This paper builds a social network employment model that divides the labor force into two groups of different relative size: a majority and minority group is introduced. Average workers from each group are otherwise completely identical, possessing an equal level of ability. However, due to social network effects, minority workers not only receive a disproportionately low fraction of job offers through referral, but also expect lower average wages. Thus, the model suggests that simply the introduction of a majority and minority group in the labor force (when workers are more likely to know others who share similar attributes) breeds inequality. Discrimination (measurable differences in expected welfare between groups, given equal ability) can persist independently of psychological prejudices like racism or sexism—discrimination may be an inherent consequence of the interaction of more- and less- “advantageous” social networks.

**Keywords:** discrimination, employment, income distribution, inequality, job referrals, labor market, merit, meritocracy, redistributive policy, social networks

*I thank Professor Kenneth Arrow for advising me throughout the development of my research. He encouraged me to explore my model on a level much deeper than I originally intended. Without his guidance, this paper would not have been possible. Thanks to Dr. Geoffrey Rothwell for providing valuable advice even before the inception of my research. Thanks to Ph.D. candidates Sriniketh Nagavarapu (for explaining Bayes’s rule) and Chenghuan Chu (for critical feedback). Thanks to Dr. Hilton Obenzinger for critiquing drafts. Thanks to my family for constant unwavering support, especially to my eldest sister, Nkasi Okafor, Esq., for invaluable feedback. Above all, thanks to God for the countless undeserved blessings before, during, and after the creation of this paper.*
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1 Introduction

Many debate the existence of psychological prejudice and its influence on society. When prejudice aligns along attributes like race, religion, or gender, discrimination (measurable differences in expected welfare between groups, given equal ability) may arise, where people in certain social groups are “unfairly” disadvantaged compared to others. However, identifying prejudice remains a difficult task, since uncovering the content of intangible thoughts often proves impossible.

As a result, the purpose of this paper is not to evaluate the impact of psychological biases like racism or sexism. This paper does not explore the role prejudice plays in exacerbating inequality. Rather, it explores inequality from a lens of social network effects, a lens that discovers a new form of discrimination divorced from prejudice. By building and analyzing a social network employment model, this paper uncovers welfare differences between majority and minority workers of equal ability, simply due to the assumption that workers more likely know others with similar characteristics (captured by the “type in-group bias” parameter of my model).

Importantly, the model predicts inequality even though no psychological prejudice exists among workers or firms. Disproportionate differences in the chance of getting a job through referral and in the average expected wage are found between majority and minority workers. Thus, this paper suggests that even by removing all inequality caused by psychological prejudice—even by fabricating a labor market free of biases like racism or sexism—a form of discrimination (caused by inherent differences in social network composition) may still persist.

No similar model currently exists (Ioannides & Loury, 2004). Though other models suggest the impact of social tie quantity (Arrow & Borzekowski, 2004) and strength (Granovetter, 1973; Montgomery, 1994), none illustrate in a similar way how inequality
increases as the size disparity between majority and minority worker populations grows, holding all else equal. This paper thus uncovers an alternate cause of welfare differences between social groups. While psychological prejudices may fuel discrimination, they are not its necessary catalyst. Even in the absence of prejudice, discrimination can still persist—as an unavoidable consequence of the interaction of more- and less- “advantageous” social networks.

## 2 Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Majority split</td>
<td>Proportion of majority workers in total worker population</td>
<td>$\delta \in \left[ \frac{1}{2}, 1 \right]$</td>
</tr>
<tr>
<td>$1 - \delta$</td>
<td>Minority split</td>
<td>Proportion of minority workers in total worker population</td>
<td>$1 - \delta \in \left[ 0, \frac{1}{2} \right]$</td>
</tr>
<tr>
<td>$\tau_{maj}$</td>
<td>Network density (majority type)</td>
<td>Probability a majority worker has a social tie</td>
<td>$\tau_{maj} \in [0, 1]$</td>
</tr>
<tr>
<td>$\tau_{min}$</td>
<td>Network density (minority type)</td>
<td>Probability a minority worker has a social tie</td>
<td>$\tau_{min} \in [0, 1]$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Ability in-group bias</td>
<td>Probability a worker’s social tie (if they have one) is with another worker of equal ability</td>
<td>$\alpha \in \left[ \frac{1}{2}, 1 \right]$</td>
</tr>
<tr>
<td>$\psi_{maj}$</td>
<td>Type in-group bias (majority type)</td>
<td>Probability a majority worker’s tie (if they have one) is with another majority worker, over the prob. already attributable to chance</td>
<td>$\psi_{maj} \in [0, 1 - \delta]$</td>
</tr>
<tr>
<td>$\psi_{min}$</td>
<td>Type in-group bias (minority type)</td>
<td>Probability a minority worker’s tie (if they have one) is with another minority worker, over the prob. already attributable to chance</td>
<td>$\psi_{min} \in [0, \delta]$</td>
</tr>
</tbody>
</table>
3 Context

My employment model predicts welfare differences between majority and minority workers by comparing whether workers find employment through a referral (when an acquaintance who is already an employee refers them to firm hirers) or through the general market (e.g., submitting a resume directly to the firm). Using job referrals as the basis for predicting inequality proves useful for a variety of reasons. First, referrals explicitly reflect the influence of social networks. Intuitively, those who are “well-connected” may have greater access to people and resources, increasing their chances of getting referred. Second, research suggests that referred workers have a greater chance of receiving a job offer and also expect higher wages when they accept one, at least initially (Granovetter, 1974; Simon, 1992). Finally, as explained below, using job referrals as a basis for predicting inequality among workers gives the model greater relevancy to the real world.

Much existing research supports the significance of social networks in job selection. Montgomery (1991) claims that approximately 50 percent of employed workers found their job through friends and relatives. Bewley (1999) corroborates this statement, using data from 24 studies to estimate the figure lies between 30 and 60 percent. Holzer (1987) reports that 36 percent of firms included in the Equal Opportunity Pilot Project data filled their last openings with referred applicants. Finally, Cambell (1990) studied 52 Indiana establishments to find that over 51 percent of jobs were filled through referral.

Firms’ preference for referred workers makes sense; referrals reduce the uncertainty of a job candidate’s ability. Simon (1992) mentions that “objective” criteria like education and work experience imperfectly communicate a candidate’s true ability to perform a particular job role. Thus, job matching theories stress that objective criteria do not provide complete information
(Barron & Bishop, 1985; Barron, Bishop, & Dunkelberg, 1985; Barron, Black, & Loewenstein, 1989). Hiring referred employees helps compensate for this shortcoming by providing an additional subjective assessment of a candidate’s “fit” for the job. Also, because a worker’s reputation with their employer is at stake, they have a vested interest in only referring well-qualified acquaintances (Montgomery, 1991).

**Table 1** – Job-finding Methods Used by Workers

<table>
<thead>
<tr>
<th>Source/data</th>
<th>Percentage of jobs found using friends/relatives</th>
<th>Sample size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rees and Shultz (1970)/Chicago labor-market study, 12 occupations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Typist</td>
<td>37.3</td>
<td>343</td>
</tr>
<tr>
<td>Keypunch operator</td>
<td>35.3</td>
<td>280</td>
</tr>
<tr>
<td>Accountant</td>
<td>23.5</td>
<td>170</td>
</tr>
<tr>
<td>Tab operator</td>
<td>37.9</td>
<td>126</td>
</tr>
<tr>
<td>Material handler</td>
<td>73.8</td>
<td>286</td>
</tr>
<tr>
<td>Janitor</td>
<td>65.5</td>
<td>246</td>
</tr>
<tr>
<td>Janitress</td>
<td>63.6</td>
<td>80</td>
</tr>
<tr>
<td>Fork-lift operator</td>
<td>66.7</td>
<td>175</td>
</tr>
<tr>
<td>Punch-press operator</td>
<td>65.4</td>
<td>133</td>
</tr>
<tr>
<td>Truck driver</td>
<td>56.8</td>
<td>67</td>
</tr>
<tr>
<td>Maintenance electrician</td>
<td>57.4</td>
<td>129</td>
</tr>
<tr>
<td>Tool and die maker</td>
<td>53.6</td>
<td>127</td>
</tr>
</tbody>
</table>

Granovetter (1974)/sample of residents of Newton, MA:

<table>
<thead>
<tr>
<th>Source/data</th>
<th>Percentage of jobs found using friends/relatives</th>
<th>Sample size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professional</td>
<td>56.1</td>
<td>132</td>
</tr>
<tr>
<td>Technical</td>
<td>43.5</td>
<td>69</td>
</tr>
<tr>
<td>Managerial</td>
<td>65.4</td>
<td>81</td>
</tr>
</tbody>
</table>

Corcoran et al. (1980)/Panel Study of Income Dynamics, 11th wave

<table>
<thead>
<tr>
<th>Source/data</th>
<th>Percentage of jobs found using friends/relatives</th>
<th>Sample size</th>
</tr>
</thead>
<tbody>
<tr>
<td>White males</td>
<td>52.0</td>
<td>1499</td>
</tr>
<tr>
<td>White females</td>
<td>47.1</td>
<td>988</td>
</tr>
<tr>
<td>Black males</td>
<td>58.5</td>
<td>667</td>
</tr>
<tr>
<td>Black females</td>
<td>43.0</td>
<td>605</td>
</tr>
</tbody>
</table>

1 Source: (Montgomery, 1991)
The referral system helps correct for incomplete information in job selection, thereby making it beneficial to many firms. However, its benefit to workers is not as clear-cut, because the benefits depend on the strength and quantity of workers’ social ties. Those having fewer social ties (or having social ties only with other unemployed workers) may suffer disadvantages, being disproportionately crowded out of consideration for certain jobs.

Arrow and Borzekowski (2004) mention that social networks’ distortion of worker opportunities can inherently give rise to inequality. Montgomery’s (1991) employment model reflects this point. The model categorizes workers solely by ability; each worker is differentiated only by high or low ability. In this model, workers with more social ties to high-ability workers not only receive more job offers through referral, but also expect higher average wages. The workers with no social ties to high-ability workers are forced to find employment through the general market and expect lower average wages. Although the model predicts inequality among individual workers, it also predicts that the average wage of high-ability workers is higher than that of low-ability workers, a finding that readily aligns with notions of “fairness.”

But what are the implications when social networks are not only aligned along ability (as in Montgomery’s (1991) model), but also along some other characteristic? Lin’s (2001) “strength of position” proposition argues that individuals are more likely to associate with others of similar social and occupational positions. Ioannides and Loury (2004) rephrase this proposition, stating that social networks form along dimensions like race, ethnicity, religious affiliation, and education. If this is the case, then one might expect individuals whose “dimensions” are most common (i.e., majority workers, or those who share the most similarities with the most people) to have the greatest probability of being referred for employment. In other words, one might expect members of a majority group to enjoy employment advantages over
members of a minority group, all else equal. While some research apparently defends this hypothesis, the prevalence of overlapping differences among workers, such as educational disparity and unequal access to information, often makes it difficult to disentangle which factors actually do cause inequality between groups. To date, very few models have explored the accuracy of this hypothesis from a theoretical standpoint (Ioannides & Loury, 2004; Montgomery, 1991).

As a result, many unanswered questions remain about the influence of social networks on inequality. Can simply the existence of majority and minority workers give rise to discrimination (measurable differences in average welfare between groups, given equal ability), even in the absence of prejudice? Do majority workers, simply as a function of having the greatest similarity with the greatest number of people, enjoy job referral advantages that translate into expected welfare benefits? Are these “unfair” advantages inescapable, or can they be mitigated if minority workers possess certain social characteristics, such as “stronger-knit” networks or more social ties?

