Using Empirically Estimated Efficiency Frontiers to Gauge Central Bank Performance and Effectiveness

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ABSTRACT

The tradeoff efficiency frontier between the variability of inflation and the variability of output has served as an important tool for evaluating monetary policy, used recently by Mervyn King (2012), the Governor of the Bank of England, and Ben Bernanke (2004), the Chairman of the Federal Reserve. However, these previous studies focus on the theoretical versions of the tradeoff curve, whereas this paper, by using dynamic stochastic optimal control methods, empirically estimates and calculates the optimal tradeoff curves between output and inflation for the three key periods of 1983-1990, 1991-2000, and 2001-2010. In the findings, an inward shift in efficiency frontiers is observed between the periods of 1983-1990 and 1991-2000, pointing to increased output and inflation stability; this macro improvement is unfortunately reversed in the 2000’s, shown by a drastic outward shift in efficiency frontiers between the periods of 1991-2000 and 2001-2010. Upon further examination, this outward shift can be attributed to more intense shocks, specifically the interest rate shocks caused by discretionary monetary policy and compromised central bank independence. Indeed, though the de jure independence of the central bank has stayed constant over time, discretionary policymaking had led to a deterioration of de facto independence in the 2000’s, explaining the higher output and inflation variabilities as illustrated by the outward shift in optimal tradeoff curves.

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I. Introduction

Output and inflation stability have long been priorities of the United States central bank, and an increase in the variabilities of these two macro performance metrics can often times serve as a signal to more subtle, underlying problems that must be addressed. To begin, it is important to remember that there is always a tradeoff between these two variables; as a central bank shifts the emphasis that it places on either output or inflation, a tradeoff curve is consequently traced out. The optimal tradeoff curve that a central bank can achieve under a set of economic circumstances becomes the output-inflation efficiency frontier, which is an important tool in studying the trends in monetary policy effectiveness. In past literatures, shifts in this efficiency frontier have been analyzed theoretically; using dynamic stochastic control methods, this paper will generate empirically the efficiency frontiers for the periods of 1983-1990, 1991-2000, and 2001-2010. Through an examination of these periods' respective tradeoff curves, a clearer picture of the changes in macroeconomic stability over the past three decades can be drawn. From this analysis will arise conclusions about the shifts in de facto independence of the U.S. Federal Reserve in addition to important implications for the monetary policymaking process of this country.
II. Literature Review

Bernanke (2004) explores the trend of low output and inflation variabilities that started in the mid-1980’s, a period of time labeled as the Great Moderation. Bernanke (2004) enumerates the benefits of low variabilities, as “lower volatility of inflation improves market functioning, makes economic planning easier, and reduces the resources devoted to hedging inflation risks” and “lower volatility of output tends to imply more stable employment and a reduction in the extent of economic uncertainty confronting households and firms” (Bernanke 2004, pg. 1). To unpack this improved level of macro stability during the Great Moderation, Bernanke alludes to the Taylor Curve, a curve that “depicts the menu of possible combinations of output volatility and inflation volatility from which monetary policymakers can choose in the long run” (Bernanke 2004, pg. 3). Using the Taylor Curve, Bernanke (2004) proposes two possible theoretical explanations for why the Great Moderation happened. The first of these explanations is that monetary policies improved during that period of time, allowing the U.S. economy to move closer to the optimal output-inflation tradeoff possibilities offered by the Taylor Curve. In Figure 1, that can be represented by the movement from point A to point B. The second of these explanations is that the inherent structure of the economy in fact changed, allowing the
overall economic environment to be more stable. In Figure 1, that would be represented by an inward shift in the Taylor Curve from the dotted frontier to the solid frontier.

Figure 1. Chart from Bernanke (2004)

Bernanke (2004) explores this potential shift in output-inflation efficiency frontiers further, posing the “intriguing possibility ... [that] better monetary policies may have resulted in what appear to be favorable shifts in the economy's Taylor curve” (Bernanke 2004, pg. 6). For instance, monetary policies may have affected the size and frequency of shocks or the sensitivity of the economy to these shocks, which in turn brought about the
structural changes necessary for an inward shift in Taylor Curves during the Great Moderation.

