X_{kt}\} \) for each island \( k \) and time \( t \) such that, for an interest rate rule \( i_t = i(\cdot) \) and given exogenous processes \( \{z_{kt}, \gamma_{kt}, \eta_{kt}\} \), they are consistent with household utility maximization and firm profit maximization and such that the following market clearing conditions hold:

\[
C_{kt} = e^{z_{kt}} (N^y_{kt})^\alpha (X_{kt})^\beta
\]

\[
N_{kt} = N^y_{kt}
\]

\[
G + \sum_k X_{kt} = \sum_k e^n_k
\]

\[
0 = \sum_k B_{kt} + B^g_t
\]

### 5.1.5 Aggregation

The first important assumption for aggregation is that all islands are identical with respect to their underlying production and utility parameters.\(^{19}\) The second assumption is that the joint distribution of island-specific shocks is such that its cross-sectional summation is zero. If \( K \), the number of islands, is large this holds in the limit because of the law of large numbers. I log-linearize the model around this steady state and show that it aggregates up to a representative economy where all aggregate variables are independent of any cross-sectional considerations to a first order approximation.\(^{20}\) I denote with lowercase letters an island variable’s log-deviation from the aggregate union equilibrium. Lowercase letters with a tilde denote deviations from the steady state. For example, \( \hat{n}_{kt} = \tilde{n}_{kt} - \tilde{n}_t \) and \( \hat{n}_t = \frac{1}{K} \sum_k \hat{n}_{kt} = \frac{1}{K} \log (N_{kt}/\bar{N}) \). I assume that the monetary authority announces the nominal interest rate rule in log-linearized form:

\[
\tilde{i}_{t+1} = \varphi \pi E_t[\tilde{\pi}_{t+1}]
\]

where \( \tilde{\pi}_t \) is the aggregate inflation rate. Finally, I assume that the endogenous component of the discount factor is \( \Phi(\cdot) = \Phi_0 n_{kt} + \Phi_1 n_{kt-1} \).\(^{21}\)

The following lemma present the aggregation result and shows that we can write the island level equilibrium in deviations from these aggregates.

**Lemma 2** For given \( \{z_{kt}, \gamma_{kt}, \eta_{kt}\} \), the behavior of \( \{w_{kt}, n_{kt}, b_{kt}, p_{kt}, c_{kt}, x_{kt}\} \) in the log-linearized economy for each island in log-deviations from aggregates is identical to that of a small open economy where

---

\(^{19}\)Given that the broad industrial composition at the state level does not differ much across states, the assumption that productivity parameters are roughly similar across states is not dramatically at odds with the data.

\(^{20}\)The model we presented has many islands subject to idiosyncratic shocks that cannot be fully hedged because asset markets are incomplete. By log-linearizing the equilibrium we gain in tractability, but ignore these considerations and the aggregate consequences of heterogeneity. As usual, the approximation will be a good one as long as the underlying volatility of the idiosyncratic shocks is not too large. If our unit of study was an individual, as for example in the precautionary savings literature with incomplete markets, the use of linear approximations would likely not be appropriate. However, since our unit of study is an island the size of a state I believe this is not too egregious of an assumption. The volatilities of key economic variables of interest at the state level are orders of magnitude smaller than the corresponding variables at the individual level.

\(^{21}\)When \( \Phi_0 > 0 \) this will be enough to induce stationary of island level variables in log-deviations from the aggregate. At the same time, since \( \Phi(\cdot) \) depends only on these deviations, the aggregate equilibrium will feature a constant endogenous discount factor \( \rho \).
the price of intermediates and the nominal interest rate are at their steady state levels, i.e. \( \bar{q}_t = \bar{i}_t = 0 \) \( \forall t \).

**Proof.** See Appendix C for a proof. ■

### 5.2 Transfer Policy in Alternative Models of Fiscal Unions

The previous section described a model where many islands (I will call them states from now on) were fiscally integrated. I showed that to a first order approximation we can study each of these states separately as if they were small open economies that receive transfers from abroad. In this section, I ask: how would the equilibrium change if the transfer policy rule changed? In particular, the case when transfers are not a function of local variables \( \theta_n = \theta_w = \theta_b = 0 \) is of interest because it correspond to the case when the states are not fiscally integrated. By comparing the equilibrium of the small open economy with and without fiscal integration, it is possible to analyze the contribution of fiscal unions to regional stabilization. As a reminder, Lemma 2 implies that the following equations are sufficient to characterize the equilibrium dynamics of wages, employment and assets at the state-level in log-deviations from the aggregate. For simplicity, I will drop the \( k \) notation indicating a given island from now on.

\[
0 = \mathbb{E}_t \left[ \frac{\sigma \beta}{\alpha \phi} (w_{t+1} + n_{t+1} - w_t - n_t) + (\alpha + \beta - 1) (n_{t+1} - n_t) \right. \\
\left. + (\beta - 1) (w_{t+1} - w_t) + \left(1 + \frac{\sigma}{\alpha \phi \phi} \right) (z_{t+1} - z_t) + \Phi_0 n_{kt} + \Phi_1 n_{kt-1} - \gamma_{t+1} \right] \tag{Euler}
\]

\[
0 = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_t - \beta w_t - z_t \tag{Labor Market (1)}
\]

\[
0 = -\frac{B}{S} b_t + \frac{B}{S} (1 + r) b_{t-1} - \frac{X}{S} (w_t + n_t) + \theta_n n_t + \theta_w w_t + \theta_b b_{t-1} + \frac{1}{S} \eta_t \tag{SB}
\]

Together with the exogenous processes

\[
\begin{align*}
    z_t &= \rho z_{t-1} + \sigma z v_t \\
    \gamma_t &= \rho \gamma_{t-1} + \sigma \gamma u_t^r \\
    \eta_t &= \rho \eta_{t-1} + \sigma \eta u_t^r
\end{align*}
\]

If parameters in this system of equations are calibrated or estimated, and the system is solved for the equilibrium, it is straightforward to construct alternative equilibria for different transfers rules simply by changing \( \{ \theta_n, \theta_w, \theta_b \} \). However, this would give an answer to the contribution of fiscal integration to regional stabilization for this particular model I described. One could imagine alternative models of fiscal unions that are reasonable a priori. For example, one could entertain the possibility that nominal wages are rigid and use a partial-adjustment model where a fraction \( \lambda \) of the gap between the actual and frictionless wage is closed every period. Formally,
this implies a different (Labor Market) equation

$$\beta \frac{1 - \lambda}{\lambda} (w_t - w_{t-1}) = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_t - \beta w_t - z_t \quad \text{(Labor Market (2))}$$

A similar specification has been used by Shimer (2010) and, more recently, by Beraja et al. (2015) and Midrigan and Philippon (2011). Shimer (2010) argues that in labor market search models there is typically an interval of wages that both the workers are willing to accept and firms willing to pay. To resolve this wage indeterminacy he considers a wage setting rule that is a weighted average of a target wage and the past wage. The target wage in this case is the value of the marginal rate of substitution.

Alternatively, one could entertain the possibility that employment is durable as in Kydland and Prescott (1982). Formally, this implies a different (Labor Market) equation

$$\nu n_{t-1} = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_t - \beta w_t - z_t \quad \text{(Labor Market (3))}$$

where parameter $\nu$ governs how much one-period lagged employment choices affect current utility. Moreover, this implies a different Euler equation as well because the marginal utility of consumption is affected.

$$0 = E_t \left[ \frac{\sigma \beta}{\alpha \phi (1 + \phi) - 1} (w_{t+1} + n_{t+1} - w_t - n_t) + (\alpha + \beta - 1)(n_{t+1} - n_t) + (\beta - 1)(w_{t+1} - w_t) ight. \\
+ (1 + \frac{\sigma}{\alpha \phi (1 + \phi) - 1})(z_{t+1} - z_t) + \left. \left( \Phi_0 - \frac{\sigma \alpha \phi}{(\alpha \phi (1 + \phi) - 1)} \right) n_{kt} + \left( \Phi_1 + \frac{\sigma \alpha \phi}{(\alpha \phi (1 + \phi) - 1)} \right) n^{-1}_{kt} - \gamma_{t+1} \right]$$

(Euler (3))

Note that one could entertain other models with ever increasing complexity as well. To name a few, models with alternative utility functions (e.g., GHH preferences, CRRA or CARA preferences, with and without habits or durability in consumption and employment, etc.), adjustment costs for assets, debt-elastic interest rates, price and wage rigidities both forward- and backward-looking, other shocks (e.g., labor wedge shocks). The last two alternative models are particularly interesting because they both amplify the response of employment to shocks relative to response of wages. This allows them to match the large employment volatility relative to wage volatility that we observe in actual economies. In this sense, some of their positive implications are observationally equivalent. I will exploit this lack of identification when constructing counterfactual equilibria for alternative transfer policy rules. I will argue that these and other models belong to a set that is indeed identified by their vector autoregression representation to the equilibrium, even if particular models are not, and that counterfactual equilibria are identical in all these cases.
models. The next section presents an example in this spirit. However, it is important to keep in mind that normative implications may be quite different, for example, between models (2) and (3) because of the presence of nominal wage rigidities in the former. This cautions against the use of observationally equivalent models for making normative statements, and it teaches us about when more data is needed to separately identify models in order to make such statements.

5.2.1 Different mechanisms, same counterfactual...

The purpose of this section is to show that some fiscal union models with very different economic mechanisms may nevertheless be observationally equivalent given data on employment, wages and assets alone, and produce identical counterfactual equilibrium for alternative transfer policy rules. This example is useful to gain some economic intuition before proceeding to the next section, which is more technical in nature and presents the main theoretical result for constructing Robust Transfer Policy Counterfactuals.