The following model addresses these issues.

4 Methodology

4.1 Model Background

This model does not evaluate the impact of psychological biases like racism or sexism; it does not explore the role prejudice plays in exacerbating discrimination. Rather, this model illustrates how discrimination can arise even in situations when majority and minority workers have equal average ability, and when no psychological prejudices exist toward either group. First, I build an employment model with two ways to gain employment, through referrals and through the general
market. Next, I analyze the model to show how there are expected differences between majority and minority workers in the chances of getting a job through referral and in the expected wage.

My employment model is an extension of Montgomery’s (1991). Whereas Montgomery’s model only divides the labor force by merit (high-ability and low-ability), my model introduces categorization of workers by majority and minority type. As a result, it allows one to analyze welfare differences between groups, keeping ability equal. Thus, my model suggests how discrimination (measurable differences in expected welfare between groups, given equal ability) can arise when employment decisions are influenced by social structure. In contrast to Montgomery’s (1991) findings, where inequality only correlates with ability, my model also predicts inequality between [majority and minority] workers of the same ability.

It must be noted that, for the purposes of this model, majority workers are not necessarily defined ethnically. “Majority” is more broadly defined, referring to workers whose group: 1) occupies more than half of the population; and 2) shares common characteristics and/or background among members to foster connectedness. Thus, a majority worker is context-specific. In many employment scenarios, majority status could refer to ethnicity; yet, in some cases, it could refer to gender, social class, or other attributes.

4.2 Social Network Employment Model

Here, I extend Montgomery’s (1991) two-period model of the labor market. The following assumptions are made on workers and firms.

Workers:

• Each worker works one period.
• There are many workers, with an equal number in each period. (Throughout the appendix, to simplify the analysis, I examine the model’s equilibrium as the number of workers approaches infinity.) To be more precise, I am thus assuming a continuum of workers, with an equal measure in each period.

• Workers may be of two types, either majority or minority. Each worker’s type is predetermined and assigned before the period in which he or she enters the market. By definition, \( \delta > \frac{1}{2} \) of all workers are majority, while \( 1 - \delta < \frac{1}{2} \) are minority. To simplify the model, I assume that (a) \( \frac{1}{2} \) of the workers within each type are high-ability, while \( \frac{1}{2} \) are low-ability, and (b) high-ability workers produce one unit of output, while low-ability workers produce zero units.

• Workers are observationally equivalent; employers are uncertain of the ability or majority/minority status of any particular worker. This prevents psychological prejudice from influencing any of the model’s findings.

Firms:

• Each firm may employ, at most, one worker.

• A firm’s profit in each period is equal to the productivity of its employee minus the wage paid. (Product price is exogenously determined and normalized to unity.)

• Each firm must set wages before learning the productivity of its worker.

• Firms are free to enter the market in either period.

As Montgomery (1991) notes, most of the model’s assumptions are standard in labor-market models of adverse selection, especially that of Greenwald (1986). Not only are workers observationally equivalent, but also they are unable to signal their ability to potential employers; if they could, the main advantage of referrals would disappear. Each firm must set its wage before learning the productivity of its employee. As Montgomery (1991) notes, although the assumption may be extreme, it “captures a plausible rationale for employer screening of job applicants (and thus the use of employee referrals)” (p. 1410).
Due to the assumption of free entry of firms, as Montgomery (1991) notes, expected profit for entering firms is driven to zero. As a result, firms offer wages equal to the expected productivity of those in the market. Due to the influence of referrals, some firms will pay a wage higher than their worker’s productivity, while others will pay a wage less than the productivity. As Montgomery (1991) states, “If the model were closed at this point, under the assumption that all workers in each period were hired through the market, all firms would offer a wage equal to ½ in each period” (p. 1410). Since the model is not closed, wage dispersion occurs.

I now introduce the following assumptions on social structure:

**Social Structure:**

- Each period-1 worker knows at most one period-2 worker. If the period-1 worker is a majority worker (which is predetermined), he or she possesses a social tie with probability $\tau_{maj} \in [0,1]$; otherwise, he or she is a minority worker and possesses a social tie with probability $\tau_{min} \in [0,1]$.

- For each period-1 worker holding a social tie, the specific period-2 individual known is selected stochastically through a three-stage process (note that the timing of the first two stages need not be sequential; timing can be simultaneous):

  1. In the first stage, the period-2 worker’s type (majority or minority) is chosen. Conditional upon holding a tie, if the period-1 worker is a majority worker, he or she knows a period-2 majority worker with probability $\psi'_{maj} = \delta + \psi_{maj}$, where “$\psi_{maj}$”\(^2\) refers to the majority “type in-group bias”.\(^3\) If the period-1 worker is a minority worker, he or she knows a period-2 minority worker with probability $\psi'_{min} = (1 - \delta) + \psi_{min}$, where “$\psi_{min}$”\(^4\) refers to the minority type in-group bias. (The

\(^2\) $0 < \psi_{maj} \leq 1 - \delta$, since $\psi'_{maj} \leq 1$

\(^3\) “Type in-group bias” refers to the probability that one type (majority/minority) of worker knows another worker of the same type, over the probability already attributable to chance. This parameter captures the assumption that workers more likely know others who share similar characteristics. The period-1 majority worker thus knows a minority worker with probability $1 - \psi'_{maj} = 1 - \delta - \psi_{maj}$

\(^4\) $0 < \psi_{min} \leq \delta$, since $\psi'_{min} \leq 1$
period-1 minority worker thus knows a majority worker with probability 
\[ 1 - \psi_{\text{min}}' = \delta - \psi_{\text{min}}. \]

2. In the second stage, a period-1 worker knows a period-2 worker of their own ability (high or low) with probability \( \alpha \ (\frac{1}{2} < \alpha \leq 1) \). This probability is independent of whether the worker is majority or minority type. (The period-1 worker thus knows a worker of the other ability with probability \( 1 - \alpha < \frac{1}{2} \).)

3. In the final stage, the specific period-2 worker is chosen randomly from those that match the appropriate majority/minority type and high/low ability.

The social structure is thus characterized by three parameters, which social-network researchers label “network density” (\( \tau_{\text{maj}} \) and \( \tau_{\text{min}} \)), type “in-group (or inbreeding) bias” (\( \psi_{\text{maj}} \) and \( \psi_{\text{min}} \)), and ability “in-group (or inbreeding) bias” (\( \alpha \)) (Wellman, 1988).

Finally, I assume the following timing:

**Timing:**

- Firms hire period-1 workers through the market, which clears at a wage of \( w_{\text{M1}} \).
- Production occurs; each firm learns the productivity of its worker.
- If a firm desires to hire through employee referral, it sets a referral offer; firm \( i \) may thus set an offer \( w_{R_i} \).
- Social ties are assigned.
- Each period-1 worker possessing a social tie relays his or her firm’s wage offer (\( w_{R_i} \)) to a period-2 acquaintance.
- Each period-2 worker compares wage offers received, either accepting one or waiting to find employment through the market.
- Those period-2 workers with no offers (or refusing all offers) go on the market, which clears at wage \( w_{\text{M2}} \).
- Production occurs.
As Montgomery (1991) notes, the timing comprises three main stages:

1. Each firm hires a period-1 worker through the market and learns his or her ability. As period-1 workers are observationally equivalent (and cannot be referred for jobs by older workers), each firm hiring through the market obtains a high-ability worker with probability $\frac{1}{2}$.

2. After learning the ability of its current worker, each firm sets a referral offer that may be relayed to an acquaintance of its worker. This depends on whether the firm’s worker holds a social tie. If they do, then the firm will only attract the acquaintance if the referral offer exceeds both the period-2 market wage and all other referral offers received by the acquaintance. A firm not wishing to hire through referral will set no referral offer (or might just offer a wage below $w_{M2}$, which has no probability of acceptance). Period-2 workers then compare all offers received, accepting the highest.

3. All period-2 workers who receive no offers must find employment through the general market. They receive a wage equal to the average expected productivity of all other workers finding employment through the general market.

5 Results

The first section below includes analysis from Montgomery’s (1991) original model that remains relevant for the model in this paper. It includes remarks on referral wage distribution and expected firm profits. The second section evaluates how the introduction of a majority and minority group in the labor force affects the distribution of job offers through referral and the expected wage between groups.
5.1 General Model Implications

Some analysis from Montgomery’s (1991) model remains relevant for the employment model built in this paper:

- Due to the assumed in-group bias between workers of similar ability, a firm will attempt to hire through referral if and only if it employs a high-ability worker in period 1. Referral wage offers are dispersed between $w_{m2}$ and some $\bar{w}_R$. Montgomery (1991) explains that, by applying theorem 4 from Burdett and Judd (1983), the density of the referral-offer distribution is positive over this entire range; wage dispersion arises because the probability that a period-2 worker receives exactly one referral offer is strictly between 0 and 1. Proposition 2.2 from Butters (1977) suggests that the wage distribution must have no “gaps.” Montgomery (1991) clarifies that if no wages were offered between $w_1$ and $w_2$ (where $w_{m2} < w_1 < w_2 < \bar{w}_R$), then “a firm offering $w_2$ could reduce its wage offer without reducing the probability that the offer is accepted, thus increasing expected profits” (p. 1411).

- Most workers receiving and accepting referral offers are of high ability. As a result, a disproportionately high number of low-ability workers find employment through the general market. This drives the market wage below the average productivity of the entire population. However, as Montgomery (1991) notes, adverse selection does not completely eliminate the market. Since some high-ability workers are not “well-connected”, they fail to receive referral wages, causing them to find employment in the general market. Thus, the market wage remains above zero.

- As Montgomery (1991) notes, due to the free entry of firms and the symmetric (lack of) information on the ability of workers, firms hiring through the market earn zero expected profit. However, due to imperfect competition for referred workers, firms making referral
offers earn positive expected period-2 profit.\textsuperscript{5} As in Greenwald (1986), the expectation of positive period-2 profits for a firm obtaining a high-ability period-1 worker drives the period-1 market wage above the average productivity of the population. Montgomery (1991) suggests that there are two components to this wage: a period-1 worker receives his expected productivity, plus the “option value” of his period-2 referral.

- If a firm’s period-1 employee is revealed to possess high ability (and a period-2 acquaintance), the firm will make a referral offer. If not, the firm hires through the market and earns zero expected profit (Montgomery, 1991).

5.2 Model Implications of Introducing Majority/Minority Groups

By incorporating majority and minority groups:

- No longer is there a single network density for the entire population of workers; rather, there are two different probabilities of having a social tie, one for majority workers and another for minority workers (\(\tau_{\text{maj}}\) and \(\tau_{\text{min}}\)).

- A new “type in-group” bias is introduced, which captures the assumption that workers are more likely to know others with similar characteristics. This bias is not assumed to be symmetric between majority and minority workers, so is captured by two parameters, \(\psi_{\text{maj}}\) and \(\psi_{\text{min}}\).

The first section below explores how the social structure affects workers’ chances of getting a job offer through referral. The second section explores the effects on wages.

\textsuperscript{5} As mentioned after appendix Equation A.4, to maintain equilibrium wage dispersion, expected profit must be constant across all referral offers made. If not, then any firm not offering the profit-maximizing referral wage value would shift accordingly.
5.2.1 Does simply the existence of a majority and minority group in the labor market cause inequality in the chance of getting a job through referral, all else equal?