Mervyn King (2012) gave a lecture at the London School of Economics, entitled “Twenty Years of Inflation Targeting.” This piece goes back through the economic history of England in order to highlight the successes and failures behind previous efforts in inflation targeting, and it once again brings the tradeoff between output stability and price stability under examination. King (2012) also uses the Taylor frontier in his analyses, where the central bank must choose the optimal “combinations of lowest volatility of inflation for a given volatility of output” (King 2012, pg. 5) on the Taylor Curve. This optimal choice, in King’s words, “leads to a policy reaction function describing how the central bank responds to shocks hitting the economy,” (King 2012, pg. 5) suggesting that the below efficiency frontier can be used to dictate monetary policy and to measure its efficiency. King (2012) goes on to propose what is called the Minsky-Taylor curve, which takes into account the tradeoff between price stability and financial stability, adding an extra level of complexity into the study of output-inflation targeting. To reach the Minsky-Taylor Curve, the original Taylor Curve is shifted outward to the right, as the Minsky-Taylor frontier reflects the volatility added by “financial instability shocks” in addition to the “aggregate demand and cost shocks” already accounted for by the Taylor Curve (King 2012, pg 8). King (2012)
proposes macro-prudential thinking and tools as a way of closing the gap between the Minsky-Taylor Curve and the Taylor Curve.

**Figure 2. Chart from King (2012)**

Taylor (2013) continues the theoretical discussion of output-inflation tradeoff curves by looking at the inflation and output variabilities of the following periods: 1965–1983, 1984–2006, and 2007–2012. These comparisons are meant to show and explain the shifts within the tradeoff diagram between the Great Moderation and the period afterwards, as illustrated in the graph below.
The above phenomenon, as explained by Taylor, was caused mainly by a “monetary policy regime change” (Taylor 2013, pg. 14), as rule-based policymaking led to the improved conditions in the 1980’s, and discretionary policymaking led to a deterioration in the 2000’s. This discretionary policymaking was accompanied by a decrease in de facto independence of the United States central bank, which led to the upward shift in output variability with no change in inflation variability.

Cecchetti, Flores-Lagunes, and Krause (2006) also study shifts in output and inflation variability, but in this case, the authors simulate the output-inflation system, which allows them to determine the efficiency frontier for output-inflation tradeoff in a
certain time frame. Similar to Bernanke’s work, Cecchetti, Flores-Lagunes, and Krause (2006) attribute the shifts in variability frontiers to shocks and changes in the structure of the economy, while movements of historical variabilities toward a frontier measure improvements in policy efficiency. By empirically calculating the output-inflation efficiency frontiers, Cecchetti, Flores-Lagunes, and Krause (2006) are able to divide improvements in macroeconomic performance into the portion that can be accounted for by more efficient policy and the part that can be attributed to shocks and changes in economic structure (Cecchetti et al 2006, pg. 2). This empirical analysis is conducted for 24 different countries and over the time periods of 1983-1990 and 1991-1998.

**III. Methodology**

This paper utilizes optimal control techniques\(^1\) to construct the output-inflation variability efficiency frontier. To do so, I first estimate a system of equations describing the overall economy, and then use these estimates to identify the efficiency frontiers. This paper focuses on the periods of 1983-1990, 1991-2000, and 2001-2010, taking a more in-depth look at both the Great Moderation and the most recent financial crisis (and its subsequent process of recovery). The above analysis is applied to the United States.