I consider a model with both wage rigidities and durability in employment—combining models (2) and (3) from the previous section. For simplicity, I focus on the special case where the exogenous processes are not persistent ($\rho_{\gamma} = \rho_z = \rho_\eta = 0$). First, consider a benchmark parameterization $\xi^0 \equiv \{\lambda^0, \nu^0, \ldots\}$ and transfer policy $\{\bar{\theta}_{n}, \bar{\theta}_{w}, \bar{\theta}_{b}\}$. The equilibrium $\{n^0_t, w^0_t, b^0_t\}$ can be written in recursive form as,

$$
\begin{bmatrix}
n^0_t \\
w^0_t \\
b^0_t
\end{bmatrix} = \rho^0 \begin{bmatrix}
n^0_{t-1} \\
w^0_{t-1} \\
b^0_{t-1}
\end{bmatrix} + \Lambda^0 \begin{bmatrix}
u^0_t \\
u^0_t \\
u^0_t
\end{bmatrix}
$$

where $\rho^0, \Lambda^0$ are functions of structural parameters and the transfer policy rule. Second, consider an alternative parameterization $\xi^1$. Is it possible to find such parameterization that is consistent with $\{\rho^0, \Lambda^0\}$? The answer is yes, as the following claim shows, and it implies that the parameters are not identified given the recursive representation of the equilibrium. In other words, there are many models that are observationally equivalent.

**Claim 2** Given $\{\rho^0, \Lambda^0\}$, there is a parameterization $\xi^1$ such that $\rho^1 = \rho^0$ and $\Lambda^1 = \Lambda^0$. Moreover, $\{\lambda^1, \nu^1\}$ are such that $\lambda^1_{1-\lambda^0} \nu^1 = \lambda^0_{1-\lambda^0} \nu^0$ for any $\xi^0$.

**Proof.** See Appendix D.

The intuition for this observational equivalence result is the following. Higher employment durability increases the elasticity of employment to shocks relative to the response of wages. Analogously, higher wage rigidity decreases the elasticity of wages to shocks relative to the response of employment. **Claim 2** formalizes this intuition by showing that models with high rigidity and low durability are observationally equivalent to models with low rigidity and high durability. However, the economic mechanisms that bring about variation in wages, employment and assets are very different.
Finally, note that the degrees of durability and rigidity only affect marginal decisions (i.e., the (Euler) and (Labor Market) equations) and not available resources (i.e., the (SB) equation). On the contrary, the transfer policy rule only affect available resources and not marginal decisions. This implies that models with different degrees of durability and rigidity will have identical (Euler), (Labor Market) and (SB) equations for any policy and any parameterization consistent with Claim 2. Hence, given a benchmark policy and parameterization that produce the "observed" benchmark equilibrium, these observationally equivalent models will have identical counterfactual equilibrium for any alternative transfer policy rule.

6 Robust Transfer Policy Counterfactual in Fiscal Union Models

In this section, I show how to identify sets of fiscal union models that are observationally equivalent under fiscal integration and yield identical counterfactual equilibrium in an economy that is not fiscally integrated. I start by describing models of fiscal unions in broader terms than what I have been doing so far, and then apply results from Section 3 to construct Robust Transfer Policy Counterfactuals.

6.1 A set of linear fiscal union models

I will focus on models that satisfy three properties. Examples of models with some of these properties include Farhi and Werning (2014), Nakamura and Steinsson (2014), Beraja et al. (2015), and Evers (2015). In particular, the models in the previous sections satisfy them all.

Assumption 4 Models of fiscal unions satisfy the following three properties:

1. **Transfer policy rule**: Tax-and-transfer system can be summarized as federal lump-sum transfers that are a function of state-level economic variables.

2. **Linear aggregation**: State-level economies in log-deviations from the aggregate union behave to a first-order approximation as if they were small open economies—indeed of other states.

3. **3 by 3**: Employment $n_t$, nominal wages $w_t$, and assets $b_t$; and exogenous processes $\{\gamma_t, \eta_t, z_t\}$ are sufficient variables for characterizing the state-level equilibrium in log-deviation from aggregates.

Property 1 excludes models where the tax-and-transfers system affects decisions at the margin, as it would be the case with distortionary taxation. In Section 7.7 I relax this assumption and discuss robustness of results. Property 2 excludes from the analysis models where member states are inherently different because of industrial composition or household preferences, for example, and/or exogenous processes correlation structures across states are such that idiosyncratic shocks do not average out. In property 3, assets might encompass both non-state-contingent nominal bonds and certain types of tradable physical capital. What is important for the property to hold is that no other variables that are necessary to describe the equilibrium are left out (e.g., other endogenous or exogenous state variables in the model).
Given the description of models of fiscal unions above, the system of matrix equations that characterizes the equilibrium in one island in log-deviations from the aggregate is written below. Without loss of generality, I will say that the first equation is the (Euler) equation in these models, the second is the (Labor Market) equation, and the third is the sequential budget constraint (SB) equation.

\[
0 = (F + \Theta_f)E_t \begin{bmatrix} n_{t+1} \\ w_{t+1} \\ b_{t+1} \end{bmatrix} + (G + \Theta_c) \begin{bmatrix} n_t \\ w_t \\ b_t \end{bmatrix} + (H + \Theta_p) \begin{bmatrix} n_{t-1} \\ w_{t-1} \\ b_{t-1} \end{bmatrix} + LE_t \begin{bmatrix} \gamma_{t+1} \\ z_{t+1} \\ \eta_{t+1} \end{bmatrix} + M \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix}
\]

\[
0 = - \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix} + N \begin{bmatrix} \gamma_{t-1} \\ z_{t-1} \\ \eta_{t-1} \end{bmatrix} + \Sigma \begin{bmatrix} u_i^{\gamma} \\ u_i^z \\ u_i^\eta \end{bmatrix} 
\]

(FiscalSME)

I will say that a particular model of fiscal unions is characterized by the structure \( \xi \) and the policy \( \Theta \), where \( \xi \equiv \{F, G, H, L, M, N, \Sigma\} \) are policy-invariant matrices, and \( \Theta \equiv \{\Theta_f, \Theta_c, \Theta_p\} \) are matrices that characterize endogenous transfer policy rule. \( \Theta_f \) contains the policy parameters associated with future expected variables, \( \Theta_c \) with contemporaneous variables, and \( \Theta_p \) past variables. The system evaluated at \( \Theta = 0 \) corresponds to a benchmark economy without a transfer policy in place.

Given Property 1 in Assumption 4, I will assume that the transfer policy is the same as in the models of fiscal unions from the previous section. Since the third equation is the sequential budget constraint, this implies:

**Assumption 5** The policy \( \Theta \) is:

\[
\Theta_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \theta_n & \theta_w & 0 \end{bmatrix}, \quad \Theta_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \theta_b \end{bmatrix}, \quad \text{and} \quad \Theta_f = 0_{3,3}
\]

6.2 Equilibrium representations

The description of models directly in terms of their equilibrium conditions is already a step forward towards constructing Robust Transfer Policy Counterfactuals. The primitives or micro-foundations of such models need not be specified. Two models with different primitives but that have the same structure and policy in (FiscalSME) are equivalent for my purposes.

**Assumption 6** \( \xi, \Theta \) are such that the system (FiscalSME) is stabilizable.

Under Assumption 6, we can derive a stable recursive solution to (FiscalSME) using the method of undetermined coefficients, which can be written as:

\[
\begin{bmatrix} n_t \\ w_t \\ b_t \end{bmatrix} = P(\xi, \Theta) \begin{bmatrix} n_{t-1} \\ w_{t-1} \\ b_{t-1} \end{bmatrix} + Q(\xi, \Theta) \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix}
\]

(FiscalRR)
Assumption 7. \( \xi, \Theta \) are such that \( Q(\xi, \Theta) \) is a non-singular square matrix.

Claim 3. If Assumptions 6 and 7 hold, there is a structural vector autoregression (SVAR) representation to the solution (RR) of the form:

\[
\begin{bmatrix}
 n_t \\
 w_t \\
 b_t
\end{bmatrix} = \rho_1(\xi, \Theta) \begin{bmatrix}
 n_{t-1} \\
 w_{t-1} \\
 b_{t-1}
\end{bmatrix} + \rho_2(\xi, \Theta) \begin{bmatrix}
 n_{t-2} \\
 w_{t-2} \\
 b_{t-2}
\end{bmatrix} + \Lambda(\xi, \Theta) \begin{bmatrix}
 u^t_n \\
 u^t_w \\
 u^t_b
\end{bmatrix}
\]

(FiscalSVAR)

where \( \rho_1(\xi, \Theta) \equiv P(\xi, \Theta) + Q(\xi, \Theta) N Q(\xi, \Theta)^{-1}; \rho_2(\xi, \Theta) \equiv (P(\xi, \Theta) - \rho_1(\xi, \Theta)) P(\xi, \Theta) \) and \( V(\xi, \Theta) \equiv Var(\Lambda(\xi, \Theta) u_t) = \Lambda(\xi, \Theta) \Lambda(\xi, \Theta)' \), where \( \Lambda(\xi, \Theta) \equiv Q(\xi, \Theta) \Sigma \).

Proof. We can write past exogenous processes as functions of past endogenous variables by inverting \( Q(\xi, \Theta) \). Next, replace this and the law of motion for the exogenous states into (FiscalRR) to obtain (FiscalSVAR) representation. ■

6.3 Robust Transfer Policy Counterfactuals

This section describes conditions under which it is possible to construct an equilibrium with \( \theta_n = \theta_w = \theta_b = 0 \) in a set of observationally equivalent models of fiscal unions. I begin by considering models of fiscal unions that can be written as in (FiscalSME) and satisfy Assumptions 5 to 7. These models are completely described by structure \( \xi \) and policy \( \Theta \).