My employment model, derived in the appendix, shows how network density ($\tau_{maj}$ and $\tau_{min}$) and type in-group bias ($\psi_{maj}$ and $\psi_{min}$) affects the chances of firms making job offers through referral. Specifically, if majority and minority workers share the same value of network density and type in-group bias—i.e., if workers in both groups have an equal chance of having a social tie and an equally “strongly-knit” network—then the probability of a firm offering a job through referral to minority workers is lower than what one would expect from chance. For example, if minority workers occupy 25% of the population ($\delta = 0.75$), less than 25% of the total job offers through referral will go to minority period-2 workers.

### Table 2.6 – Job Offers Through Referral for Majority and Minority Workers, Holding Network Density and Type In-Group Bias Constant

<table>
<thead>
<tr>
<th>Proportion of majority workers in labor force ($\delta$)</th>
<th>Fraction of job offers going to majority workers</th>
<th>Majority job offer disparity from proportionality</th>
<th>Minority job offer disparity from proportionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.500</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>0.55</td>
<td>0.555</td>
<td>0.9%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>0.60</td>
<td>0.610</td>
<td>1.7%</td>
<td>-2.5%</td>
</tr>
<tr>
<td>0.65</td>
<td>0.665</td>
<td>2.3%</td>
<td>-4.3%</td>
</tr>
<tr>
<td>0.70</td>
<td>0.720</td>
<td>2.9%</td>
<td>-6.7%</td>
</tr>
<tr>
<td>0.75</td>
<td>0.775</td>
<td>3.3%</td>
<td>-10.0%</td>
</tr>
<tr>
<td>0.80</td>
<td>0.830</td>
<td>3.8%</td>
<td>-15.0%</td>
</tr>
<tr>
<td>0.85</td>
<td>0.885</td>
<td>4.1%</td>
<td>-23.3%</td>
</tr>
<tr>
<td>0.90</td>
<td>0.940</td>
<td>4.4%</td>
<td>-40.0%</td>
</tr>
<tr>
<td>0.95</td>
<td>0.995</td>
<td>4.7%</td>
<td>-90.0%</td>
</tr>
</tbody>
</table>

---

${\tau_{maj}} = 1.0$, ${\tau_{min}} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$
This disproportionality in job offers makes sense. Even though the majority type in-group bias equals the minority one, the biases unequally skew the distribution of jobs offers, due to the difference in relative group size. In other words, since majority workers occupy more of the population, majority type in-group bias ($\psi_{maj}$) will have a stronger “pull” on the distribution of job offers, skewing the distribution in favor of majority (and to the detriment of minority) workers.

For minority workers to have a proportional chance of receiving job offers through referral, they must have either (or both) a stronger network density or type in-group bias. The “compensating” network density ($\tau^\text{nu}_\text{min}$) refers to the network density needed for minority workers to achieve this proportionality in job offers received. Below are the final steps from appendix Equation A.1 in deriving the compensating network density.

\[
Pr \left\{ \text{firm makes offer to high ability majority} \right\} \propto \frac{1}{\delta \left(\delta + \psi_{maj} \alpha \tau_{maj}\right) + (1-\delta) \left(\delta - \psi_{min} \alpha \tau_{min}\right)}
\]

\[
Pr \left\{ \text{firm makes offer to high ability minority} \right\} \propto \frac{1}{\delta \left(1-\delta - \psi_{maj} \alpha \tau_{maj}\right) + (1-\delta) \left(1-\delta + \psi_{min} \alpha \tau_{min}\right)}
\]

\[
(1-\delta) \times \left[ \frac{\delta \left(\delta + \psi_{maj} \alpha \tau_{maj}\right) + (1-\delta) \left(\delta - \psi_{min} \alpha \tau_{min}\right)}{\left[\delta^3 + \delta^2 - \delta^2 \psi_{maj} + \delta \psi_{maj} \tau_{maj}\right] + \left(\delta^3 - 2\delta^2 + \delta - \delta^2 \psi_{min} + 2\delta \psi_{min} - \psi_{min} \tau_{min}\right]} \right] = \delta \times \left[ \frac{\delta \left(1-\delta - \psi_{maj} \alpha \tau_{maj}\right) + (1-\delta) \left(1-\delta + \psi_{min} \alpha \tau_{min}\right)}{\left[-\delta^3 + \delta^2 - \delta^2 \psi_{maj}\right]_{maj} + \left(\delta^3 - 2\delta^2 + \delta - \delta^2 \psi_{min} + 2\delta \psi_{min} - \psi_{min} \tau_{min}\right]} \right]
\]

only when

\[
\tau^\text{nu}_\text{min} = \frac{\delta \psi_{maj}}{1-\delta \psi_{min}} \tau_{maj}
\]
The value \( \rho_\tau = \left( \frac{\delta}{1-\delta} \right) \left( \frac{\psi_{maj}}{\psi_{min}} \right) \) is a multiplier that demonstrates how many times stronger the network density of minority workers must be for them to receive a proportional amount of job offers through referral. For example, if majority workers comprise 80% of the population \((\delta = 0.8)\) and both majority and minority workers share the same strength of type in-group bias \((\psi_{min} = \psi_{maj})\), then \( \rho_\tau = 4 \). In other words, for minority workers to receive a proportional amount (or 20%) of all job offers through referral in the next period, the probability of them having a tie in the current period must be four times greater than that of majority workers \((\tau_{min} = 4\tau_{maj})\). Similarly, the value \( \rho_\psi = \left( \frac{\delta}{1-\delta} \right) \left( \frac{\tau_{maj}}{\tau_{min}} \right) \) is a multiplier that demonstrates how many times stronger the minority type in-group bias must be for minority workers to receive a proportional amount of all job offers. This demonstrates that both network density and type in-group bias have equal ability in “compensating” for disproportionality in the distribution of job offers through referral—minority workers can either have more social ties \((\tau_{min} >> \tau_{maj})\) or a “stronger-knit” social network \((\psi_{min} >> \psi_{maj})\).

The compensating network density increases in \( \tau_{maj}, \psi_{maj}, \) and \( \delta \). It decreases in \( \psi_{min} \), which makes sense intuitively. The greater the probability of majority workers having social ties (or the greater the strength of their network), the greater minority workers’ compensating parameters \((\tau_{min}^\sim \) and \( \psi_{min}^\sim \)) must be to achieve a proportional amount of all job offers through referral. Likewise, since both minority parameters affect job offer distribution similarly, as either one increases, the required value of the other parameter to reach proportionality falls.
The above equation for network density and type in-group bias multipliers, \( \rho_r \) and \( \rho_y \), shows a linear relationship between the compensating parameters \( (\tau_{min}^m \text{ and } \psi_{min}^m) \) and the majority and minority network parameters \( (\tau_{maj}^m, \psi_{maj}^m, \tau_{min}^m, \psi_{min}^m) \). However, as Graph 1 illustrates, a nonlinear relationship exists between the compensating parameters and \( \delta \), the fraction of the population majority workers occupy.

**Graph 1**: Network Density / Type In-Group Bias Multiplier for Minority Workers to Receive Proportional Job Referrals

The nonlinear relationship suggests that as majority workers increasingly saturate the labor force, minority workers’ disadvantage multiplies.

With regard to job offers through referral, my employment model shows that, all else equal, majority workers have a disproportionate advantage over minority workers. Importantly, this disadvantage does not depend on the existence of psychological prejudices like racism or

\[ \tau_{maj} = \tau_{min}, \quad \psi_{maj} = \psi_{min} \]
sexism; it arises as an inherent consequence of the interaction of more- and less- “advantageous” social networks. The only assumed difference between majority and minority workers is the relative size of their group. However, because of this difference, not only do majority workers receive a higher proportion of job offers through referral (relative to their population size), but also, as explained below, majority workers receive higher expected wages.

5.2.2 Does simply the existence of a majority and a minority group in the labor market cause inequality in the expected wage between groups, all else equal?

The analysis of expected wage must be broken into two components, the period-2 market wage ($w^{M2}$) and the referral wage ($w^R$). As mentioned earlier, those workers who do not receive any jobs through referral find employment through the market. The first section discusses the effects of introducing a majority and minority group on the market wage. The second section uses these findings, combined with analysis on the referral wage, to identify wage gaps between majority and minority workers.

5.2.2A Market wage ($w^{M2}$)

This section shows how changes in the social network parameters affect market wage. Already, this paper has shown that minority workers, all else equal, receive a disproportionately low fraction of job offers through referral. Workers who do not receive jobs through referral must find employment through the market. As a result, the model predicts that minority workers disproportionately find employment through the general market. Decreases in the market wage ($w^{M2}$) thereby hurt the average welfare of minority workers, relative to that of majority workers.
To arrive at this conclusion, first I derive an expression for period-2 market wage using Bayes’s rule (refer to appendix Equation A.3 for more detail on this derivation) (H\textsubscript{maj} = high-ability major worker and H\textsubscript{min} = high-ability minority worker):

\[
\begin{align*}
w_{M_2} & \left( \delta, \alpha, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min} \right) \\
& = E(\text{productivity} \mid \text{market}) \\
& = \text{Productivity}(H_{maj}) \times \text{Probability}(H_{maj} \mid \text{market}) + \text{Productivity}(H_{min}) \times \text{Probability}(H_{min} \mid \text{market}) \\
& = \exp \left\{ -\frac{1}{2} \left[ \delta(\delta + \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min}) \alpha \tau_{min} \right] \right\} \times \left( \frac{\delta}{2} \right) + \\
& \quad + \exp \left\{ -\frac{1}{2} \left[ \delta(1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(\delta + \psi_{min}) \alpha \tau_{min} \right] \right\} \times \left( \frac{1 - \delta}{2} \right) + \\
& \quad + \exp \left\{ -\frac{1}{2} \left[ \delta(\delta + \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha) \tau_{min} \right] \right\} \times \left( \frac{\delta}{2} \right) + \\
& \quad + \exp \left\{ -\frac{1}{2} \left[ \delta(1 - \delta - \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta + \psi_{min})(1 - \alpha) \tau_{min} \right] \right\} \times \left( \frac{1 - \delta}{2} \right) 
\end{align*}
\]

Recall that the productivity of high-ability workers equals one, while that of low-ability workers equals zero. Given \( \alpha > \frac{1}{2} \) and \( \tau > 0 \), \( w_{M_2} \) is always less than \( \frac{1}{2} \), the average productivity of the population. Graph 2 shows that \( w_{M_2} \) is decreasing in \( \delta, \alpha, \tau_{maj}, \tau_{min} \), and \( \psi_{maj} \), but is increasing in \( \psi_{min} \).
Graph 2.1\(^8\) – \(w_{M2}\) as a Function of \(\delta\)

Graph 2.2\(^0\) – \(w_{M2}\) as a Function of \(\alpha\)

Graph 2.3\(^10\) – \(w_{M2}\) as a Function of \(\tau_{maj}\)

Graph 2.4\(^11\) – \(w_{M2}\) as a Function of \(\tau_{min}\)

\(^8\) \(\alpha = 0.8, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05\)

\(^9\) \(\delta = 0.75, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05\)

\(^10\) \(\delta = 0.75, \ \alpha = 0.8, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05\)

\(^11\) \(\delta = 0.75, \ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05\)
The most novel finding is that \( w_{M2} \) is decreasing in \( \delta, \tau_{maj}, \) and \( \psi_{maj} \). The previous section shows that the proportion of job offers through referral going to minority workers also decreases in \( \delta, \tau_{maj}, \) and \( \psi_{maj} \). This suggests a compounding disadvantage for minority workers. Not only do they receive a lower chance of receiving a job offer through referral (so are disproportionately forced to find employment through the [lower-paying] market), but also the wage they face in that market (\( w_{M2} \)) is driven further down as majority workers occupy a greater fraction of the population (\( \delta \)), have a greater number of social ties (\( \tau_{maj} \)), or have a stronger-“knit” network (\( \psi_{maj} \)). This makes sense, because as these social network parameters increase, an increasing number of high-ability [majority] workers are “taken out” of the market by receiving job offers through referral. This, in turn, drives down the average expected productivity of the remaining workers, decreasing equilibrium market wage.

\[ \delta = 0.75, \alpha = 0.8, \tau_{maj} = 1.0, \tau_{min} = 1.0, \psi_{min} = 0.05 \]

\[ \delta = 0.75, \alpha = 0.8, \tau_{maj} = 1.0, \tau_{min} = 1.0, \psi_{maj} = 0.05 \]
Graph 2 shows $w_{M2}$ is also decreasing in $\tau_{\text{min}}$. However, whether the relationship between market wage and network density is unfavorable for minority workers requires further scrutiny, because the proportion of job offers received by minority workers increases with $\tau_{\text{min}}$. As a result, although an increase in $\tau_{\text{min}}$ increases high-ability minority workers’ chances of receiving a job through referral (and drives up the referral wage ($w_R$)), it also drives down the market wage ($w_{M2}$) for the remaining high-ability minority workers who do not receive a referral. This is because the remaining workers in the market are of lower average ability (due to the effects of ability in-group bias).