Estimating a Macroeconomic Model of Output and Inflation

I assume that the following system of equations can describe the overall U.S. economy, similar to the one suggested in Cecchetti et al (2006).²

\[
y_t = \sum_{i=1}^{2} \alpha_{1i} t_{i,t-1} + \sum_{i=1}^{2} \alpha_{3(i+2)} y_{t-1} + \sum_{i=1}^{2} \alpha_{4(i+4)} \pi_{t-1} + \varepsilon_{1t} \tag{1}
\]

\[
\pi_t = \sum_{i=1}^{2} \alpha_{2i} y_{t-1} + \sum_{i=1}^{2} \alpha_{5(i+2)} \pi_{t-1} + \varepsilon_{2t} \tag{2}
\]

where

\[
y = \text{GDP}
\]

\[
i = \text{short-term interest rate}
\]

\[
\pi = \text{inflation}
\]

\[
\varepsilon = \text{error terms}
\]

In the above system of equations, I assume that the error terms are serially uncorrelated random variables with zero mean and constant variance-covariance matrices. The GDP is measured by the GDP gap (the percentage deviation of real GDP from potential GDP), calculated by subtracting the natural log of potential real GDP from the natural log of the real GDP. Inflation is calculated by taking the annual percentage change in the CPI, and then subtracting the mean from each entry. Short-term interest rate is represented by the federal fund rate.

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² In Cecchetti et al (2006), there is an extra term, demeaned external price inflation, which is meant to "account for the inter-relation between the economy of interest and its main trading partner" (Cecchetti et al 2006, pg. 14). I simplify the system by not including this term.
Identifying the Efficiency Frontiers

Once I arrive at the estimates of the parameters for Equations (1) and (2), I can obtain the efficiency frontiers using optimal control techniques. The below minimization problem must first be solved; it replicates the problem faced by the central banker, who attempts to minimize the loss function with the estimated output-inflation system as constraints. I assume that the main concern of a central bank is output and price stability as measured by the squared deviation of these variables from desired levels. This leads to the following loss function and minimization problem, which allows the central bank to find the best policy rule for the interest rate. The $E[x]$ notation in the following equations stands for expected value; the expected value must be used here since both the output and inflation are random variables, and this way, an optimal policy path for interest rates can be calculated for one steady state.

$$\min (E[L] = E[\lambda (\pi_t - \pi^*)^2 + (1 - \lambda)(y_t - y^*)^2])$$ (3)

such that

$$y_t = \sum_{l=1}^{2} \alpha_{1l} \pi_{t-l} + \sum_{l=1}^{2} \alpha_{4l+2} y_{t-l} + \sum_{l=1}^{2} \alpha_{6l+4} \pi_{t-l} + \varepsilon_{1t}$$

$$\pi_t = \sum_{l=1}^{2} \alpha_{2l} y_{t-l} + \sum_{l=1}^{2} \alpha_{4l+2} \pi_{t-l} + \varepsilon_{2t}$$

This system of equations can be written in matrix form as
The minimization is taken over the coefficients of the central bank reactionary function, as shown by the equation below.

\[ i_t = \Gamma Y_t \quad (5) \]

where \( \Gamma \) = reaction coefficient of the monetary authority

From Chow (1975), the solution (\( \Gamma \)) to the above dynamic stochastic control problem will take the form:

\[ \Gamma = -(c'Hc)^{-1}c'Hb \quad (6) \]

\[ H = \Lambda + (B + c\Gamma)'H(B + c\Gamma) \quad (7) \]

This is a system of Ricatti equations, and in order to solve for \( \Gamma \), an iteration process must be used; a convergence in the iteration process points to the solution of \( \Gamma \). Once \( \Gamma \) is found, I can substitute Equation (5) into Equation (1), resulting in a first order vector difference equation. From there, the following equations can be used to obtain the variance-covariance matrix that holds the variances of output and inflation in its diagonal elements.
\[ Y_t = (B + c\Gamma)Y_{t-1} + v_t \quad (8) \]

\[
v_t = \begin{bmatrix} 0 \\ \epsilon_{1t} \\ 0 \\ \epsilon_{2t} \\ 0 \end{bmatrix}
\]

\[
E(Y_t)(Y_t)' = (B + c\Gamma)E(Y_{t-1})(Y_{t-1})'(B + c\Gamma) + Ev_t v_t'
\]

\[
\Omega = (B + c\Gamma)\Omega(B + c\Gamma)' + V 
\quad (9)
\]

I assume that the mean of \( Y_t \) is 0, and \( \Omega \) represents the variance-covariance matrix of \( Y_t \).