Next, I say that a set of the models is characterized by semi-structure \( \xi^s \subseteq \xi \). In this application, I will consider only semi-structures that take the form of exclusion restrictions, i.e., semi-structures that are described by certain elements in \( \xi \) being zeros. For example, one such exclusion restriction is that assets \( b_t \) do not enter in the (Labor Market) equation. Section 3 generalizes the semi-structures to linear restrictions on the elements of \( \xi \).

Then, I ask: what are the sets of models of fiscal unions that are (i) consistent with a (FiscalSVAR) generated by a given \( \{\xi, \Theta\} \), and (ii) deliver the same counterfactual (FiscalSVAR') if the policy changes from \( \Theta \) to \( \Theta' \)? In particular, I am interested in \( \Theta' = 0 \) which corresponds to an economy without fiscal integration.

Proposition 3. Given \( \{\rho_1, \rho_2, \Lambda\} \) and \( \Theta \), if models of fiscal unions can be written as in (FiscalSME), satisfy Assumptions 5 to 7 and:

1. have identical (FiscalSVAR) representation \( \{\rho_1, \rho_2, \Lambda\} \) when policy is \( \Theta \).

2. have identical semi-structure \( \xi^s \), described by at least 5 zeros in each equation in (FiscalSME).

then counterfactual (FiscalSVAR') representation \( \{\rho_1(\xi^*, \Theta'), \rho_2(\xi^*, \Theta'), \Lambda(\xi^*, \Theta')\} \) is identical for all these models. Moreover, this Robust Transfer Policy Counterfactual can be constructed with knowledge of \( \{\rho_1, \rho_2, \Lambda, \Theta, \Theta'\} \) alone.
Proof. Apply the general results from Proposition 1 to the special case where there are 3 variables in (SME) and we only consider linear restrictions that take the form of exclusion restrictions. ■

The proposition is not obvious. This is because the structural vector autoregression representation is found as a solution to a highly non-linear system of equations given a structure and policy \( \{ \xi, \Theta \} \). Specifying a particular model, parameterizing it, and solving this system for the equilibrium under alternative policies is the standard way of evaluating them. However, fixing a SVAR representation, the mapping is linear back to the structure \( \xi \) that generated it. This is the key insight that allows identification of a set of models that yield identical counterfactuals by specifying only 5 zeros per equation in (FiscalSME). The reason why 5 zeros are sufficient is because there are 14 unknown elements per equation in structure \( \xi \) and there are 9 known elements per equation in structural vector autoregression representation (FiscalSVAR). Since the mapping is linear, specifying 5 of these unknown elements is enough to infer the remaining 9 unknown elements in structure \( \xi \) given the 9 known elements in (FiscalSVAR) and \( \Theta \). In Section 3, I showed a general version of this proposition in linear dynamic stochastic models.

7 A Counterfactual United States Economy without Fiscal Integration

I start this section by describing all inputs to construct the Robust Transfer Policy Counterfactual using Proposition 3. I need to: (i) specify a semi-structure \( \xi^s \) describing a set of fiscal union models, (ii) estimate a regional VAR using data for a fiscal union, (iii) identify the impulse response matrix, and (iv) estimate the transfer policy rule. Next, I present primary findings from the counterfactual exercise. I conclude the section by discussing the sensitivity of these findings to alternative ways of specifying the transfer policy rule.

7.1 The sufficient semi-structure \( \xi^s \)

The zeros in the matrices below describe a set of fiscal union models by specifying a semi-structure \( \xi^s \) that is sufficient in the sense of Proposition 3.

Assumption 8 A set of fiscal union models that can be written as in (FiscalSME) is described by:

\[
F = \begin{bmatrix} f_{11} & f_{12} & 0 \\ f_{21} & f_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}; \\
G = \begin{bmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix}; \\
H = \begin{bmatrix} h_{11} & 0 & 0 \\ h_{21} & h_{22} & 0 \\ 0 & 0 & h_{33} \end{bmatrix}; \\
L = \begin{bmatrix} 1 & l_{12} & l_{13} \\ 0 & 1 & l_{23} \\ 0 & 0 & 0 \end{bmatrix}; \\
M = \begin{bmatrix} 0 & m_{12} & m_{13} \\ 0 & m_{22} & m_{23} \\ 0 & 0 & m_{33} \end{bmatrix}; \\
N = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ 0 & n_{22} & n_{23} \\ 0 & n_{32} & n_{33} \end{bmatrix}
\]

The absence of expected and lagged terms beside assets makes the third equation consistent with most log-linearized, incomplete market models that include a sequential budget constraint (SB). Moreover, I assume that the only exogenous shifter in the sequential budget constraint is
"wealth" process $\eta_t$. The other two exogenous processes do not appear in the sequential budget constraint, which further restricts the set models that I analyze. Finally, it is assumed that the only exogenous shifters are "wealth" shocks, and the other two exogenous processes do not appear in the sequential budget constraint. In terms of the (Euler) and (Labor Market) equations—the first and second equations—I assume that (i) assets (future, contemporaneous, or lagged) $\{b_{t+1}, b_t, b_{t-1}\}$ do not appear in them, (ii) lagged wages $w_{t-1}$ does not appear in the first equation, (iii) contemporaneous "discount rate" process $\gamma_t$ do not shift these equations, and (iv) future "discount rate" process $\gamma_{t+1}$ do not appear in the second equation. Finally, I assume that past "discount rate" process $\gamma_{t-1}$ does not cause movements in "productivity" $z_t$ and "wealth" $\eta_t$ as is evidenced by autoregressive matrix $N$. These assumptions are consistent with all models of fiscal unions described in Section 5.2 as well as many others. The key feature of models described by this semi-structure is that the (Labor Market) and (Euler) equations dependence on future and lagged variables is relatively unconstrained, as well as the exogenous processes and their correlation structure. This is important for the question of regional stabilization in fiscal unions because it means that this set encompasses many models with rich features.

### 7.2 Identifying $\Lambda(\xi, \Theta)$ and structural shocks

An important input in the construction of Robust Policy Counterfactuals is the impulse response matrix $\Lambda(\xi, \Theta)$. The literature proposes myriad ways to identify it, ranging from simple ordering assumptions to more sophisticated sign and long-run restrictions. I follow Lemma 1, which extends the methodology proposed in Beraja et al. (2015). In essence, the procedure uses elements of theory to identify underlying shocks in a VAR. These theoretical restrictions imply a series of particular linear restrictions linking the reduced form errors to the structural shocks. I use the sequential budget constraint (SB) to generate these theoretical restrictions because, as I argued in Section 5, many fiscal union models are consistent with it.\footnote{In Beraja et al. (2015), we used a different equation to identify aggregate shocks—a wage-setting equation.}

The first step in the procedure consists of estimating the reduced form (FiscalSVAR) to obtain the autoregressive matrices $\{\rho_1, \rho_2\}$, and the reduced form errors covariance matrix $V$. Second, derive identification restrictions that will allow us to infer $\Lambda$ and the shocks.\footnote{The shocks are identified simply by multiplying the matrix of reduced form errors from the estimated VAR by the inverse of the matrix $\Lambda$.} Applying the conditional expectation operator $\mathbb{E}_{t-1}(\cdot)$ on both sides of the (SB) and constructing the reduced form expectational errors, we obtain:

$$0 = \begin{bmatrix} g_{31} + \vartheta_n & g_{32} + \vartheta_w & g_{33} \end{bmatrix} \Lambda \begin{bmatrix} u_t^\gamma \\ u_t^z \\ u_t^\eta \end{bmatrix} + m_{33} \sigma_{33} u_t^\eta \quad (\text{Id1})$$

This equation must hold for all realizations of the shocks. Whenever there is an innovation to $u_t^\gamma$ or $u_t^\eta$ or $u_t^z$, employment, wages, and debt must co-move on impact in a way that satisfies...
this linear relationship. Hence, it gives us two linear restrictions in the second and third columns' elements of $\Lambda$ for a given parameterization of (SB) when there are either contemporaneous $u_t^\gamma$ or $u_t^\eta$ shocks. These linear restrictions are the same as those implied by equation (6) in Section ??.

Similarly, constructing $\mathbf{E}_{t-1}(\cdot) - \mathbf{E}_{t-2}(\cdot)$, we obtain:

\[
0 = \left( \begin{bmatrix} \varrho_{31} + \vartheta_n & \varrho_{32} + \vartheta_w & \varrho_{33} \end{bmatrix} \right) \begin{bmatrix} \rho_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & h_{33} + \vartheta_b \end{bmatrix} \begin{bmatrix} u_{t-1}^\gamma \\ u_{t-1}^\eta \\ u_{t-1}^\eta \end{bmatrix} + m_{33} \sigma_{33} n_{33} u_{t-1}^\eta + m_{33} \sigma_{33} n_{32} u_{t-1}^\eta
\]

(Id2)

This gives us one extra linear restriction in the second column's elements of $\Lambda$ for a given parameterization of (SB) when there are $u_{t-1}^\gamma$ shocks. These three linear restrictions, combined with six non-linear restrictions coming from the orthogonalization of the shocks, are sufficient to identify all nine elements in $\Lambda$. Intuitively, equation (Id1) separates the "wealth" shock from the other two shocks. If the unexpected component of employment, wages, and assets does not co-move in the linear way implied by equation (Id1), when $u_{t-1}^\eta = 0$, a "wealth" shock must have occurred. Analogously, equation (Id2) separates "discount rate" and "productivity" shocks. If the unexpected component of employment, wages, and assets does not co-move in the linear way implied by equation (Id2), when $u_{t-1}^\gamma = u_{t-1}^\eta = 0$, a "discount rate" shock occurred. For completeness, matrix $\Lambda$ solves the system:

\[
\begin{bmatrix} \varrho_{31} + \vartheta_n & \varrho_{32} + \vartheta_w & \varrho_{33} \end{bmatrix} \Lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
\left( \begin{bmatrix} \varrho_{31} + \vartheta_n & \varrho_{32} + \vartheta_w & \varrho_{33} \end{bmatrix} \rho_1 + \begin{bmatrix} 0 & 0 & h_{33} + \vartheta_b \end{bmatrix} \right) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0
\]

\[
V = \Lambda \Lambda^t
\]

This is the same system implied by Lemma 1 for this particular semi-structure.