**Graph 3** – Referral Proportion and Market Wage Effects as Minority Worker Network Density Increases

\[ \delta = 0.75, \quad \alpha = 0.8, \quad \tau_{\text{maj}} = 1.0, \quad \psi_{\text{maj}} = 0.05, \quad \psi_{\text{min}} = 0.05 \]
Graph 3 shows that the “positive” effects on referral proportion outweigh the negative effects on the market wage, for all possible values of $\tau_{min}$. Thus, high-ability minority workers, on average, are increasingly better off as their probability of having a social tie ($\tau_{min}$) rises. However, by driving down $w_{m2}$ while driving up $w_R$, higher values of network density cause a greater gap between high-ability minority workers who receive offers and those who do not. In other words, minority workers who are “well-connected” become even better off than those who are not, widening wage extremes among high-ability minority workers.

5.2.2B Wage gap

So far, this paper has shown that, all else equal, minority workers are at a disadvantage in getting jobs through referral. As mentioned earlier, for a referral wage to be accepted, it must be greater than or equal to the market wage (otherwise, the period-2 worker would simply forgo the referral and receive a job through the [higher-paying] market). Therefore, one might expect for minority workers to receive lower average wages than majority workers.

To confirm this, one must derive an expression for expected wage (in the expression below, for greater detail on the probability of accepting market wage $w_{m2}$, see appendix Equation A.2; for detail on calculating the average referral wage $w_R^{average}$, see appendix Equation A.6\textsuperscript{15}) ($H_{maj} =$ high-ability majority worker and $L_{maj} =$ low-ability majority worker):

\begin{equation}
\end{equation}

\textsuperscript{15} Since there is no closed-form solution for the referral-offer distribution, $F(\bullet)$ from appendix Equation A.6, $w_R^{average}$ is approximated by averaging $w_R$ for $F(\bullet)$ equaling 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.
\[
E(w_{\text{Majority}}) = \frac{1}{2} E(w_{\text{High ability majority}}) + \frac{1}{2} E(w_{\text{Low ability majority}})
\]

\[
= \frac{1}{2} \left( (1 - \text{Probability}\{H_{\text{maj}} \text{ accepts } w_{M2}\}) \times w_{\text{average}}^{R} + \text{Probability}\{H_{\text{maj}} \text{ accepts } w_{M2}\} \times w_{M2} \right) + \frac{1}{2} \left( (1 - \text{Probability}\{L_{\text{maj}} \text{ accepts } w_{M2}\}) \times w_{\text{average}}^{R} + \text{Probability}\{L_{\text{maj}} \text{ accepts } w_{M2}\} \times w_{M2} \right)
\]

Similarly (\(H_{\text{min}} = \text{high-ability minority worker and } L_{\text{min}} = \text{low-ability minority worker}):

\[
E(w_{\text{Minority}}) = \frac{1}{2} E(w_{\text{High ability minority}}) + \frac{1}{2} E(w_{\text{Low ability minority}})
\]

\[
= \frac{1}{2} \left( (1 - \text{Probability}\{H_{\text{min}} \text{ accepts } w_{M2}\}) \times w_{\text{average}}^{R} + \text{Probability}\{H_{\text{min}} \text{ accepts } w_{M2}\} \times w_{M2} \right) + \frac{1}{2} \left( (1 - \text{Probability}\{L_{\text{min}} \text{ accepts } w_{M2}\}) \times w_{\text{average}}^{R} + \text{Probability}\{L_{\text{min}} \text{ accepts } w_{M2}\} \times w_{M2} \right)
\]

Appendix Equation A.5 shows that with a non-perfectly competitive labor market (due to the effects of social network distortion), the average firm expects to make profits in the second period. In other words, total worker productivity exceeds total worker compensation. Analysis not only confirms this point, but also shows that a wage gap exists between majority and minority workers.16

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16Since there is no closed-form solution for the referral-offer distribution, \(F(\bullet)\) from appendix Equation A.6, all figures in Graph 4 use an approximation of \(w_{\text{average}}^{R}\), which is calculated by averaging \(w_{R}\) for \(F(\bullet)\) equaling 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.
Graph 4.1\textsuperscript{17} – $E(w)$ as a Function of $\delta$

Graph 4.2\textsuperscript{18} – $E(w)$ as a Function of $\alpha$

Graph 4.3\textsuperscript{19} – $E(w)$ as a Function of $\tau_{maj}$

Graph 4.4\textsuperscript{20} – $E(w)$ as a Function of $\tau_{min}$

\textsuperscript{17} $\alpha = 0.8$, $\tau_{maj} = 1.0$, $\tau_{min} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$

\textsuperscript{18} $\delta = 0.75$, $\tau_{maj} = 1.0$, $\tau_{min} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$

\textsuperscript{19} $\delta = 0.75$, $\alpha = 0.8$, $\tau_{min} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$

\textsuperscript{20} $\delta = 0.75$, $\alpha = 0.8$, $\tau_{maj} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$
My model predicts a gap in expected average wages between majority and minority workers. This prediction holds even under the assumption of equivalent ability between both groups. These results strongly suggest that discrimination (measurable differences in average welfare between groups, given equal ability) can exist independently of psychological biases like racism or sexism (which explicitly are not introduced in the employment model). Discrimination may be an inherent consequence of the interaction of more- and less- advantageous social networks.

6 Implications

This paper introduces a new cause of discrimination, one independent from prejudice. Intuitively, social networks provide unequal benefits based on how “well-connected” an individual worker is. However, the model in this paper presents a novel point: when social networks form along characteristics (like race or gender) that create majority and minority

\[ \delta = 0.75, \quad \alpha = 0.8, \quad \tau_{maj} = 1.0, \quad \tau_{min} = 1.0, \quad \psi_{maj} = 0.05 \]

\[ \delta = 0.75, \quad \alpha = 0.8, \quad \tau_{maj} = 1.0, \quad \tau_{min} = 1.0, \quad \psi_{maj} = 0.05 \]
workers—and when workers are more likely to know others with similar characteristics—welfare inequality arises, even given equal average ability among all workers. A new dialogue begins when discrimination is defined as mathematical differences in expected welfare between groups of equal ability, and not necessarily as societal manifestations of psychological prejudice. This distinction links discrimination with concrete actions and states of the world, which can be measured, not with intangible thoughts and states of mind (which cannot be). Discrimination may thus be seen as an unavoidable consequence of the interaction of more- and less-“advantageous” social networks.

This finding does not undermine traditional thoughts on discrimination; rather, it adds to the understanding of discrimination’s roots. Justification for redistributive policies that promote equality often centers on correcting “past wrongs,” or combating continued prejudice. The model in this paper does not challenge these justifications, but instead shows that neither scenario need exist for discrimination to arise. Thus, another justification for redistributive policies surfaces: compensating for recurrent disadvantages inherent in social structure.

This point must not be overlooked. Even though the model does not specify which characteristic creates the majority/minority split (and therefore could theoretically be attributed to gender or social class), research suggests that the dominating characteristic in the United States is ethnicity (Marsden, 1987). Racial minorities not only occupy a smaller fraction of the labor force, but also have, on average, lower network densities (Marsden, 1987). In both aspects, the expected wage and the chance of getting a job through referral, inequality persists between groups. One-fifth of the total difference in the probability of gaining employment between white and black youth is tied to the difference in gaining employment through friends and relatives (Ioannides & Loury, 2004). This is an issue because only 15 percent of unemployed blacks in a
1991 study who used informal contacts (like friends and relatives) in any given month received jobs; the value for whites was 24 percent (Bortnick & Ports, 1992). Also, Korenman and Turner (1996) identify wage differences between whites and blacks who found jobs through contacts. Among workers in inner-city Boston, whites received 19 percent higher wage gains than blacks with similar characteristics. By no means does this research prove that the model in this paper perfectly reflects reality; however, it does suggest, at the very least, that further work may eventually provide a robust empirical defense of how social networks can inherently exacerbate welfare differences between groups. This model takes an important step in introducing a new lens through which to view observed inequality—as a direct consequence of social structure.

More research must be done to bridge the theoretical implications of my model with reality. The predicted inequality between majority and minority workers is based on several assumptions: majority and minority workers have no welfare disparity in the initial state, period 1; the only distinguishable difference between groups is relative size (i.e., ability, network density, and in-group biases can be set as equal); and, workers are more likely to know others with similar characteristics. These assumptions are critical when considering historical examples where minority workers enjoy greater welfare than majority workers (e.g., white South Africans). These historical cases do not necessarily undermine the accuracy of the model, not only because their circumstances clearly violate the model’s assumptions, but also because these cases intimately involve the distorting influence of psychological prejudice, which the employment model does not incorporate.

However, one may still be able to manipulate, albeit crudely, the model’s parameters to [roughly] reflect exceptional cases where minority workers fare better than majority workers. The expressions used to derive expected welfare, appendix Equations A.2 and A.6, defend this
point. By assuming 1) sufficiently low majority network density, because majority workers may not be able to successfully refer acquaintances (prejudiced employer’s may not trust majority workers’ opinions) coupled with 2) high minority in-group bias, because prejudice may even more strongly polarize social networks along the majority/minority split, the model predicts that the expected welfare of minority workers will be greater than that of majority workers.\footnote{If $\delta = 0.6$, $\alpha = 0.8$, $\tau_{maj} = 0$, $\tau_{min} = 1.0$, $\psi_{maj} = 0$, $\psi_{min} = 0.4$, $E(\text{WageMin}) > E(\text{WageMaj})$, or $0.489 > 0.487$}

Further research must also be done to explore how the model’s findings shift as other simplifying assumptions are relaxed. The model aggregates one network density and type in-group bias for all majority workers. However, at higher levels of ability (which often correlates with wealth), class cohesion may dominate. In other words, high-ability minority workers may have closer ties with high-ability majority workers than with low-ability minority workers. Yet, graph 5 shows that the model’s prediction of inequality remains intact and does not fundamentally change even if this simplification is removed.\footnote{Even by assuming that workers have no type cohesion (setting both type in-group biases to zero), inequality between majority and minority workers persist. In graph 5, “class cohesion” dominates for all values.}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{graph5.png}
\caption{$E(w)$ as a Function of $\delta$}
\end{figure}

\footnote{$\alpha = 0.8$, $\tau_{maj} = 1.0$, $\tau_{min} = 1.0$, $\psi_{maj} = 0$, $\psi_{min} = 0$}
The employment model in this paper suggests that the introduction of a majority and minority group (where workers more likely know others who share similar characteristics) inherently gives rise to discrimination, all else equal. Of course, while my model mainly differentiates groups by relative size, there may be other ways that group differences give rise to discrimination. For example, the strength-of-weak ties hypothesis holds that an individual benefits more from a variety of weak ties (acquaintances) than from similar strong ones (close family and friends) (Granovetter, 1973). Weak ties foster greater diversity in knowledge and resource dispersion. As a result, social networks that consist of more weak ties may prove more “advantageous.” Similarly, introducing into the model groups formed along other attributes may also cause measurable differences in welfare.