Because the variances are being solved for a steady state, \( \Omega(Y_t) = \Omega(Y_{t-1}) = \Omega \). Once again, \( \Omega \) can be determined by using an iterative process, and the second and fourth elements in the diagonal of the 5x5 matrix \( \Omega \) are the variabilities of output and inflation, respectively.

By re-running the above analyses for values of \( \lambda \), or the “inflation variability aversion” (Cecchetti et al 2006, pg. 7) parameter, between 0 and 1, an efficiency frontier can then be produced. For instance, the first \( \lambda \) can be set at 0.01, and from there, a corresponding \( \Omega \) can be calculated. This \( \Omega \) will provide a set of variabilities for output and inflation, which are the coordinates of one point out of many on the efficiency frontier.

When a central bank shifts its attention from inflation targeting to output targeting or vice versa, the value of \( \lambda \) changes. Hence, by increasing \( \lambda \) from 0.01 to 0.99 in increments of 0.01, I can trace out an empirical plot of the optimal output-inflation tradeoff curve.
IV. Results

Estimation of the Output-Inflation System

To estimate the coefficients of the output-inflation system of equations, the ordinary least squares (OLS) estimation procedure is used. The results are shown below, where the numbers are the coefficients corresponding to each of the variables and the standard deviations are shown in parentheses.

Table 1. Output-inflation system estimation (OLS coefficients)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Dependent Variable GDP</td>
<td></td>
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<td></td>
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<tr>
<td>CONSTANT</td>
<td>-0.025</td>
<td>0.505</td>
<td>-0.086</td>
</tr>
<tr>
<td>(0.767)</td>
<td>(0.478)</td>
<td>(0.266)</td>
<td></td>
</tr>
<tr>
<td>FEDFUND(-1)</td>
<td>0.088</td>
<td>-0.324</td>
<td>0.212</td>
</tr>
<tr>
<td>(0.177)</td>
<td>(0.221)</td>
<td>(0.204)</td>
<td></td>
</tr>
<tr>
<td>FEDFUND(-2)</td>
<td>-0.090</td>
<td>0.239</td>
<td>-0.228</td>
</tr>
<tr>
<td>(0.163)</td>
<td>(0.200)</td>
<td>(0.196)</td>
<td></td>
</tr>
<tr>
<td>GDPGAP(-1)</td>
<td>1.055</td>
<td>0.759</td>
<td>1.515</td>
</tr>
<tr>
<td>(0.297)</td>
<td>(0.167)</td>
<td>(0.169)</td>
<td></td>
</tr>
<tr>
<td>GDPGAP(-2)</td>
<td>-0.233</td>
<td>0.220</td>
<td>-0.482</td>
</tr>
<tr>
<td>(0.235)</td>
<td>(0.172)</td>
<td>(0.183)</td>
<td></td>
</tr>
<tr>
<td>INFLATION(-1)</td>
<td>-0.035</td>
<td>-0.243</td>
<td>-0.248</td>
</tr>
<tr>
<td>(0.166)</td>
<td>(0.226)</td>
<td>(0.109)</td>
<td></td>
</tr>
<tr>
<td>INFLATION(-2)</td>
<td>-0.011</td>
<td>-0.072</td>
<td>0.024</td>
</tr>
<tr>
<td>(0.178)</td>
<td>(0.227)</td>
<td>(0.118)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.894</td>
<td>0.960</td>
<td>0.969</td>
</tr>
</tbody>
</table>
As can be seen in the GDP equation, the sum of the coefficients of the two lagged fed fund rate series is negative, and the sum of the coefficients of the two lagged GDP series add up to be approximately 1. Additionally, inflation has a negative impact on GDP as well. These are all in line with traditional theory:

1. An increase in interest rates can lead to GDP contraction since people will be less likely to invest. In other words, there will be a movement along the I-S (investment-savings) curve, arriving at a new equilibrium of higher interest rates and decreased investments; this in turn will lower the aggregate demand.