### 7.3 Data description

I exclude Alaska, District of Columbia, and Hawaii from analysis, leaving 48 observations (one for each remaining state) per year, and 6 years (2006-2011) of data.

To make state-level nominal wages indices, I use data from the 2000 US Census and the 2001-2012 American Community Surveys (ACS)\(^{24}\).

The 2000 Census includes 5 percent of the US population. The 2001-2012 ACS's include

\(^{24}\)I access the data through the IPUMS-USA website https://usa.ipums.org/usa/. See Ruggles et al. (1997).
approximately 600,000 respondents between 2001-2004, and about 2 million after 2004. The large sample sizes allow detailed labor market information at the state level. I begin by using the data to make individual hourly nominal wages. I restrict the sample to only individuals who are employed, who report usually working at least 30 hours per week, and who worked at least 48 weeks during the prior 12 months. For each individual, I divide total labor income earned during the prior 12 months by a measure of annual hours worked during the prior 12 months. The composition of workers differs across states and within a state over time, which might explain some variation in nominal wages across states over time. To account for this, I run the following regression:

$$\ln(w_{itk}) = K_t + \Gamma_t X_{itk} + u_{itk}$$

where \(\ln(w_{it})\) is log-nominal wages for household \(i\) in period \(t\) residing in state \(k\), and \(X_{itk}\) is a vector of household specific controls. The vector of controls include a series of dummy variables for usual hours worked (30-39, 50-59, and 60+), a series of five-year age dummies (with 40-44 being the omitted group), 4 educational attainment dummies (with some college being the omitted group), three citizenship dummies (with native born being the omitted group), and a series of race dummies (with white being the omitted group). I run these regressions separately for each year such that both constant \(K_t\) and the vector of coefficients on the controls, \(\Gamma_t\), can differ for each year. I then take the residuals from these regressions for each individual, \(u_{itk}\), and add back constant \(K_t\). Adding back the constant from the regression preserves differences over time in average log wages. To compute average log wages within a state, holding composition fixed, I average \(u_{itk} + K_t\) across all individuals in state \(k\). I refer to this measure as the demographically adjusted, log-nominal wage in time \(t\) in state \(k\).

The measure of employment at the state level is the employment rate for each state, calculated using data from the US Bureau of Labor Statistics. The BLS reports annual employment counts and population numbers for each state and year. I divide employment counts by population to make an annual employment rate measure for each state.

Data on federal transfers net of taxes paid come from the Bureau of Economic Analysis. Transfers include retirement and disability insurance benefits, medical benefits, income maintenance benefits, unemployment insurance compensation, veterans benefits, federal education and training assistance, and other transfer receipts of individuals from governments. Federal taxes are the sum of personal income taxes that are withheld, usually by employers, from wages and salaries, quarterly payments of estimated taxes on income that is usually not subject to withholding, and final settlements, which are additional tax payments made when tax returns for a year

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25Total labor income during the prior 12 months is the sum of both wage and salary earnings and business earnings. Total hours worked during the previous 12 months is a multiple of total weeks worked during the prior 12 months and the respondents’ reports of their usual hours worked per week. For some years, bracketed reports are provided for weeks worked during the prior 12 months, and the usual hours per week worked. In those cases, I take the midpoint of the brackets.

26I access the data through the BEA website on regional GDP and personal income: [http://www.bea.gov/iTable/index_regional.cfm](http://www.bea.gov/iTable/index_regional.cfm)
are filed, or as a result of audits by the Federal Government\footnote{Excise, Medicare and Social security federal taxes are not included in this measure.}

Given the unavailability of official state-level data on asset positions, I construct a measure of state-level assets as the sum of physical and financial assets. From national account identities, we can derive the law of motion for assets $B_t$ in a given state as:

$$
B_t = B_{t-1}(1 + r_t) + Y_t - C_t + S_t - G^{local}_t + v_t
$$

where $Y_t$ is nominal gross domestic product, $C_t$ is private consumption expenditures, $S_t$ are net transfers (i.e., expenditures minus taxes) from the federal government, $G^{local}_t$ are expenditures from the local government, and $r_{t-1}$ captures the change in asset valuation between $t - 1$ and $t$. Finally, error term $v_t$ includes income receipts from abroad minus income payments to foreigners, federal government expenditures not counted as federal transfers (e.g., salaries and wages), and differences in returns between physical and financial assets for which no data are available\footnote{Error term $v_t$ accounts for most of the “wealth” exogenous process. The remainder is the error term $e_t$ in the difference between observed net transfers and estimated policy rule in equation Section 7.4.}

I obtain $Y_t$ and $C_t$ directly from the Bureau of Economic Analysis website. $S_t - G^{local}_t$ also comes from several variables in the BEA. I calculate it as (personal current transfers receipts) - (personal current taxes paid + taxes on production and imports net of subsidies).\footnote{As long as local government expenditures plus transfers are close enough to local tax revenues (i.e., local governments have a nearly balanced budget), the calculation is accurate. If not, the difference is absorbed by error term $e_t$.}

The revaluation of assets term $r_t$ is obtained residually to ensure that the growth rate of the sum of local assets across states is consistent with the growth rate of aggregate net worth in the US economy. Having all components in the law of motion for $B_t$, I calculate assets at each point in time for each state simply by iterating forward with 2006 as the initial observation. I obtain initial assets in 2006 by aggregating at the state level, the zip code total net worth data from Mian et al. (2013). In order to construct financial assets at the zip code level\footnote{They use data on dividends and interest income from the IRS Statistics of Income (SOI). They assume that households hold identical shares of stocks and bonds (they hold the market index portfolio). Given the share of total dividends and interest income received by a zip code they can construct the share of total stocks and bonds held by that zip code. Then, they total financial assets from the Federal Reserve’s Flow of Funds data to zip codes based on these shares. For the value of nominal debt owed by households they use data based on information from Equifax Predictive Services. Then they match the Federal Reserve Flow of Funds data by using the share of Equifax total debt in a zip code to allocate Flow of Funds debt. The final component of the asset measure is the value of housing wealth which they estimate using the 2000 Decennial Census data. They construct total home value as of 2000 in a zip code as the product of the number of home owners and the median home value. Then, they project it forward into later years using the CoreLogic zip code level house price index and an aggregate estimate of the change in homeownership and population growth.} Mian et al. (2013), they use data on dividends and interest income from the IRS Statistics of Income (SOI). They assume that households hold identical shares of stocks and bonds (they hold the market index portfolio). Given the share of total dividends and interest income received by a zip code they can construct the share of total stocks and bonds held by that zip code. Then, they total financial assets from the Federal Reserve’s Flow of Funds data to zip codes based on these shares. For the value of nominal debt owed by households they use data based on information from Equifax Predictive Services. Then they match the Federal Reserve Flow of Funds data by using the share of Equifax total debt in a zip code to allocate Flow of Funds debt. The final component of the asset measure is the value of housing wealth which they estimate using the 2000 Decennial Census data. They construct total home value as of 2000 in a zip code as the product of the number of home owners and the median home value. Then, they project it forward into later years using the CoreLogic zip code level house price index and an aggregate estimate of the change in homeownership and population growth.} I calculate it as (personal current transfers receipts) - (personal current taxes paid + taxes on production and imports net of subsidies).\footnote{As long as local government expenditures plus transfers are close enough to local tax revenues (i.e., local governments have a nearly balanced budget), the calculation is accurate. If not, the difference is absorbed by error term $e_t$.}
7.4 Estimating the transfer policy rule

Figure 1 shows a scatter plot of net federal transfers growth (direct federal transfers minus federal taxes growth) between 2006 and 2010 against nominal wage income growth (wage plus employment growth) between 2006 and 2010 in the United States. There is a very strong, negative relationship between the two. In particular, a one percentage point increase in nominal wage income associates with a 1.25 percentage point decrease in net transfers. If the tax-and-transfer system helps stabilize regional economies, it is because a state whose economy temporarily worsens relative to the average receives some temporary relief through transfer payments or lower tax payments to the federal government—a type of insurance against temporary negative shocks. The opposite is true for states whose economies are in a relative boom. The negative relationship is a consequence of both the progressivity of the tax system in the United States and the presence of automatic stabilizers like unemployment insurance, and particularly during this period, federal emergency unemployment compensation and food stamps.

Figure 1: Net Transfers growth v. Wage Income growth (2006-2010)

![Scatter plot](image)

Note: Figure shows a simple scatterplot of the log-growth of net transfers in a state between 2006 and 2010 against nominal wage income log-growth during the same period. See text for details of variable construction. The population size in 2006 of the underlying state is represented by the size of the circle in the figure. The line represents a weighted regression line from the bivariate regression, where the weights are the population in 2006.