Overall, the employment model in this paper makes an important step in broadening the dialogue on welfare disparity. No longer is it simply a discussion of how independent groups breed inequality throughout the social structure. Now, it is a discussion of how the social structure breeds inequality, independent of the groups.

7 Conclusion

This paper introduces an alternate cause of discrimination: distortion of the referral market from the introduction of majority and minority workers. The “fairness” of equilibrium market conditions in the model depends on different aspects of network formation, grouping around ability and grouping around majority/minority type. While the former aspect promotes fairness (by increasing the expected wage of high-ability workers), the latter hinders it (by creating an expected wage gap between workers of equal ability, but unequal personal attributes).
Ensuring that job offers purely correlate with ability proves difficult, if not impossible, to realize when social networks so closely impact employment decisions. On the micro level, an individual who is not “well-connected,” either by intention or misfortune, suffers disadvantages. On the macro level, since social networks are not purely correlated with ability, welfare differences arise between entire cross-sections of the labor force, given equal ability. Not only do workers with less “advantageous” social networks expect lower average wages and a smaller chance of getting a job through referral, but also firms’ expected profits decline, due to the distortion in the referral market (see appendix Equation A.5).

Hopefully, more work will be done to better grasp the extent to which social networks breed inequality between majority and minority workers. From this, the justifications for (and implementation of) redistributive responses to inequality should adapt—to acknowledge how inequality is unavoidably reinforced by the pervasiveness of non-identical social networks.

Although this paper does not solve discrimination, hopefully it does provide a useful step in disentangling one root of such disparity.
Appendix: Model Derivation

In this appendix, I formally analyze my social network employment model, which extends Montgomery’s (1991) model. I begin by offering the following propositions to be proved later:

**PROPOSITIONS:** A firm makes a referral offer if and only if it employs a high-ability worker in period 1; the average expected referral wage received by majority workers is greater than that of minority workers of equal ability; referral wage offers are dispersed over the interval $[w_{M2}, \bar{w}_R]$.

First consider a given high-ability period-2 worker (H). Since all referral wage offers exceed the period-2 market wage, the probability that H would accept a referral wage offer $w_{R,i}$ from firm $i$ can be written:

$$
\Pr\{H \text{ accepts } w_{R,i}\} = \Pr\{H \text{ receives no higher offer } w_{R,j} \forall \text{ firm } j \neq i\}
$$

As referral offers are allocated independently,

$$
\Pr\{H \text{ accepts } w_{R,i}\} = \prod_{j \neq i} \Pr\{H \text{ receives no higher offer } w_{R,j}\}
$$

$$
= \prod_{j \neq i} \left[1 - \Pr\{H \text{ receives an offer } w_{R,j} > w_{R,i}\}\right]
$$

The probability that firm $j$ offers a wage $w_{R,j} > w_{R,i}$ to H is in turn the product of two independent probabilities:

$$
\Pr\{H \text{ receives an offer } w_{R,j} > w_{R,i}\} = \Pr\{\text{firm } j \text{ makes offer to } H\} \times \Pr\{w_{R,j} > w_{R,i}\}
$$
If there were $2N$ workers in period-1, free entry implies that $N$ firms employ high-ability workers. Now we will analyze both parts of the expression from the perspective of a high-ability majority ($H_{maj}$) and minority ($H_{min}$) worker. The probability that firm $j$ offers a referral to $H_{maj}$ is a weighted average of whether firm $j$ hired a majority or minority worker in period-1.

Remember, $\delta = \left(\frac{N_{majority}}{N}\right)$:

$$\Pr \begin{cases} \text{firm } j \text{ makes offer to } H_{maj} \\ \text{firm } j \text{ makes offer to } H_{min} \end{cases}$$

$$= \delta \left(\frac{\psi'_{maj} \alpha \tau_{maj}}{2N}\right) + (1 - \delta) \left(\frac{(1 - \psi'_{min}) \alpha \tau_{min}}{2N}\right)$$

$$= \delta \left(\frac{\delta + \psi'_{maj} \alpha \tau_{maj}}{2N}\right) + (1 - \delta) \left(\frac{\delta - \psi'_{min} \alpha \tau_{min}}{2N}\right)$$

Likewise, for a period-2 minority high-ability worker:

$$= \delta \left(\frac{(1 - \psi'_{maj}) \alpha \tau_{maj}}{2N}\right) + (1 - \delta) \left(\frac{\psi'_{min} \alpha \tau_{min}}{2N}\right)$$

$$= \delta \left(\frac{(1 - \delta - \psi'_{maj}) \alpha \tau_{maj}}{2N}\right) + (1 - \delta) \left(\frac{(1 - \delta + \psi'_{min}) \alpha \tau_{min}}{2N}\right)$$

Now, one can compare the probability of receiving a wage in period-2 for a high-ability majority worker versus a high-ability minority worker:
The network density of minority workers must be for them to have a proportional chance of receiving a job offer through referral. For example, if majority workers comprise 80% of the population (δ = 0.8) and both majority and minority workers share the same strength of type ingroup bias (ψ_min = ψ_maj), then ρτ = 4. In other words, for minority workers to receive a proportional amount (or 20%) of all job offers through referral in the next period, the probability of them having a tie in the current period must be four times greater than that of majority workers (τ_min = 4τ_maj). Similarly, the value ρ_ψ = \left(\frac{\delta}{1-\delta}\right)\left(\frac{\tau_maj}{\tau_min}\right) is a multiplier that demonstrates how many times stronger the minority type ingroup bias must be for minority workers to receive a proportional amount of all job offers. This demonstrates that both network
density and type in-group bias have equal ability in “compensating” for disproportionality in the
distribution of job offers through referral—minority workers can either have more social ties
($\tau_{\text{min}} \gg \tau_{\text{maj}}$) or a “stronger-knit” social network ($\psi_{\text{min}} \gg \psi_{\text{maj}}$).

The compensating network density increases in $\tau_{\text{maj}}$, $\psi_{\text{maj}}$, and $\delta$. It decreases in $\psi_{\text{min}}$, which makes sense intuitively. The greater the probability of majority workers having social ties
(or the greater the strength of their network), the greater minority workers’ compensating
parameters ($\tau_{\text{min}}$ and $\psi_{\text{min}}$) must be to achieve a proportional amount of all job offers through
referral. Likewise, since both minority parameters increase job dispersion similarly, as either
one increases, the required value of the other parameter to reach proportionality falls.

**Network Density / Type In-Group Bias Multiplier for Minority Workers to Receive Proportional Amount of Job Offers Through Referral**

![Graph showing the relationship between $\delta$ (Majority/Total) and $\rho$ (Parameter Multiplier)]
All else equal, the multiplier is nonlinear with respect to the majority’s fraction of the population \((\delta)\). In other words, the compensating parameters must increasingly get larger as majority workers occupy a greater fraction of the population.

If there were \(2N\) workers in period-1, free entry implies that \(N\) firms employ high-ability workers. If each firm chooses its referral wage offer by randomizing over the equilibrium wage distribution \(F(o)\) (to be derived below),

\[
\Pr \{ H_{maj} \text{ receives an offer } w_{Rj} > w_{R,i} \} = \left[ \delta \left( \frac{\delta + \psi_{maj}}{2N} \right) \right] + \left(1-\delta\right) \left( \frac{\psi_{maj} - \psi_{min}}{2N} \right) \times \left[ 1 - F(w_{R,i}) \right]
\]

and

\[
\Pr \{ H_{min} \text{ receives an offer } w_{Rj} > w_{R,i} \} = \left[ \delta \left( \frac{1 - \delta - \psi_{maj}}{2N} \right) \right] + \left(1-\delta\right) \left( \frac{1 - \delta + \psi_{min}}{2N} \right) \times \left[ 1 - F(w_{R,i}) \right]
\]

for all firms \(j\) employing a high-ability worker in period-1. Substitution yields:

\[
\Pr \{ H_{maj} \text{ accepts } w_{R,i} \} = \left\{ 1 - \left[ \delta \frac{\psi_{maj}}{N} + (1-\delta) \frac{(\delta - \psi_{min})}{2N} \right] \times \left[ 1 - F(w_{R,i}) \right] \right\}^{N-1}
\]

and

\[
\Pr \{ H_{min} \text{ accepts } w_{R,i} \} = \left\{ 1 - \left[ \delta \frac{(1 - \delta - \psi_{maj})}{N} + (1-\delta) \frac{(1 - \delta + \psi_{min})}{2N} \right] \times \left[ 1 - F(w_{R,i}) \right] \right\}^{N-1}
\]
Since the model assumes a large number of workers, as \( N \) approaches \( \infty \),

\[
\Pr \{ H_{\text{maj}} \text{ accepts } w_{R,i} \} \\
= \exp\left\{-\frac{1}{2} \left[ \delta (\delta + \psi_{\text{maj}}) \alpha \tau_{\text{maj}} + (1-\delta)(\delta - \psi_{\text{min}}) \alpha \tau_{\text{min}} \right] [1 - F(w_{R,i})] \right\}
\]

and

\[
\Pr \{ H_{\text{min}} \text{ accepts } w_{R,i} \} \\
= \exp\left\{-\frac{1}{2} \left[ \delta (1 - \delta - \psi_{\text{maj}}) \alpha \tau_{\text{maj}} + (1-\delta)(1 - \delta + \psi_{\text{min}}) \alpha \tau_{\text{min}} \right] [1 - F(w_{R,i})] \right\}
\]

Details on this step can found in existing labor market papers (Rapoport, 1963). Through similar analysis, one obtains the probability that firm \( i \)'s offer is accepted by a given low-ability worker (L):

\[
\Pr \{ L_{\text{maj}} \text{ accepts } w_{R,i} \} \\
= \exp\left\{-\frac{1}{2} \left[ \delta (\delta + \psi_{\text{maj}})(1 - \alpha) \tau_{\text{maj}} + (1-\delta)(\delta - \psi_{\text{min}})(1-\alpha) \tau_{\text{min}} \right] [1 - F(w_{R,i})] \right\}
\]

and

\[
\Pr \{ L_{\text{min}} \text{ accepts } w_{R,i} \} \\
= \exp\left\{-\frac{1}{2} \left[ \delta (1 - \delta - \psi_{\text{maj}})(1-\alpha) \tau_{\text{maj}} + (1-\delta)(1 - \delta + \psi_{\text{min}})(1-\alpha) \tau_{\text{min}} \right] [1 - F(w_{R,i})] \right\}
\]