2. Higher inflation will hurt output, as it leads to increased uncertainty in the market. Because prices in the future are now harder to predict, people will have a more
difficult time planning their consumption and investment patterns. Additionally, the efforts spent on managing high inflation will divert attention from true productivity.  

**Calculation of the Tradeoff Curves**

In the below graphs, the x-axis is the variance of inflation, and the y-axis is the variance of output; the red line symbolizes the earlier period in the comparison, whereas the blue line symbolizes the later period. Figure 4 shows an inward shift between the periods of 1983-1990 and 1991-2000. This is in line with Cecchetti et al's (2006) findings, and also in line with the discussion of the Great Moderation in Bernanke (2004).

*Figure 4. Inward shift in efficiency frontiers (1983-1990 v. 1991-2000)*
Now, comparing the more recent periods of 1991-2000 and 2001-2010, Figure 5 shows that there is a drastic outward shift in the efficiency frontier between the earlier period and the later period.

Figure 5. Outward shift in efficiency frontiers (1991-2000 v. 2001-2010)

Causes of the Outward Shift

This observed shift can be due to two different reasons: 1) change in the structure of the economy (as modeled by changes in coefficients of the output-inflation system of equations) and 2) change in the shocks that the economy experienced (as modeled by the residual covariance matrix from the output-inflation system estimation). Indeed, the
financial crisis in the 2001-2010 period could have led to both of the above, and hence, it is important to delve deeper into the exact cause of the outward shift in the efficiency frontiers.

To do so, variances of the shocks from the 1991-2000 period, as measured by the residual covariance matrix, can be applied to the system of equations for output and inflation estimated for the 2001-2010 period; this shock test generates a new efficiency frontier, which allows us to determine whether it was a change in structure or an increase in shocks that produced the outward shift.

Figure 6. Exploration of the 1991-2010 outward shift (with 2001-2010 shock test)
As shown above, though the structure of the 2001-2010 economy is kept constant, the outward shift in efficiency frontier is much less drastic when the shocks of the 1991-2000 period are applied. In other words, shocks in the economy exacerbated the outward shift in efficiency frontiers between the 1990’s and the 2000’s. In Bernanke (2004), the Great Moderation is partially attributed to improved monetary policy, and here, the situation is comparable but reversed, where a deterioration in monetary policymaking led to shocks that unfavorably shifted the efficiency frontier in the 2000’s. In the output-inflation system of equations, the three inputs consist of GDP, inflation, and federal fund rate, and in the 2001-2010 period, the one variable that shifted significantly as a result of monetary action was the federal fund rate, which produced an interest rate shock.

V. Analysis

Unpacking the Interest Rate Shock

When it comes to monetary policy, it may make sense to divide the 2001-2010 period into two: 2003-2005 and 2008-2010. As described in Taylor (2009), interest rates were kept relatively low by the Federal Reserve between 2003-2005, and this loose credit partially fueled the housing bubble, leading to rapid market acceleration followed by a devastating bust. In 2007, the United States economy collapsed, and the recession began.
In 2008, the United States Federal Reserve had dropped its interest rates to near 0, and turned to quantitative easing in order to stimulate the economy. The Fed rapidly increased its reserves 100-fold within two months, from 8 billion dollars to 800 billion dollars, by actively absorbing unwanted loans and securities within the private sector, bailing out institutions such as Bear Sterns and AIG. Taylor (2009) coins this occurrence as mondustrial policy, where monetary policy was used to aid certain industries and firms.