30 See Section 7.3 for a detailed description of the construction of these data.
31 If agents understand that federal emergency unemployment compensation would always be passed in situations of economic malaise, it is correct to include them as part of the implicit net federal transfer policy rule in place in a fiscal union.
As a reminder, the transfer policy rule is:

\[ s_t = \theta_n n_t + \theta_w w_t + \theta_b b_{t-1} + e_t \]

For regional data to be used to estimate \( \Theta \), one of the following must hold: (1) the innovations to the policy rule have no regional component \( (e_t = 0) \)—in which case, a simple OLS regression produces consistent estimates—or (2) valid instruments can be found that isolate movements in \( n_t, w_t, b_{t-1} \) that are orthogonal to \( e_t \). The issue of endogeneity arises because of reverse causality. When the innovation in the policy rule is part of the "wealth" shock \( u_t^\eta \), employment and wages both cause and are caused by net transfers in the equation above. To deal with the endogeneity of \( n_t, w_t, b_{t-1} \), I proceed variously. First, I estimate a regression of net transfers onto nominal wage income alone (assuming \( \theta_w = \theta_n \)) using house price growth between 2006 and 2010 as an instrument. This accords with many recent papers, including Mian and Sufi (2014). Contemporaneous housing price growth strongly predicts contemporaneous nominal wage income growth. The instrument is valid as long as local housing prices are orthogonal to the transfer policy rule shock, which appears plausible. In the second approach, I use "discount rate" and "productivity" shocks in 2008, estimated from (FiscalSVAR), as instruments for wages and employment. They are linear combinations of wages, employment, and assets in 2008 that are orthogonal to the "wealth" shock, and hence \( e_t \), by construction.

Table 3 in Appendix E presents results for several specifications. The dependent variable is the log-growth rate of transfers minus the growth rate of taxes between 2006 and 2010 for each state. The independent variables are the log-growth rate of nominal wages between 2006 and 2010 and the log-growth rate of employment between 2006 and 2010 in the first two columns, and the log-growth rate of assets between 2006 and 2009 in the third column. In the fourth column, the independent variable is the sum of wage and employment growth. The first line is a simple OLS regression. The second presents two-stage, least-squares results using the "discount rate" and "productivity" shocks in 2008 \( u_t^\gamma, u_t^z \). The third uses house price log-growth between 2006 and 2008 as an instrument instead. The fourth uses all three instruments. For all specifications, and when possible, I consider case (1) when \( b_{t-1} \) is not endogenous, and case (2) when \( b_{t-1} \) might be endogenous.

I find that the policy rule estimates have the expected sign and are significant in all specifications. They are also similar in magnitude, ranging from -1.3 to -1.6 for \( \theta_n \) and -0.9 to -1.4 for \( \theta_w \). Lagged assets have nearly no independent explanatory power for net transfers across all specifications. To give a sense of the magnitudes involved, when net transfers increase by 30 percent for every 1 percent decrease in nominal wage income, and the average income tax rate is 0.17, for every 1 dollar decrease in nominal wage income, a state receives 0.22 dollars in federal transfers. This result is similar to findings by Feyrer and Sacerdote (2013), who find a 0.25 decrease, and Bayoumi and Masson (1995), who find a 0.31 decrease.
7.5 Main Findings

I estimate the vector autoregression (FiscalSVAR) via weighted OLS where the weights are the 2006 population in the state, using data described in subsection 7.3. For each variable and year, I take the cumulative log-growth between 2006 and 2011 and express it in log-deviations from the average across states. I pool all data between 2006 and 2011, leaving 240 observations (5 years \( \times 48 \) states), and estimate common autoregressive coefficients \( \rho_1, \rho_2 \) and reduced form errors \( U \). I follow 7.2 to identify \( \Lambda \), setting \( \tilde{g}_{33} = \tilde{b} = 2.25 \) to match the median net worth to revenues ratio across states in the United States in 2006 and \( \vartheta_n = -1.6, \vartheta_w = -0.9, \vartheta_b = -0.03 \), which correspond to the OLS policy rule estimates in Table 3. Following Proposition 3, I construct a Robust Transfer Policy Counterfactual for \( \vartheta_n = \vartheta_w = \vartheta_b = 0 \) which corresponds to an economy without a federal transfer policy rule.

Figure 2 in Appendix E shows the impulse response functions of employment, wages, and assets to a one-standard-deviation "discount rate" shock \( \gamma \), both in the actual and the counterfactual economy without transfers. I find that employment and wages both decrease on impact, whereas assets increase in response to a "discount rate" shock. This accords with the theoretical impulse response functions in models in Section 5.2. As for the effects of fiscal integration, I find that amplification and persistence of "discount rate" shocks are mitigated by the transfer policy rule—the employment response (after two years) is -1.2 percent in the actual economy, whereas it is -2.1 percent in the counterfactual economy without transfers.

Table 4 presents moments of the employment distribution in the actual and counterfactual economies without transfers. The cross-state employment standard deviation in the US data in 2010, \( \sigma_{n}^{2010} \), was 2.6 percent (this corresponds to the first line and third column in the table, when all shocks are present). I then consider the following thought experiment. At the end of 2007, it is announced that from 2008 onwards the United States federal government will cease to give transfers and collect taxes to/from the states in the union. How would employment, wages, and assets have evolved were regional economies hit by the same sequence of shocks? I find that, absent a federal transfer policy rule, the standard deviation of employment in 2010 would have been 3.5 percent. The skewness of the distribution, \( sk_{n}^{2010} \), would have been -0.3 instead of -0.2. To give some context to these numbers, aggregate output volatility during the pre-Volcker period (1960:1 to 1979:2) was 2.7, whereas during the post-Volcker-disinflation period (1982:4 to 1996:4) volatility was 2.06. Much literature examines causes of this "Great Moderation". The consequences of the US federal tax-and-transfer system are in the same order of magnitude.

Results above imply that the federal tax-and-transfer system helped stabilize regional economies by redistributing resources from regions that were doing relatively well to regions that were doing relatively poorly. Figure 3 in Appendix E elaborates on this point, showing the employment gain (or loss) from fiscal integration for each state in 2010, where states are sorted according to

32 Mian and Sufi (2014) and Beraja et al. (2015), among others, argue that this type of shocks were key drivers of regional business cycles during the Great Recession in the United States.

33 These numbers come from Clarida et al. (2000).
Table 4: Employment statistics: Fiscal Integration v. Fiscal Autonomy

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$</th>
<th>$(\gamma, z)$</th>
<th>$(\gamma, z, \eta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{n}^{2010}$</td>
<td>$\Theta \neq 0$</td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>$\Theta = 0$</td>
<td>3</td>
<td>3.4</td>
</tr>
<tr>
<td>$sk_{n}^{2010}$</td>
<td>$\Theta \neq 0$</td>
<td>-0.26</td>
<td>-0.29</td>
</tr>
<tr>
<td></td>
<td>$\Theta = 0$</td>
<td>-0.39</td>
<td>-0.37</td>
</tr>
<tr>
<td>$\overline{\sigma}_{n}$</td>
<td>$\Theta \neq 0$</td>
<td>2.3</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>$\Theta = 0$</td>
<td>3.9</td>
<td>4.5</td>
</tr>
<tr>
<td>$\overline{sk}_{n}$</td>
<td>$\Theta \neq 0$</td>
<td>-0.03</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>$\Theta = 0$</td>
<td>-0.02</td>
<td>-0.07</td>
</tr>
<tr>
<td>$\sqrt{s_{n}(0)}$</td>
<td>$\Theta \neq 0$</td>
<td>1.5</td>
<td>6.3</td>
</tr>
<tr>
<td></td>
<td>$\Theta = 0$</td>
<td>5.5</td>
<td>9.7</td>
</tr>
</tbody>
</table>

Note: $\sigma_{n}^{2010}$ is the standard deviation of the distribution of employment $n_{t}$ across states in 2010 in percentages. $sk_{n}^{2010}$ is the skewness of the distribution in 2010. $\overline{\sigma}_{n}$ and $\overline{sk}_{n}$ are the standard deviation and skewness in the stationary distribution. $s_{n}(0)$ is the spectrum at zero frequency (the long-run variance of $n_{t}$). Line $\Theta \neq 0$ corresponds to the fiscal union economy, and line $\Theta = 0$ to results from the counterfactual. Column $\gamma$ corresponds to the case with only "discount rate" shocks, and column $(\gamma, z)$ to the case with "discount rate" and "supply shocks. Column $(\gamma, z, \eta)$ corresponds to the case with all shocks.

their employment in 2010 from lowest to highest. We observe that fiscal integration increased employment in states with the worst employment outcomes, and the opposite is true for states with the best employment outcomes.

In the first and second columns of the table, I calculate the same statistics if regional economies had been hit by only "discount rate" shocks or both "discount rate" and "productivity" shocks. Comparing the first and last columns in the first line, I find that most of the employment variation across states in 2010 is accounted for by "discount rate" shocks (approximately 90 percent). This accords with findings in Beraja et al. (2015), in which data, and particularly the identification strategy, are different. Similarly, the federal transfer policy rule reduced employment dispersion primarily by stabilizing regional "discount rate" shocks. This is evidenced by comparing the first column in the second line of the table to the other columns. Of the 0.9 volatility reduction, 0.7 is achieved because of stabilization of "discount rate" shocks, and only 0.2 because of "productivity" shocks.34

In the lower half of Table 4, I present Monte Carlo estimates of the standard deviation $\sigma_{n}$ and skewness $sk_{n}$ of employment in the stationary distribution. I construct them by sampling with replacement 1,000,000 observations from the empirical distribution of shocks, feeding them to the (FiscalSVAR) and calculating the corresponding statistic. In the last line of the table, I present an estimate of the square root of the long-run variance of employment. The long-run

34The volatility reduction of 0.9 is the difference between the actual and counterfactual volatilities of 3.5 and 2.6. Analogously, 0.7 is the difference between 3 and 2.3, and 0.2 is the difference between 3.4 minus 2.5 and 0.7.
variance is constructed as the diagonal element corresponding to employment in the spectrum at frequency zero of the multivariate (FiscalSVAR). Its square root is a common measure of the persistence in the series, as proposed by [Pesaran et al. (1993)]. It associates with the speed at which the impulse response function to a given set of shocks decays, and hence their persistence. Results for the reduction in stationary employment volatility are qualitatively similar to the ones during the Great Recession. Finally, I find that the persistence of employment increases from 7.8 to 10.9 in the counterfactual economy without a federal transfer policy rule. Interestingly, most of the persistence in employment in the fiscally integrated economy is due to "productivity" shocks. In the counterfactual economy, "discount rate" shocks generate much larger persistence in employment.