Montgomery (1991) states that, conditional upon the offer being received by a given worker, high-ability workers (and majority workers) are less likely to accept any offer \( w_{R,i} < \bar{w}_R \) than low-ability workers (and minority workers). This is because both high-ability and majority workers tend to receive more offers. (For \( w_{R,i} = \bar{w}_R \), however, both probabilities are equal to 1: since no firm offers a higher wage, workers always accept.)
Since a period-2 worker finds employment through the market only if he receives no offers (or rejects all referral offers):

\[
\Pr \{ \text{market} \mid H_{\text{maj}} \} = \Pr \{ H_{\text{maj}} \text{ accepts } w_{M2} \}
\]

\[
\Pr \{ \text{market} \mid H_{\text{min}} \} = \Pr \{ H_{\text{min}} \text{ accepts } w_{M2} \}
\]

\[
\Pr \{ \text{market} \mid L_{\text{maj}} \} = \Pr \{ L_{\text{maj}} \text{ accepts } w_{M2} \}
\]

\[
\Pr \{ \text{market} \mid L_{\text{min}} \} = \Pr \{ L_{\text{min}} \text{ accepts } w_{M2} \}
\]

The market wage coincides with the bottom of the referral wage distribution, \( F(\bullet) \), because any referral wage below the market wage will be rejected by period-2 workers, to gain employment through the market. Thus, given that \( F(w_{M2}) = 0 \), Equation A.2 shows::

\[
\text{A.2) } \Pr \{ \text{market} \mid H_{\text{maj}} \} = \exp \left\{ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj}) \tau_{maj} + (1 - \delta)(\delta - \psi_{min}) \alpha \tau_{min} \right] \right\}
\]

\[
\Pr \{ \text{market} \mid H_{\text{min}} \} = \exp \left\{ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min}) \alpha \tau_{min} \right] \right\}
\]

\[
\Pr \{ \text{market} \mid L_{\text{maj}} \} = \exp \left\{ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha) \tau_{min} \right] \right\}
\]

\[
\Pr \{ \text{market} \mid L_{\text{min}} \} = \exp \left\{ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha) \tau_{min} \right] \right\}
\]
Since I have assumed a continuum of workers, one may use Bayes’s rule, as in Equation A.3 below, to calculate the period-2 market wage:

\[ w_{M2} (\delta, \alpha, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) \]

\[ = E(\text{productivity} \mid \text{market}) \]

\[ = \text{Productivity}(H_{maj}) \times \Pr\{H_{maj} \mid \text{market}\} + \text{Productivity}(H_{min}) \times \Pr\{H_{min} \mid \text{market}\} \]

\[ = \left\{ \begin{array}{c} \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} \\ \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} + \Pr\{\text{market} \mid H_{min}\} \times \Pr\{H_{min}\} \end{array} \right\} \]

\[ + \left\{ \begin{array}{c} \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} \\ \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} + \Pr\{\text{market} \mid L_{min}\} \times \Pr\{L_{min}\} \end{array} \right\} \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} \\ \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} + \Pr\{\text{market} \mid H_{min}\} \times \Pr\{H_{min}\} \end{array} \right) \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} \\ \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} + \Pr\{\text{market} \mid L_{min}\} \times \Pr\{L_{min}\} \end{array} \right) \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} \\ \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} + \Pr\{\text{market} \mid H_{min}\} \times \Pr\{H_{min}\} \end{array} \right) \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} \\ \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} + \Pr\{\text{market} \mid L_{min}\} \times \Pr\{L_{min}\} \end{array} \right) \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} \\ \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} + \Pr\{\text{market} \mid H_{min}\} \times \Pr\{H_{min}\} \end{array} \right) \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} \\ \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} + \Pr\{\text{market} \mid L_{min}\} \times \Pr\{L_{min}\} \end{array} \right) \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} \\ \Pr\{\text{market} \mid H_{maj}\} \times \Pr\{H_{maj}\} + \Pr\{\text{market} \mid H_{min}\} \times \Pr\{H_{min}\} \end{array} \right) \]

\[ \left( \begin{array}{c} \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} \\ \Pr\{\text{market} \mid L_{maj}\} \times \Pr\{L_{maj}\} + \Pr\{\text{market} \mid L_{min}\} \times \Pr\{L_{min}\} \end{array} \right) \]
Let

\[ e^{HMAJ} = \exp\left\{ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj}) \alpha \tau_{maj} + (1 - \delta) (\delta - \psi_{min}) \alpha \tau_{min} \right] \right\} \]

\[ e^{HMIN} = \exp\left\{ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta) (1 - \delta + \psi_{min}) \alpha \tau_{min} \right] \right\} \]

\[ e^{LMAJ} = \exp\left\{ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj}) (1 - \alpha) \tau_{maj} + (1 - \delta) (\delta - \psi_{min}) (1 - \alpha) \tau_{min} \right] \right\} \]

\[ e^{LMIN} = \exp\left\{ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj}) (1 - \alpha) \tau_{maj} + (1 - \delta) (1 - \delta + \psi_{min}) (1 - \alpha) \tau_{min} \right] \right\} \]

So to simplify,

\[
\begin{align*}
 w_{M2} (\delta, \alpha, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) &= \frac{e^{HMAJ} \left( \frac{\delta}{2} \right) + e^{HMIN} \left( \frac{1 - \delta}{2} \right)}{e^{HMAJ} \left( \frac{\delta}{2} \right) + e^{LMAJ} \left( \frac{\delta}{2} \right) + e^{HMIN} + e^{LMIN} \left( \frac{1 - \delta}{2} \right)}
\end{align*}
\]

Montgomery (1991) mentions that if \( N \) were finite, “the use of Bayes’s rule would be inappropriate: the market wage would be stochastic and would depend upon the realized allocation of social ties” (p. 1415). Given \( \alpha > \frac{1}{2} \) and both network densities (\( \tau_{maj} \) and \( \tau_{min} \)) are greater than zero, \( w_{M2} \) is always less than \( \frac{1}{2} \), the average productivity of the population. Analysis shows that \( w_{M2} \) is decreasing in \( \delta, \alpha, \tau_{maj}, \tau_{min}, \) and \( \psi_{maj} \), but increasing in \( \psi_{min} \).
$w_{M2}$ as a function of $\delta^{26}$

$w_{M2}$ as a function of $\alpha^{27}$

$w_{M2}$ as a function of $\tau_{maj}^{28}$

$w_{M2}$ as a function of $\tau_{min}^{29}$

$\delta = 0.8$, $\tau_{maj} = 1.0$, $\tau_{min} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$

$\delta = 0.75$, $\tau_{maj} = 1.0$, $\tau_{min} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$

$\delta = 0.75$, $\alpha = 0.8$, $\tau_{min} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$

$\delta = 0.75$, $\alpha = 0.8$, $\tau_{maj} = 1.0$, $\psi_{maj} = 0.05$, $\psi_{min} = 0.05$
Now consider the expected period-2 profit earned by a firm employing a high-ability majority worker and setting a referral wage $w_R$ (remember the productivity of high-ability workers equals one, while that of low-ability workers equals zero):

$$E \Pi_{H_{maj}}(w_R) = \left( \Pr \{ \text{high ability majority period2 referred hired } | \ w_R \} \times (1 - w_R) + \Pr \{ \text{high ability minority period2 referred hired } | \ w_R \} \times (1 - w_R) + \Pr \{ \text{low ability majority period2 referred hired } | \ w_R \} \times (-w_R) + \Pr \{ \text{low ability minority period2 referred hired } | \ w_R \} \times (-w_R) \right)$$

(If no referred worker is hired, either because the period-1 worker holds no social tie or because the referral receives a better offer, the firm hires through the market and receives zero expected profit.)

$^{30} \delta = 0.75, \ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05$

$^{31} \delta = 0.75, \ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05$
The probability of hiring a high-ability period-2 referred worker is the product of two independent probabilities:

\[
\Pr \{ \text{high ability period-2 majority referral hired} \mid w_R \} = \Pr \{ \text{offer made to a high ability majority referral} \} \times \Pr \{ H_{\text{maj}} \text{ accepts } w_R \}
\]

\[
= \left( \left( \delta (\delta + \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min}) \alpha \tau_{min} \right) \right)
\times \exp\left\{ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min}) \alpha \tau_{min} \right] \left[1 - F(w_{R,i})\right] \right\}
\]

\[
\Pr \{ \text{high ability period-2 minority referral hired} \mid w_R \} = \Pr \{ \text{offer made to a high ability minority referral} \} \times \Pr \{ H_{\text{min}} \text{ accepts } w_R \}
\]

\[
= \left( \left( \delta (1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min}) \alpha \tau_{min} \right) \right)
\times \exp\left\{ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min}) \alpha \tau_{min} \right] \left[1 - F(w_{R,i})\right] \right\}
\]

Similarly, the probability of hiring a low-ability worker may be written

\[
\Pr \{ \text{low ability period-2 majority referral hired} \mid w_R \}
\]

\[
= \left( \left( \delta (\delta + \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha) \tau_{min} \right) \right)
\times \exp\left\{ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha) \tau_{min} \right] \left[1 - F(w_{R,i})\right] \right\}
\]

\[
\Pr \{ \text{low ability period-2 minority referral hired} \mid w_R \}
\]

\[
= \left( \left( \delta (1 - \delta - \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha) \tau_{min} \right) \right)
\times \exp\left\{ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha) \tau_{min} \right] \left[1 - F(w_{R,i})\right] \right\}
By substitution,

\[ E \Pi_H (w_R) \]

\[
\left( \left( \delta (\delta + \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min}) \alpha \tau_{min} \right) 
\times \exp \left[ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min}) \alpha \tau_{min} \right] \right] \right) 
\times (1 - w_R)
\]

\[
\left( \left( \delta (1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min}) \alpha \tau_{min} \right) 
\times \exp \left[ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min}) \alpha \tau_{min} \right] \right] \right) 
\times (1 - w_R)
\]

\[
\left( \left( \delta (\delta + \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha) \tau_{min} \right) 
\times \exp \left[ -\frac{1}{2} \left[ \delta (\delta + \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha) \tau_{min} \right] \right] \right) 
\times (-w_R)
\]

\[
\left( \left( \delta (1 - \delta - \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha) \tau_{min} \right) 
\times \exp \left[ -\frac{1}{2} \left[ \delta (1 - \delta - \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha) \tau_{min} \right] \right] \right) 
\times (-w_R)
\]

Let

\[
p^{HM} = \delta (\delta + \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min}) \alpha \tau_{min}
\]

\[
p^{HN} = \delta (1 - \delta - \psi_{maj}) \alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min}) \alpha \tau_{min}
\]

\[
p^{LM} = \delta (\delta + \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha) \tau_{min}
\]

\[
p^{LN} = \delta (1 - \delta - \psi_{maj})(1 - \alpha) \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha) \tau_{min}
\]
So, to simplify in Equation A.4:

\[
A.4) \quad E \Pi_H (w_R) = \left\{ p_{HMAJ} \times \exp \left( -\frac{1}{2} p_{HMAJ} [1 - F(w_{R,i})] \right) \times (1 - w_R) \right\} + \left\{ p_{HMIN} \times \exp \left( -\frac{1}{2} p_{HMIN} [1 - F(w_{R,i})] \right) \times (1 - w_R) \right\} \\
+ \left\{ p_{LMAJ} \times \exp \left( -\frac{1}{2} p_{LMAJ} [1 - F(w_{R,i})] \right) \times (1 - w_R) \right\} + \left\{ p_{LMIN} \times \exp \left( -\frac{1}{2} p_{LMIN} [1 - F(w_{R,i})] \right) \times (1 - w_R) \right\}
\]