Indeed, the above instances of monetary policy differ from monetary policymaking during the Great Moderation in a significant way. In Taylor (2012), the author labels the period of 1985-2003 as the “Rule-Based Era,” and the period of 2003-present as the “Ad Hoc Era” (Taylor 2012, pg. 1023). Under rule-based policymaking, “decisions about policy instruments are more predictable and systematic,” whereas discretionary policymaking leads to “decisions that are less predictable and more ad hoc, and they tend to focus on short-term fine tuning” (Taylor 2012, pg. 1018). Indeed, the decisions behind low interest rates in the 2003-2005 period and behind quantitative easing in 2008 could be seen as somewhat discretionary in nature; while the effectiveness of this particular round of quantitative easing remains up for debate, such discretionary policymaking did lead to more shocks and increased instability in the United States economy.
Deterioration in *De Facto* Central Bank Independence

The above discussions around discretionary policymaking have interesting implications on central bank independence as well. The output-inflation tradeoff curve has also been associated with the study of central bank independence; theoretically, an optimal point on the output-inflation efficiency frontier can only be achieved when a central banker has the independence to set policy without political backlash. In my analysis, this translates to the central banker being able to choose the appropriate inflation variability aversion parameter $\lambda$, solve the minimization problem, and make policy decisions based on the resulting policy reaction coefficient. When a central banker has the independence to do the above, theory states that there will be positive influences on a country’s stability and growth.

In traditional literature, central bank independence usually involves assigning an independence score to a country, based on a set of predetermined criteria. By answering questions such as who appoints the central bank governor and how long is his or her term, researchers have been able to simplify the concept of independence into just a number, with this number representing a country’s *de jure* independence. These independence scores, calculated across a number of countries, can then be regressed with various economic performance metrics to find correlations and relationships.
Interestingly enough, throughout the three periods studied in this paper from 1983 to 2010, the *de jure* independence of the central bank did not change. However, the outward shift in efficiency frontiers hints at shifts in *de facto* independence. For instance, the low interest rates in the 2003-2005 period point to a deviation from the traditional means of setting the interest rate path. Empirically, one good rule-based approach for defining the fed fund rate would be the Taylor Rule, but as can be seen below, there appeared to be a clear gap between the dictations of the Taylor Rule and the historical rates from 2003 to 2005. Contrarily, the federal fund rates tracked the Taylor Rule much more closely during the Great Moderation.

*Figure 7. Taylor Rule diagram, used by Taylor (2009) and The Economist (2007)*
Some may argue that discretionary policymaking can be attributed to the true independence of the central bank, as the Federal Reserve has the prerogative to act or react in the ways that it deems fit or necessary. Nevertheless, the severe outward shift of the efficiency frontier proves troublesome, as even the most independent central banks should only be redistributing the emphasis placed on output and inflation depending on changing monetary needs. This redistribution, which is represented by a change in $\lambda$, should lead to movements along the curve, as opposed to inward or outward shifts in the curve. When an outward shift is observed, it highlights a decrease in the de facto independence of the central bank in question.

VI. Conclusion

In this paper, stochastic optimal control methods are used to generate the output-inflation efficiency frontiers for the periods of 1983-1990, 1991-2000, and 2001-2010. The results show an inward shift between the periods of 1983-1990 and 1991-2000 and a substantial outward shift from 1991-2000 to 2001-2010. Through further analyses, I have determined that the latter outward shift can be attributed to an increase in shocks in the economy, especially interest rate shocks as brought about by discretionary policymaking in the 2000’s. These shifts in the efficiency frontiers align with discussions surrounding the Great Moderation and the most recent financial crisis, and provide added insights to
existing research on how monetary policy has contributed to varying levels of stability in these two time frames. The implications of the above findings are telling, both for future monetary policymaking and for central bank independence. Once again, it is important to put aside the debate around effectiveness of specific monetary policies, because the work here is focused on the output-inflation tradeoff curve. Nevertheless, the interest rate shocks and the other measures of discretionary policymaking may in fact be compromising the de facto independence of our central bank, despite a consistent level of de jure independence. Should more discretionary policy be used going forward, this could lead to a negative feedback loop of higher and higher overall variability, and when this happens, the mission of the central bank – “to maintain the stability of the financial system” (“About the Fed: Mission”) – will inevitably be undermined.
Reference List