7.6 Channels of regional stabilization

To elucidate channels behind measured benefits from fiscal integration, I examine the contribution of what I define as a rigidity channel, which includes nominal and real rigidities in comparison to other broad frictions like asset market incompleteness and/or labor mobility costs. The idea is to construct an economy without nominal and/or real rigidities using the methodology from Section ??, and conduct the transfer policy counterfactual in this alternative economy. Then, follow a diff-in-diff approach to evaluate contributions of nominal/real rigidities to the benefits of fiscal integration by using differences across all four combinations; that is, an economy with rigidities and transfers \((r, s)\), rigidities and no transfers \((r, ns)\), no rigidities and transfers \((nr, s)\), and no rigidities and no transfers \((nr, ns)\). Any observation \(\omega\) can be written as:

\[
\omega = \beta_{nr, ns} + \beta_r \mathbb{I}_r + \beta_s \mathbb{I}_s + \beta_{r,s} \mathbb{I}_r \mathbb{I}_s
\]

where \(\mathbb{I}_r\) is an indicator variable that is turned on when rigidities are present, and \(\mathbb{I}_s\) is an indicator variable turned on when transfers are present. For example, in the previous section, I identify \(\beta_s + \beta_{r,s}\) for the employment distribution. This corresponds to the total difference between economies with and without transfers (the transfers counterfactual). The purpose of this section is to identify \(\beta_{r,s}\), which measures the contribution of nominal/real rigidities to the total difference in the transfers counterfactual \(\beta_s + \beta_{r,s}\) (i.e., the rigidity channel). Following diff-in-diff logic, we can identify \(\beta_{r,s}\) by comparing the transfers counterfactual in economies with and without rigidities. That is:

\[
\beta_{r,s} = (\omega_{r,s} - \omega_{r,ns}) - (\omega_{nr,s} - \omega_{nr,ns})
\]

To construct counterfactual economies without nominal or real rigidities, we can follow the same methodology that allows to construct Robust Policy Counterfactual. Given the semi-structure specified in Section 7.1, the second equation in (FiscalSME) is a combination of labor demand, labor supply, wage-setting, and/or price-setting schedules. This point was made in the aforementioned section. For the purpose of this counterfactual exercise, I further assume
that only the second equation in \([\text{FiscalSME}]\) is affected by parameters related to the degree of forward- and backward-lookingness in labor demand, price-setting, labor supply, or wage-setting schedules in the presence of nominal and real rigidities. Then, the methodology is also useful for defining a counterfactual economy in which there are no nominal and real rigidities. This counterfactual economy is consistent with models in which labor supply and demand are static. The way to construct such counterfactual economy is by imposing counterfactual policy matrices that cancel the forward- and backward-looking terms in the equation. Given the way we specified the semi-structure \(\zeta^s\), this is achieved by setting \(\Theta_f'_{21} = -f_{21}; \Theta_f'_{22} = -f_{22}; \Theta_p'_{22} = -h_{22}; \Theta_p'_{21} = -h_{21}\).

Table 5 presents results for the \(20^{th}\), \(50^{th}\), and \(80^{th}\) percentiles of the distribution of employment growth between 2008 and 2010. Figure 4 in Appendix E presents kernel density estimates for all four distributions used in the construction of Table 5 and summary statistics.

Table 5: Employment in the Great Recession: Channels Decomposition

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>(20^{th})</th>
<th>(50^{th})</th>
<th>(80^{th})</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rigidity channel</strong></td>
<td>0.4</td>
<td>-0.2</td>
<td>0</td>
</tr>
<tr>
<td><strong>Transfers counterfactual</strong></td>
<td>1.3</td>
<td>0</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

Findings for the Robust Transfer Policy Counterfactual are as mentioned in the previous section. The increase in employment (1.3 percent) because of fiscal integration in the bottom part of the employment distribution (below the 20th percentile) is about the same as the decrease (-1.1 percent) in the upper part (above the 80th percentile). The degree to which nominal and real rigidities amplify (or attenuate) the consequences of fiscal integration, the rigidity channel, is not the same across the employment distribution. I find that about one-third of the 1.3 percent employment increase because of fiscal integration in states with the worst employment outcomes during the Great Recession was due to this channel. On the other hand, the channel was irrelevant for states with the best employment outcomes, meaning that at least for these states, the employment reduction because of fiscal integration involved frictions other than nominal and real rigidities.

7.7 Sensitivity to alternative policy specifications

Results from the previous section are based on a particular transfer policy specification. As a reminder, the benchmark policy matrices were:

\[
\Theta_f = 0; \Theta_c = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\vartheta_n & \vartheta_w & 0
\end{bmatrix}; \Theta_p = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \vartheta_b
\end{bmatrix}
\]
where $\theta_n = -1.6, \theta_w = -0.9, \theta_b = 0.03$, which corresponds to the OLS estimates of the transfer policy rule in Table 3 in Appendix E.

First, we would like to evaluate the sensitivity of results to estimates of the transfer policy rule other than the benchmark OLS estimates. Second, the specification is consistent with households in each state receiving lump-sum transfers from the federal government because the only affected equation that characterizes the equilibrium is the sequential budget constraint (the third equation in this case). In practice, lump-sum transfers are rare, and instead, the federal government uses distortionary taxes. This section explores the sensitivity of the counterfactual exercise to both of these issues.

Table 6 in Appendix E shows the counterfactual employment standard deviation in an economy without transfers, both in 2010 and in the stationary distribution, for alternative initial policy parameterizations corresponding to the instrumental variable estimates in Table 3. Results are qualitatively similar to those reported in the previous section for the benchmark policy rule. Although quantitatively reduced somewhat for some of the parameterizations, gains from fiscal integration in terms of the reduction of the dispersion in employment across states remain large. The largest quantitative difference is for the case in which I restrict coefficients on employment and wages in the policy rule to be identical ($\theta_n = \theta_w = -1.1$). As the last column in the table shows, the counterfactual employment standard deviation in 2010 would have been 3.1 percent (instead of 2.6 percent in the data), and the counterfactual standard deviation in the stationary distribution is 4.5 percent (instead of 3.5). The counterfactual using the benchmark policy estimates instead resulted in counterfactual standard deviations of 3.5 and 4.9 percent, respectively.

Evaluating sensitivity to an alternative policy that accounts for the potentially distortionary effects of taxes is less straightforward. For simplicity, consider the transfer policy without lagged assets in log-deviations from the aggregate:

$$ s_t = \theta_n n_t + \theta_w w_t $$

This implies that tax rate $\tau_t$ per unit of nominal labor income $w_t + n_t$ in log-deviations from the aggregate can be written as:

$$ \tau_t = -(1 + \theta_n) n_t - (1 + \theta_w) w_t $$

The potential labor supply (or wage-setting) tax distortion relates to $\tau_t$, not total transfers $s_t$ for which we estimated elasticities $\theta_w, \theta_n$. If the federal tax-and-transfer system affects equilibrium equations beyond the sequential budget constraint, $(1 + \theta_w), (1 + \theta_n)$ would appear in these equations, not $\theta_w, \theta_n$.

I consider the case in which the second equation in (FiscalSME) is a wage-setting equation, as in Section 5.2. If the federal tax-and-transfer system is distortionary, and given semi-structure

35Since the estimated coefficient on lagged assets is so small, results are identical whether we include it.
ξ_s from Section 7.1 we can write it as:

\[ 0 = f_{21}E_t[n_{t+1}] + f_{22}E_t[w_{t+1}] + \left( g_{21} + \frac{\tau \theta_n}{1 - \tau \theta_n} (1 + \theta_n) \right) n_t + \left( g_{22} + \frac{\tau \theta_n}{1 - \tau \theta_n} (1 + \theta_w) \right) w_t \]

\[ + h_{21}n_{t-1} + h_{22}w_{t-1} + E_t[z_{t+1}] + I_{23}E_t[\eta_{t+1}] + m_{22}z_t + m_{23}\eta_t \]

The equation above accords with the tax rate that affects the target wage in the wage-setting equation by distorting the marginal rate of substitution. The distortion is given by terms \(1 + \theta_n\) and \(1 + \theta_w\). For example, the case \(\theta_w = \theta_n = -1\) is such that the tax schedule is flat (i.e., a proportional labor income tax). Due to the lack of curvature, it would not affect the island’s log-deviations of the marginal rate of substitution from the aggregate.

I construct a Robust Transfer Policy Counterfactual using this alternative policy specification, in which I set \(\bar{\tau} = 0.17\) to match the average tax rate in the US economy. Results from the previous section are unchanged because estimates of transfer policy rule \(\theta_n, \theta_w\) are close to \(-1\). Thus, policy-related terms that distort this equation are very small in absolute magnitude, in comparison with terms in the policy-invariant structure. To see this, consider the case in which the second equation is interpreted as a static labor supply equation, and the policy rule depends on employment alone: \(w_t = \left( -g_{21} - \frac{\tau \theta_n}{1 - \tau \theta_n} (1 + \theta_n) \right) n_t \). Plausible calibrations of labor supply Frisch elasticity \(-g_{21}\) are in the range 0.5 to 4.\(^{36}\) The policy-related term for the case when \(\bar{\tau} = 0.17, \theta_n = -1.6\) is -0.08, which is an order of magnitude smaller than the Frisch elasticity.