To maintain equilibrium wage dispersion, firms must earn the same expected profit on each referral wage offered:

\[
E \Pi_H (w_R) = c \quad \forall \, w_R \in \left[ w_{M2}, \bar{w_R} \right]
\]

To determine this constant, note that the firm could potentially deviate from its specified strategy by offering a wage of \( w_{M2} \); the referred worker accepts the firm’s offer only if no other offers have been received.
Remember that $F(w_{M2}) = 0$. The firm’s expected profit is therefore given by:

$$E\Pi_H(w_{M2})$$

\[
\begin{align*}
&\left(\delta(\delta + \psi_{maj})\alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min})\alpha \tau_{min}\right) \\
&\times \exp\left\{-\frac{1}{2}\left[\delta(\delta + \psi_{maj})\alpha \tau_{maj} + (1 - \delta)(\delta - \psi_{min})\alpha \tau_{min}\right]\right\} \times (1 - w_{M2}) \\
&\left(\delta(1 - \delta - \psi_{maj})\alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})\alpha \tau_{min}\right) \\
&\times \exp\left\{-\frac{1}{2}\left[\delta(1 - \delta - \psi_{maj})\alpha \tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})\alpha \tau_{min}\right]\right\} \times (1 - w_{M2}) \\
&\left(\delta(\delta + \psi_{maj})(1 - \alpha)\tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha)\tau_{min}\right) \\
&\times \exp\left\{-\frac{1}{2}\left[\delta(\delta + \psi_{maj})(1 - \alpha)\tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha)\tau_{min}\right]\right\} \times (-w_{M2}) \\
&\left(\delta(1 - \delta - \psi_{maj})(1 - \alpha)\tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha)\tau_{min}\right) \\
&\times \exp\left\{-\frac{1}{2}\left[\delta(1 - \delta - \psi_{maj})(1 - \alpha)\tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})(1 - \alpha)\tau_{min}\right]\right\} \times (-w_{M2})
\end{align*}
\]

= c.

Or, to simplify:

$$E\Pi_H(w_{M2})$$

\[
= (p^{HMAJ}_H)(e^{HMAJ}_H)(1 - w_{M2}) + (p^{HMIN}_H)(e^{HMIN}_H)(1 - w_{M2}) + (p^{LMAJ}_e)(e^{LMAJ}_e)(-w_{M2}) + (p^{LMIN}_e)(e^{LMIN}_e)(-w_{M2})
\]

= c
Substituting for \( w_{M2} \) in Equation A.5 below gives:

\[
A.5) \quad c(\delta, \alpha, \tau_{\text{maj}}, \tau_{\text{min}}, \psi_{\text{maj}}, \psi_{\text{min}}) = \left( p_{\text{HMAJ}} e_{\text{HMAJ}} \right) \left( 1 - \frac{e_{\text{HMAJ}} \left( \frac{\alpha}{2} \right) + e_{\text{HMIN}} \left( 1 - \frac{\alpha}{2} \right)}{e_{\text{HMAJ}} + e_{\text{LMAJ}}} \right) + \\
\left( p_{\text{HMIN}} e_{\text{HMIN}} \right) \left( 1 - \frac{e_{\text{HMAJ}} \left( \frac{\alpha}{2} \right) + e_{\text{HMIN}} \left( 1 - \frac{\alpha}{2} \right)}{e_{\text{HMAJ}} + e_{\text{LMAJ}}} \right) + \\
\left( p_{\text{LMAJ}} e_{\text{LMAJ}} \right) \left( -\frac{e_{\text{HMAJ}} \left( \frac{\alpha}{2} \right) - e_{\text{HMIN}} \left( 1 - \frac{\alpha}{2} \right)}{e_{\text{HMAJ}} + e_{\text{LMAJ}}} \right) + \\
\left( p_{\text{LMIN}} e_{\text{LMIN}} \right) \left( -\frac{e_{\text{HMAJ}} \left( \frac{\alpha}{2} \right) - e_{\text{HMIN}} \left( 1 - \frac{\alpha}{2} \right)}{e_{\text{HMAJ}} + e_{\text{LMAJ}}} \right) \right)
\]

Given \( \alpha > \frac{1}{2} \) and \( c > 0 \), firms with high-ability workers with social ties earn positive expected profits. Below shows that \( c \) is increasing in \( \alpha, \tau_{\text{maj}}, \) and \( \tau_{\text{min}} \). \( c \) is decreasing in \( \delta \) and \( \psi_{\text{maj}} \).

---

32 \( \alpha = 0.8, \tau_{\text{maj}} = 1.0, \tau_{\text{min}} = 1.0, \psi_{\text{maj}} = 0.05, \psi_{\text{min}} = 0.05 \)

33 \( \delta = 0.75, \tau_{\text{maj}} = 1.0, \tau_{\text{min}} = 1.0, \psi_{\text{maj}} = 0.05, \psi_{\text{min}} = 0.05 \)
As the number of social ties increases, so do firms’ profits, because there is a greater chance of hiring a high-ability worker. Conversely, the negative correlations with $\delta$ and $\psi_{maj}$ suggest that as distortion from the introduction of majority and minority workers grows, so does firms’ loss.

$34 \delta = 0.75, \alpha = 0.8, \tau_{min} = 1.0, \psi_{maj} = 0.05, \psi_{min} = 0.05$

$35 \delta = 0.75, \alpha = 0.8, \tau_{maj} = 1.0, \psi_{maj} = 0.05, \psi_{min} = 0.05$

$36 \delta = 0.75, \alpha = 0.8, \tau_{maj} = 1.0, \tau_{min} = 1.0, \psi_{min} = 0.05$

$37 \delta = 0.75, \alpha = 0.8, \tau_{maj} = 1.0, \tau_{min} = 1.0, \psi_{maj} = 0.05$
of profits. Initially, $\psi_{\text{min}}$ helps correct for referral market distortions by increasing the chances of minority workers receiving referrals; however, eventually $\psi_{\text{min}}$ “overcompensates.” At this point, any marginal increase in $\psi_{\text{min}}$ drives down expected profits.

Given the preceding expressions for $c$, the equilibrium referral-offer distribution $F(\bullet)$ may be determined by setting $E \Pi_H (w_R)$ equal to $c$ for all potential wage offers $w_R$:

$$E \Pi_H (w_R) = c$$

$$\left[ p_{\text{HMAJ}}^{\psi_{\text{HMAJ}}} \times \exp\left\{ -\frac{1}{2} \left[ p_{\text{HMAJ}}^{\psi_{\text{HMAJ}}} \mathbb{E}[1 - F(w_{R,i})] \right] \times (1 - w_R) \right\} + \left( p_{\text{HMIN}}^{\psi_{\text{HMIN}}} \times \exp\left\{ -\frac{1}{2} \left[ p_{\text{HMIN}}^{\psi_{\text{HMIN}}} \mathbb{E}[1 - F(w_{R,i})] \right] \times (1 - w_R) \right\} \right]$$

$$\left[ p_{\text{LMAJ}}^{\psi_{\text{LMAJ}}} \times \exp\left\{ -\frac{1}{2} \left[ p_{\text{LMAJ}}^{\psi_{\text{LMAJ}}} \mathbb{E}[1 - F(w_{R,i})] \right] \times (1 - w_R) \right\} + \left( p_{\text{LMIN}}^{\psi_{\text{LMIN}}} \times \exp\left\{ -\frac{1}{2} \left[ p_{\text{LMIN}}^{\psi_{\text{LMIN}}} \mathbb{E}[1 - F(w_{R,i})] \right] \times (1 - w_R) \right\} \right]$$

$$= \left\{ p_{\text{HMAJ}}^{\psi_{\text{HMAJ}}} \left( 1 - \frac{e^{\psi_{\text{HMAJ}} \left( \frac{\delta}{2} \right)}}{e^{\psi_{\text{HMAJ}} \delta / 2} + e^{\psi_{\text{HMIN}} \left( 1 - \frac{\delta}{2} \right)}} \right) + \right\}$$

$$\left\{ p_{\text{HMIN}}^{\psi_{\text{HMIN}}} \left( 1 - \frac{e^{\psi_{\text{HMIN}} \left( 1 - \frac{\delta}{2} \right)}}{e^{\psi_{\text{HMAJ}} \delta / 2} + e^{\psi_{\text{HMIN}} \left( 1 - \frac{\delta}{2} \right)}} \right) + \right\}$$

$$\left\{ p_{\text{LMAJ}}^{\psi_{\text{LMAJ}}} \left( \frac{e^{\psi_{\text{LMAJ}} \left( \frac{\delta}{2} \right)} - e^{\psi_{\text{MIN}} \left( 1 - \frac{\delta}{2} \right)}}{e^{\psi_{\text{LMAJ}} \delta / 2} + e^{\psi_{\text{LMIN}} \left( 1 - \frac{\delta}{2} \right)}} \right) + \right\}$$

$$\left\{ p_{\text{LMIN}}^{\psi_{\text{LMIN}}} \left( \frac{e^{\psi_{\text{MIN}} \left( 1 - \frac{\delta}{2} \right)} - e^{\psi_{\text{LMAJ}} \left( \frac{\delta}{2} \right)}}{e^{\psi_{\text{LMAJ}} \delta / 2} + e^{\psi_{\text{LMIN}} \left( 1 - \frac{\delta}{2} \right)}} \right) \right\}$$

\[ \forall w_R \in [\underline{w}_2, \bar{w}_R] \]
Unfortunately, the previous equation does not yield a closed-form solution for $F(w_r)$. As a result, the expression for any given referral wage can only be expressed as:

\[
\begin{align*}
\text{A.6) } \quad w_r(\delta_0, \alpha, \tau_{maj}, \tau_{min}, \Psi_{maj}, \Psi_{min}, F(w_r)) & = \left\{ \begin{array}{l}
\pi^{\text{HMAJ}} \times \exp \left\{ -\frac{1}{2} \left( \pi^{\text{HMAJ}} \right) \left[ 1 - F(w_r) \right] \right\} + \pi^{\text{HMIN}} \times \exp \left\{ -\frac{1}{2} \left( \pi^{\text{HMIN}} \right) \left[ 1 - F(w_r) \right] \right\} - \\
\left( \pi^{\text{HMAJ}} \right) \left( 1 - \frac{\pi^{\text{HMAJ}} (1 - \delta_0)}{2} + \frac{\pi^{\text{HMIN}} (1 - \delta_0)}{2} \right) - \\
\left( \pi^{\text{HMIN}} \right) \left( 1 - \frac{\pi^{\text{HMAJ}} (1 - \delta_0)}{2} + \frac{\pi^{\text{HMIN}} (1 - \delta_0)}{2} \right) - \\
\left( \pi^{\text{LMAJ}} \right) \left( 1 - \frac{\pi^{\text{HMAJ}} (1 - \delta_0)}{2} + \frac{\pi^{\text{HMIN}} (1 - \delta_0)}{2} \right) - \\
\left( \pi^{\text{LMIN}} \right) \left( 1 - \frac{\pi^{\text{HMAJ}} (1 - \delta_0)}{2} + \frac{\pi^{\text{HMIN}} (1 - \delta_0)}{2} \right)
\end{array} \right. \\
\left\{ \begin{array}{l}
\pi^{\text{HMAJ}} \times \exp \left\{ -\frac{1}{2} \left( \pi^{\text{HMAJ}} \right) \left[ 1 - F(w_r) \right] \right\} + \pi^{\text{HMIN}} \times \exp \left\{ -\frac{1}{2} \left( \pi^{\text{HMIN}} \right) \left[ 1 - F(w_r) \right] \right\} + \\
\left( \pi^{\text{LMAJ}} \right) \left( 1 - \frac{\pi^{\text{HMAJ}} (1 - \delta_0)}{2} + \frac{\pi^{\text{HMIN}} (1 - \delta_0)}{2} \right) - \\
\left( \pi^{\text{LMIN}} \right) \left( 1 - \frac{\pi^{\text{HMAJ}} (1 - \delta_0)}{2} + \frac{\pi^{\text{HMIN}} (1 - \delta_0)}{2} \right)
\end{array} \right. \\
\end{align*}
\]