8 Conclusions

In this paper, I propose a methodology to analyze systematic policy changes in widely used models in macroeconomics with minimal \textit{a-priori} structural assumptions. In particular, I show how to construct policy counterfactuals that are both robust across many observationally-equivalent-models and immune to Lucas critique. While I have emphasized benefits and feasibility of the methodology, it also has some limitations when compared to fully structural or ”sufficient statistics” approaches. The most important is that it is an inherently \textit{linear} methodology. Counterfactual questions that involve non-linearities are not easily, or at all, accommodated.\(^{37}\) A further limitation is that the methodology does not directly lend itself to welfare comparisons or optimal policy analysis.

I apply the methodology to quantify how fiscal unions, through their federal tax-and-transfer system, contribute to regional stabilization. This quantitative question has received surprisingly little attention in the literature beyond reduced-form calculations and simple calibration exercises in specific models, despite existing theoretical work on fiscal unions and its relevance for current discussions about European fiscal integration. My primary quantitative finding is that during

\(^{36}\)See Chetty et al. (2011).

\(^{37}\)For example, in Beraja et al. (2016), we study monetary policy in an environment where the pass-through from posted interest rates to effective interest rates is influenced by collateralized lending, a type of lending that naturally introduces non-linearities.
the Great Recession fiscal integration significantly reduced cross-state employment differences by redistributing resources from states that were doing relatively well to states that were doing relatively poorly. On the one hand, this finding may overstate regional stabilization benefits because fiscal integration could partially displace existing private risk-sharing arrangements. On the other hand, measured gains from fiscal integrations would be larger if the reduction in state-level volatility reduces within-state individual risk exposure by more than it reduces per-capita (or average) risk exposure of the state’s "representative agent".

In future research, the methodology could help answering other questions in macroeconomics where stochasticity and dynamics are important and linear models provide a good approximation—for example, questions related to stabilization via monetary policy, financial integration, tax reform, exchange rate dynamics. Moreover, the methodology could be used in applications in other fields, like industrial organization and labor economics, where models typically involve simultaneous systems of equations of first order conditions that need to be estimated. Finally, I discussed how to combine the methodology with observed “policy experiments” in order to falsify a set of models.
References


A Proof of Proposition 1

The proof is by construction. Applying the method of undetermined coefficients to (SME1) implies that under policy \( \Theta^0 \), the recursive solution \( \{ P^0, \Lambda^0 \} \) satisfies,

\[
((F + \Theta^0_f)P^0 + G + \Theta^0_c)P^0 + H + \Theta^0_p = 0
\]

\[
(LN + M)\Sigma + (F + \Theta^0_f)\Lambda^0\Sigma^{-1}N\Sigma + ((F + \Theta^0_f)P^0 + G + \Theta^0_c)\Lambda^0 = 0
\]

Also, from Claim 1 we have,

\[
\Lambda^0\Sigma^{-1}N\Sigma + (\rho^0_1 - P^0)\Lambda^0 = 0
\]

\[
(P^0 - \rho^0_1)P^0 - \rho^0_2 = 0
\]

Then, given \( \{ \rho^0_1, P^0, \Lambda^0 \} \) and Assumption 3, let \( P^0 \) be the unique solution with all eigenvalues inside the unite circle to the matrix quadratic equation \( 0 = (P^0 - \rho^0_1)P^0 - \rho^0_2 \).

Replacing the above in the first system, we obtain a new system

\[
((F + \Theta^0_f)P^0 + G + \Theta^0_c)P^0 + H + \Theta^0_p = 0
\]

\[
(LN + M)\Sigma + ((F + \Theta^0_f)\rho^0_1 + G + \Theta^0_c)\Lambda^0 = 0
\]

Then, take equations in this system involving the first line of structure \( \xi \). This is without loss of generality because the proof is identical for all other lines. Given \( \{ \rho^0_1, P^0, \Lambda^0 \} \), this can be written as

\[
\begin{bmatrix}
F_{1.} + \Theta^0_{f,1.} & G_{1.} + \Theta^0_{c,1.} & H_{1.} + \Theta^0_{\rho,1.} & ((LN + M)\Sigma)_{1.}
\end{bmatrix}_{1 \times 4k} \begin{bmatrix}
(p^0)^2 & \rho^0_1 & \Lambda^0 \\
p^0 & \rho^0_1 & \Lambda^0 \\
I & 0 & \Lambda^0 \\
0 & I & \Lambda^0
\end{bmatrix}_{4k \times 2k} = 0_{1 \times 2k}
\]

There are 4\( k \) unknowns in this system involving elements of structure \( \xi \). There are 2\( k \) equations. One of the unknown elements can be normalized to 1 if \( \text{rank}(\Theta^1) = 0 \). This implies that if we specify a set \( R_{n_1} \) of \( n_1 = 2k - 1 + \text{rank}(\Theta^1) \) independent linear restrictions on the elements of structure \( \xi \), we obtain a linear system of equations that is exactly determined (as long as the \( 4k \times 2k \) matrix containing \( \{ P, \Lambda, \rho_1 \} \) has full rank). In particular, from this exactly determined linear system, we identify semi-structure \( \xi^* \) that is both consistent with the imposed linear restrictions and the SVAR representation under the benchmark policy.

Finally, we solve the following system for the counterfactual equilibrium under the alternative policy \( \Theta^1 \). That is, we solve for \( \{ P(\xi^*, \Theta^1), \Lambda(\xi^*, \Theta^1) \} \) given \( \{ P^0, \rho^0_1, \Lambda^0, \Theta^0, \Theta^1 \} \) and \( \{ F^*, G^* \} \in \xi^* \). The system results from subtracting (SME1) for policy \( \Theta^0 \) and \( \Theta^1 \).
\[ 0 = (F^* + \Theta_1^1)(P(\xi^*, \Theta^1)^2 - (P^0)^2) + (G^* + \Theta_1^1)(P(\xi^*, \Theta^1) - P^0) \]
\[ + (\Theta_1^1 - \Theta_0^1)(P^0)^2 + (\Theta_1^1 - \Theta_0^1)P^0 + \Theta_1^p - \Theta_0^p \]
\[ 0 = \left((F^* + \Theta_1^1)P(\xi^*, \Theta^1) + G^* + \Theta_1^1\right)\left(\Lambda(\xi^*, \Theta^1)(\Lambda^0)^{-1} - I\right) \]
\[ + (F^* + \Theta_1^1)\left(\Lambda(\xi^*, \Theta^1)(\Lambda^0)^{-1} - I\right)(\rho_1^0 - P^0) \]
\[ + (F^* + \Theta_1^1)(P(\xi^*, \Theta^1) - P^0) + (\Theta_1^1 - \Theta_0^1)\rho_0^1 + \Theta_1^c - \Theta_0^c \]

And obtain \(\{\rho_1(\xi^*, \Theta^1), \rho_2(\xi^*, \Theta^1)\}\) as:

\[ \rho_1(\xi^*, \Theta^1) = P(\xi^*, \Theta^1) + \Lambda(\xi^*, \Theta^1)(\Lambda^0)^{-1}(\rho_1^0 - P^0) \left(\Lambda(\xi^*, \Theta^1)(\Lambda^0)^{-1}\right)^{-1} \]
\[ \rho_2(\xi^*, \Theta^1) = (P(\xi^*, \Theta^1) - \rho_1(\xi^*, \Theta^1))P(\xi^*, \Theta^1) \]

This concludes the proof because the counterfactual SVAR representation under the alternative policy \(\Theta^1\) is unique and identical for all models that have semi-structure \(\xi^*\). ■

**B Proof Lemma 1**

There are \(k^2\) elements in \(\Lambda(\xi, \Theta)\) that we would like to identify. The proof of the lemma consist in deriving \(k^2\) independent equations to generate a system that is exactly determined.

From the orthogonalization conditions in 4., we obtain \(k(k+1)/2\) equations. Since \(V(\xi, \Theta)\) is symmetric we do not obtain \(k^2\) equations, but only a number of equations equal to the lower (or upper) triangle. Conditions 1. to 3. imply that the system derived from picking ‘s’ lines in equations (6) and (7) and specifying \(r(k) = k(k - 1)/2\) elements in \(A(s)\)

\[ A(s) + B(s)\Lambda(\xi, \Theta) = 0 \]

generates \(r(k)\) independent equations that includes all the elements in all the columns of \(\Lambda(\xi, \Theta)\).

Finally, we have described a total of \(k(k + 1)/2 + r(k) = k^2\) equations to identify the \(k^2\) parameters in \(\Lambda(\xi, \Theta)\).
C Proof of Lemma 2

The following equations characterize the log-linearized equilibrium

\[
\tilde{w}_{kt} - \tilde{p}_{kt} = \frac{1}{\phi} \tilde{n}_{kt}
\]

\[
\tilde{w}_{kt} - \tilde{p}_{kt} = (\alpha - 1) \tilde{n}_{kt} + \beta \tilde{x}_{kt} + \tilde{z}_{kt}
\]

\[
\tilde{q}_{t} - \tilde{p}_{kt} = \alpha \tilde{n}_{kt} + (\beta - 1) \tilde{x}_{kt} + \tilde{z}_{kt}
\]

\[
0 = E_t \left( m_{u_{kt+1}} - m_{u_{kt+1}} - (\tilde{p}_{kt+1} - \tilde{p}_{kt}) - \gamma_{kt+1} - \Phi_0 (\tilde{c}_{kt} - \tilde{c}_t) + \tilde{i}_{t+1} \right)
\]

\[
m_{u_{kt+1}} = -\frac{\sigma}{C - \frac{\phi}{1 + \phi} N^{1 + \frac{\phi}{1 + \phi}}} \left( C \tilde{c}_{kt+1} - N^{1 + \frac{\phi}{1 + \phi}} \tilde{n}_{kt+1} \right)
\]

\[
\tilde{c}_{kt} = \tilde{w}_{kt} - \tilde{p}_{kt} + \tilde{n}_{kt}
\]

\[
B \tilde{b}_{kt} = B (1 + r) (\tilde{b}_{kt-1} + \tilde{i}_t) + \eta_{kt} - X (\tilde{q}_t + \tilde{x}_kt) + S \tilde{s}_{kt}
\]

\[
\sum_k \tilde{x}_{kt} = \sum_k \tilde{\eta}_{kt}
\]

\[
B \tilde{b}_{it}^* + S \sum_k \tilde{s}_{kt} + G \tilde{q}_t = B \tilde{x}_t^* (1 + r) (\tilde{i}_{t-1}^* + \tilde{i}_t)
\]

\[
\tilde{s}_{kt} = \theta_w \tilde{w}_{kt} + \theta_n \tilde{n}_{kt} + \theta_b \tilde{b}_{kt-1}
\]

\[
\tilde{i}_{t+1} = \Phi_p E_t [\tilde{p}_{t+1} - \tilde{p}_t]
\]

After adding up, the aggregate log-linearized equilibrium evolution of \{\tilde{w}_t - \tilde{p}_t, \tilde{n}_t\} is characterized by

\[
0 = E_t (m_{u_{t+1}} - m_{u_t}) + (\phi_p - 1) (\tilde{p}_{t+1} - \tilde{p}_t) + \gamma_{t+1}
\]

\[
0 = \frac{1}{\phi} \tilde{n}_t - (\tilde{w}_t - \tilde{p}_t)
\]

\[
\tilde{w}_t - \tilde{p}_t = (\alpha - 1) \tilde{n}_t + \tilde{z}_t + \beta \tilde{\eta}_t
\]

\[
m_{u_{t+1}} = -\frac{\sigma}{C - \frac{\phi}{1 + \phi} N^{1 + \frac{\phi}{1 + \phi}}} \left( C (\tilde{w}_{t+1} - \tilde{p}_{t+1} + \tilde{n}_{t+1}) - N^{1 + \frac{\phi}{1 + \phi}} (\tilde{n}_{t+1}) \right)
\]

which is equivalent to the system of equations characterizing the log-linearized equilibrium in a representative agent economy with a production technology that utilizes labor alone with an elasticity of \(\alpha\), no endogenous discounting and only 2 exogenous processes \{\tilde{z}_t + \beta \tilde{\eta}_t, \gamma_t\}.

Next, take log-deviations from the aggregate in the original system. This results in the system
characterizing the evolution of \( \{ p_{kt}, w_{kt}, n_{kt}, b_{kt}, c_{kt} \} \) for given \( \{ z_{kt}, \eta_{kt}, \gamma_{kt} \} \)

\[
0 = \mathbb{E}_t \left[ \frac{\sigma \beta}{\alpha \phi - 1 \gamma} (w_{kt+1} + n_{kt+1} - w_{kt} - n_{kt}) + (\alpha + \beta - 1)(n_{kt+1} - n_{kt}) \right] \\
+ (\beta - 1)(w_{kt+1} - w_{kt}) + (1 + \frac{\sigma}{\alpha \phi - 1 \gamma}) (z_{kt+1} - z_{kt}) + \Phi_0 c_{kt} - \gamma_{kt+1} \]

\[
\beta w_{kt} = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_{kt} - z_{kt} \\
\frac{B}{S} b_{kt} = \frac{B}{S} (1 + r) b_{kt-1} - \frac{X}{S} (w_{kt} + n_{kt}) + \vartheta_n n_{kt} + \vartheta_w w_{kt} + \vartheta_b b_{kt-1} + \frac{1}{\pi} \eta_{kt} \\
c_{kt} = w_{kt} - p_{kt} + n_{kt} \\
p_{kt} = 1 - \beta w_{kt} - (z_{kt} + (\alpha + \beta - 1)n_{kt})
\]

This system is independent of all aggregate variables and is analogous to the system characterizing the equilibrium in a small open economy without movements in the terms of trade and nominal interest rate, proving the Lemma.

D Proof of Claim 2

For any \( \nu^1 \) we can find an alternative parameterization,

\[
\begin{align*}
\frac{1 - \lambda_1}{\lambda_1} &= \frac{1 - \lambda^0}{\lambda^0} \nu^1, b_1 &= b_0 \nu^1, \alpha_1 = \alpha_0, \frac{1 + \phi_1}{\nu^1}, \phi_1 = \frac{1 + \phi_0}{\nu^0} \\
1 + \frac{\sigma_1}{\frac{\sigma_0}{\nu^0} - 1} &= 1 + \frac{\sigma_0}{\frac{\sigma_0}{\nu^0} - 1} \nu^1, \phi_1 = \phi_0 - \frac{(\sigma_1 \nu^0 - \sigma_0) \phi_0}{\alpha_0 - \frac{1 + \phi_0}{\nu^0}} \\
\Phi_1 &= \Phi_0 - \frac{(\sigma_1 \nu^0 - \sigma_0) \alpha_0}{\alpha_0 - \frac{1 + \phi_0}{\nu^0}}
\end{align*}
\]

Replacing these in the system of equations characterizing the equilibrium we obtain the exact same system of equations corresponding to \( \nu^0 \). Since \( \nu^1 \) was arbitrarily chosen, this implies that there are many parameterizations consistent with the same \( \rho^0, \Lambda^0 \). Moreover, in all of these parameterizations \( \frac{1 - \lambda_1}{\lambda_1} = \frac{1 - \lambda^0}{\lambda^0} \nu^1 \).
### Table 2: Employment Impact Elasticity to $\gamma$ in (%)

<table>
<thead>
<tr>
<th>$\vartheta_n$</th>
<th>$\kappa$</th>
<th>$\lambda$</th>
<th>1</th>
<th>0.7</th>
<th>0.5</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vartheta_n = -1.6$</td>
<td>0</td>
<td>0.7</td>
<td>1.6</td>
<td>2</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>$\vartheta_n = 0$</td>
<td>0.5</td>
<td>0.7</td>
<td>2.1</td>
<td>2.9</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>$\vartheta_n = -1.6$</td>
<td>0</td>
<td>0.7</td>
<td>1.8</td>
<td>2.1</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>$\vartheta_n = 0$</td>
<td>0.9</td>
<td>0.7</td>
<td>2.3</td>
<td>3.1</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>$\vartheta_n = -1.6$</td>
<td>0</td>
<td>0.7</td>
<td>1.9</td>
<td>2.2</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>$\vartheta_n = 0$</td>
<td>0.9</td>
<td>0.7</td>
<td>2.5</td>
<td>3.3</td>
<td>3.9</td>
<td></td>
</tr>
</tbody>
</table>

Note: Table shows the employment impact elasticity to a discount rate shock, in percentage deviations from the steady state, for alternative calibrations of $\kappa, \lambda$ in the model in Section ??.
Table 3: Policy rule baseline estimates

<table>
<thead>
<tr>
<th></th>
<th>$\vartheta_n$</th>
<th>$\vartheta_w$</th>
<th>$\vartheta_b$</th>
<th>$\vartheta_{w+n}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>-1.6***</td>
<td>-0.9*</td>
<td>-0.03</td>
<td>-0.03</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.5)</td>
<td>(0.7)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV w/ shocks (1)</td>
<td>-1.3*</td>
<td>-1.4*</td>
<td>-0.02</td>
<td></td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td>(0.8)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV w/ house prices (1)</td>
<td>.</td>
<td>.</td>
<td>-0.03</td>
<td>-1.1***</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.4)</td>
<td></td>
</tr>
<tr>
<td>IV w/ house prices and shocks (1)</td>
<td>-1.4*</td>
<td>-1.2*</td>
<td>-0.02</td>
<td></td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(0.6)</td>
<td>(0.6)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>-1.3*</td>
<td>-1.4*</td>
<td>0.01</td>
<td></td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td>(0.7)</td>
<td>(0.08)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers in parenthesis are OLS (or second stage) standard errors. Variables with ‘*’ are significant at a 5% level. Variables with ‘**’ are significant at 1%. All variables are state log-growth rates between 2006 and 2010. $b_{t-1}$ is exogenous in (1) and endogenous in (2).

Table 6: Counterfactual Employment Dispersion without Fiscal Integration

<table>
<thead>
<tr>
<th></th>
<th>$\vartheta_n, \vartheta_w, \vartheta_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-1.6,-0.9,0.03) (-1.4,-1.2,0) (-1.2,-1.4,0) (-1.1,-1.1,0)</td>
</tr>
<tr>
<td>$\sigma_n^{2010}$</td>
<td>3.5 3.4 3.3 3.1</td>
</tr>
<tr>
<td>$\overline{\sigma}_n$</td>
<td>4.9 4.9 4.7 4.5</td>
</tr>
</tbody>
</table>

Note: Table shows selected results of the counterfactual exercise for alternative parameterizations of the prevalent transfer policy rule. $\sigma_n^{2010}$ is the standard deviation of the counterfactual distribution of employment across states in the year 2010 in percents in an economy without transfers. $\overline{\sigma}_n$ is the counterfactual standard deviation in the stationary distribution.
Note: Figure shows the response of employment, wages and assets to a one-standard-deviation "discount rate" shock; both in the actual and counterfactual economy without transfers. The units are a state’s percentage deviation from the aggregate.

Note: For each state, the figure shows the employment difference between the counterfactual economy without a federal transfer policy rule and the actual economy in 2010. The states were sorted according to their actual employment in 2010 in ascending order. To the left, the states with the worst employment realizations; and, to the right, the states with best.
Figure 4: Employment in the Great Recession: Channels Decomposition

Note: Figure shows kernel density estimates of the distribution of employment growth between 2008 and 2010 across states. This is done for the actual economy with rigidities and transfers, and for all other counterfactual economies constructed in Section 7.6.