Given a continuum of firms, the equilibrium referral-offer distribution $F(\bullet)$ in Equation A.6 may be interpreted in two ways: either each firm randomizes over the entire distribution, or else a fraction $f(w_r)$ of firms offer each wage for sure. From the second interpretation, one can estimate the average referral wage received by a majority worker vs. a minority worker, for any given $\delta_0$, $\alpha$, $\tau_{maj}$, $\tau_{min}$, $\Psi_{maj}$, and $\Psi_{min}$.
To calculate this wage,

\[ E(w_{Hmaj}) = \frac{1}{n} \sum_{i=1}^{n} \left( \Pr \{ \text{high ability majority period referral hired} \mid w_{Ri} \} \times w_{Ri} \right) \]

\[ = \frac{1}{n} \sum_{i=1}^{n} \left( \Pr \{ \text{offer made to a high ability majority referral} \} \times \Pr \{ H_{maj} \text{ accepts } w_{Ri} \} \times w_{Ri} \right) \]

\[ = \frac{1}{n} \sum_{i=1}^{n} \left( \delta(\delta + \psi_{maj}) \alpha \tau_{maj} + (1-\delta)(\delta - \psi_{min}) \alpha \tau_{min} \right) \times \exp\left( -\frac{1}{2} \left[ \delta(\delta + \psi_{maj}) \alpha \tau_{maj} + (1-\delta)(1-\delta + \psi_{min}) \alpha \tau_{min} \right] \right) \times \left( 1 - F(w_{Ri}) \right) \times w_{Ri} \]

\[ = \frac{1}{n} \sum_{i=1}^{n} \left( p^{HMAJ} \times \exp\left( -\frac{1}{2} \left[ \delta(\delta + \psi_{maj}) \alpha \tau_{maj} + (1-\delta)(\delta - \psi_{min}) \alpha \tau_{min} \right] \right) \times \left( 1 - F(w_{Ri}) \right) \right) \times w_{Ri} \]

Similarly,

\[ E(w_{Hmin}) = \frac{1}{n} \sum_{i=1}^{n} \left( p^{HMIN} \times \exp\left( -\frac{1}{2} \left[ (1-\delta - \psi_{maj}) \alpha \tau_{maj} + (1-\delta)(1-\delta + \psi_{min}) \alpha \tau_{min} \right] \right) \times \left( 1 - F(w_{Ri}) \right) \right) \times w_{Ri} \]

\[ E(w_{Lmaj}) = \frac{1}{n} \sum_{i=1}^{n} \left( p^{LMAJ} \times \exp\left( -\frac{1}{2} \left[ \delta(\delta + \psi_{maj}) (1-\alpha) \tau_{maj} + (1-\delta)(\delta - \psi_{min}) (1-\alpha) \tau_{min} \right] \right) \times \left( 1 - F(w_{Ri}) \right) \right) \times w_{Ri} \]

\[ E(w_{Lmin}) = \frac{1}{n} \sum_{i=1}^{n} \left( p^{LMIN} \times \exp\left( -\frac{1}{2} \left[ (1-\delta - \psi_{maj}) (1-\alpha) \tau_{maj} + (1-\delta)(1-\delta + \psi_{min}) (1-\alpha) \tau_{min} \right] \right) \times \left( 1 - F(w_{Ri}) \right) \right) \times w_{Ri} \]

Analysis shows that, all else equal, as long as \( \alpha > 0 \) and \( \delta > \frac{1}{2} \), \( E(w_{Hmaj}) > E(w_{Hmin}) \). In other words, the expected referral wage received by high-ability majority workers is greater than that received by high-ability minority workers. This inequality holds even when minority workers have the compensating network parameters (\( \tau_{\min}^n \) and \( \psi_{\min}^n \)) to receive a proportional amount of all job offers through referral. Similarly, \( E(w_{Lmaj}) > E(w_{Lmin}) \). These results show
how the introduction of majority and minority workers (where people more likely know others with similar qualities) causes unequal expected wages among workers of equal ability, thereby suggesting how discrimination can exist even in the absence of psychological prejudices like racism or sexism.

To complete the proposition, we must derive an expression for the maximum wage offered. Expanding and manipulating the above equation for equilibrium referral wage and solving for $w_R$ (where $F(w_R)=1$, by definition) gives:

$$w_R(\delta, \alpha, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})$$

$$= \frac{\delta \alpha \tau_{maj} + (1 - \delta) \alpha \tau_{min}}{\delta \tau_{maj} + (1 - \delta) \tau_{min}} - \frac{c}{\delta \tau_{maj} + (1 - \delta) \tau_{min}}$$
Intuitively, a firm offering a referral wage of \( w_R \) attracts a referred worker with probability 1 (conditional upon its worker holding a social tie). The firm’s expected profit, \( c \), is thus equal to
\[- w_R \left( \delta \tau_{maj} + (1 - \delta) \tau_{min} \right) + \delta \alpha \tau_{maj} + (1 - \delta) \alpha \tau_{min}. \]
\( w_R \) is decreasing in \( \psi_{maj} \) and increasing in \( \delta, \alpha, \tau_{maj}, \) and \( \tau_{min}. \) Its relationship with \( \psi_{min} \) depends on the magnitude of \( \psi_{min}. \)

\[ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05 \]
\[ \delta = 0.75, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05 \]
\[ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05 \]
\[ \delta = 0.75, \ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \psi_{maj} = 0.05, \ \psi_{min} = 0.05 \]
The preceding analysis has already established that firms employing high-ability workers in period 1 will make referrals offers: while hiring through the market generates zero expected profit, a referral offer generates constant positive profit over the range \([w_{M2}, \bar{w}_R]\). A lower offer will never be accepted, while a higher offer increases the wage without increasing the probability of attracting a worker. To complete the proof of the initial proposition, I now demonstrate that firms employing low-ability workers in period 1 will hire through the market.

\[\delta = 0.75, \ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{min} = 0.05\]

\[\delta = 0.75, \ \alpha = 0.8, \ \tau_{maj} = 1.0, \ \tau_{min} = 1.0, \ \psi_{maj} = 0.05\]
If such a firm were to deviate from the proposed equilibrium, making a referral offer \( w_r \),
its expected profit would be written

\[
E \Pi_L (w_r)
\]

\[
= \left( \left( \delta \left( \delta + \psi_{maj} \right) \left( 1 - \alpha \right) \tau_{maj} + \left( 1 - \delta \right) \left( \delta - \psi_{min} \right) \left( 1 - \alpha \right) \tau_{min} \right) \times \exp \left\{ -\frac{1}{2} \left[ \rho_{LMAJ} \left[ 1 - F (w_r) \right] \right] \right\} \times (1 - w_r) \right)
\]

\[
+ \left( \left( \delta \left( 1 - \delta - \psi_{maj} \right) \left( 1 - \alpha \right) \tau_{maj} + \left( 1 - \delta \right) \left( 1 - \delta + \psi_{min} \right) \left( 1 - \alpha \right) \tau_{min} \right) \times \exp \left\{ -\frac{1}{2} \left[ \rho_{LMIN} \left[ 1 - F (w_r) \right] \right] \right\} \times (1 - w_r) \right)
\]

\[
+ \left( \left( \delta \left( \delta + \psi_{maj} \right) \left( \alpha \right) \tau_{maj} + \left( 1 - \delta \right) \left( \delta - \psi_{min} \right) \left( \alpha \right) \tau_{min} \right) \times \exp \left\{ -\frac{1}{2} \left[ \rho_{LMAJ} \left[ 1 - F (w_r) \right] \right] \right\} \times (-w_r) \right)
\]

\[
+ \left( \left( \delta \left( 1 - \delta - \psi_{maj} \right) \left( \alpha \right) \tau_{maj} + \left( 1 - \delta \right) \left( 1 - \delta + \psi_{min} \right) \left( \alpha \right) \tau_{min} \right) \times \exp \left\{ -\frac{1}{2} \left[ \rho_{LMIN} \left[ 1 - F (w_r) \right] \right] \right\} \times (-w_r) \right)
\]

\[
\forall \ w_r \in \left[ w_{m2}, w \right]
\]

This equation differs from the one for \( E \Pi_H (w_r) \) because in the coefficient before the “\( \exp \{ \} \)”
term, all values of \( \alpha \) have been replaced with \( (1 - \alpha) \), and vice versa. Analysis shows that
\( \partial E \Pi_L (w_r) / \partial w_r < \partial E \Pi_H (w_r) / \partial w_r \). Since \( \partial E \Pi_H (w_r) / \partial w_r \) is (by construction) equal to zero
for all \( w_r \in \left[ w_{m2}, w \right] \), \( \partial E \Pi_L (w_r) / \partial w_r \) is negative; \( E \Pi_L \) is maximized at \( w_r = w_{m2} \).
However, even at this wage, expected profits for firms that employ a low-ability worker are negative.
Substituting for $w^*_{M2}$ (remembering \( F(w^*_{M2}) = 0 \) because \( w^*_{M2} \) is at the bottom of the equilibrium wage distribution \( F(\bullet) \)) gives:

\[
E \Pi_L(w^*_{M2}) = \left\{ \begin{array}{l}
(\delta(\delta + \psi_{maj})(1 - \alpha)\tau_{maj} + (1 - \delta)(\delta - \psi_{min})(1 - \alpha)\tau_{min})e^{HMAJ}(1 - w^*_{M2}) + \\
(\delta(1 - \delta - \psi_{maj})(1 - \alpha)\tau_{maj} + (1 - \delta)(\delta + \psi_{min})(1 - \alpha)\tau_{min})e^{MIN}(1 - w^*_{M2}) + \\
(\delta(\delta + \psi_{maj})\alpha\tau_{maj} + (1 - \delta)(\delta - \psi_{min})\alpha\tau_{min})e^{LMAJ} - w^*_{M2} + \\
(\delta(1 - \delta - \psi_{maj})\alpha\tau_{maj} + (1 - \delta)(1 - \delta + \psi_{min})\alpha\tau_{min})e^{LMIN} - w^*_{M2} \end{array} \right.
\]

\( E \Pi_L(w^*_{M2}) \) is negative given \( \alpha > \frac{1}{2} \). The proposition is thus proved: a firm employing a low-ability worker in period-1 prefers to hire through the market, maximizing expected profit.

Finally, consider the period-1 market. Firms hiring in this market earn an expected period-2 profit equal to the probability of obtaining a high-ability period-1 worker times the expected profit from a referral. Free entry thus drives the wage above expected period-1 productivity:

\[
\begin{align*}
\omega^*_{M1} & = \frac{1}{2} + \left( \frac{1}{2} \right)c(\delta, \alpha, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) \\
& = \frac{1}{2} \left[ 1 + c(\delta, \alpha, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) \right]
\end{align*}
\]

Given the previous comparative-statics results on \( c \), \( \omega^*_{M1} \) is increasing in \( \alpha, \tau_{maj} \), and \( \tau_{min} \) but is decreasing in \( \delta \) and \( \psi_{maj} \). Its relationship with \( \psi_{min} \) depends on the magnitude of \( \psi_{min} \).
References


