DEFAULT OPTIONS AND NON-STANDARD 401(K) CHOICE

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ABSTRACT

As policy makers search for ways to boost savings rates, automatic enrollment 401(k) plans are gaining popularity among employers in the United States. Such plans enroll new hires into default 401(k) funds at a default contribution rate with the option to opt out. The growing field of Behavioral Economics, meanwhile, has begun to incorporate framing effects such as default options into economic modeling. These effects have important implications for public policy. This paper presents a model of 401(k) choice that accounts for non-standard responses to default contribution rates. The parameters of this model are estimated using maximum likelihood estimation. To infer the policy implications of the model, this paper adopts a generalized normative framework that defines welfare in terms of choice. The observed data generate a parameterization that is consistent with the notion that defaults influence contribution rates through both anchoring bias and status quo bias. Moreover, welfare gains from automatic enrollment may not be as large as previously thought.

Keywords: behavioral, retirement, anchor, status quo, choice-based, framing

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1. Introduction

In 2008, net saving as a percentage of gross national income in the United States was negative for the first time since 1934. With the rise of consumer credit and willing foreign lenders, Americans have consistently been saving less with each recent decade. The 1960s witnessed double-digit annual saving rates. The saving rate fell below five percent twice in the 1970s and 80s. In the last seven years, the annual national saving rate has never exceeded two percent (Bureau of Economic Analysis, NIPA Table 5.1).

This trend is potentially alarming. People cannot successfully smooth consumption over their lifetimes if they do not save adequately for retirement. Moreover, Hurd and Rohwedder (2003) report a significant and consistent drop in Americans’ consumption at the retirement age. Such an observation leads to crucial policy questions. Why are people saving so little? What factors influence savings choices, and what policy adjustments can make people better off?

These questions were on the government’s mind in 2006 when President Bush signed a bill that instituted a reform of 401(k) provision. This bill is best known for encouraging employers to automatically enroll their employees in 401(k) plans at a default contribution rate (Wall Street Journal 2008). Such automatic enrollment plans have been gaining significant popularity in recent years. According to an annual survey of 401(k) plans by Deloitte Consulting LLP, the number of automatic enrollment plans has increased by as much as 50% since 2004. In last year’s survey, 23% of employers polled had implemented such a program.

The structure of an automatic enrollment system is straightforward. When a new hire joins a company with such a policy, he or she is automatically enrolled in the company’s 401(k) plan. He or she can opt out at any moment, typically by calling the
Human Resources Department, but failure to do so results in contributing to a default fund at a default, nonzero contribution rate. The employer sets both the default fund allocation and the default contribution rate. An automatic enrollment regime is an “opt out” policy, as opposed to an “opt in” policy.

The trend towards automatic enrollment, however, has not generated the boost in retirement plan contributions that many policy makers had hoped for. Recent research by Vanguard, a 401(k) provider, has actually shown a decrease in contribution rates since the 2006 legislation was introduced (Wall Street Journal 2008). Automatic enrollment has not done its part to reverse the trend of a sinking national saving rate. To understand why, policy makers must be able to model employee response to a change in the default option. This paper analyzes the issue by evoking principles and developments from the growing field of Behavioral Economics.

Behavioral Economics has emerged in response to growing doubt in the standard model of choice and its ability to explain human behaviors such as myopia, mistakes, and the desire to restrict choice sets. Behavioral models typically relax assumptions such as unbounded willpower, unbounded rationality, and coherent preferences (see Bernheim and Rangel (2005)). According to the standard model of choice, an employee should make the same 401(k) enrollment decision whether her firm has an “opt in” or “opt out” policy. After all, the economic saving decision is identical in the two cases. Empirically, however, choices that respond to different default options often differ. A behavioral, or non-standard, model is thus needed to understand such choice reversals.

Such a model, however, complicates welfare analysis. If peoples’ choices reflect mistakes and variation based upon framing effects and biases, policy makers cannot comfortably rely on the principles of revealed preference. Suppose I choose to contribute 4% to my 401(k) when the decision is framed one way, and to not contribute
at all when the decision is framed another way. How can one possibly tell which of these choices reveals my true savings preference? Moreover, once revealed preference is overturned, welfare analysis opens the door to paternalistic intervention. Even if one accepts that myopia or procrastination often causes suboptimal saving for retirement, he or she may be uncomfortable with a policy maker ensuring that employees maintain high savings rates against their will, no matter how weak it may be.

With such concerns in mind, this paper adopts from Bernheim and Rangel (2009) a choice-based normative framework that accounts for non-standard decision making. To better understand the implications of automatic enrollment plans, as well as similar default features in other walks of life, I will estimate and analyze a model of 401(k) choice in response to a default. Then, using the choice based framework, I will discuss the model’s policy implications, focusing on how 401(k) policies succeed or fail in making employees better off. I will conclude by discussing how and when these results may generalize to other situations in which default options affect choice patterns.

2. Literature Review

2.1 Default Options and 401(k) Contribution

A default option is defined as the choice that results from inaction, and the effect of default options on decision-making has recently infiltrated the discussion of choice in Economics, Psychology, and Marketing. The presence or lack of a default option is one of many examples of framing, or the manner in which a choice problem is presented. In a series of papers in the late 1970s and early 1980s, Amos Tversky and Daniel Kahneman introduced the notion of framing, and presented its effects on decision making under uncertainty (see Kahneman and Tversky (1979, 1981, 1986)). Specifically, Kahneman and Tversky showed that stating the same problem in different ways often
led to choice reversals. This finding sparked great interest in non-standard choice and its potential effects on behavior and public policy.

Samuelson and Zeckhauser (1988) were among the first to look deeply into choice reversals arising from the presence of default options. Using a wide range of survey data and two field experiments regarding health care and retirement decisions by Harvard University employees, Samuelson and Zeckhauser (1988) show that people disproportionately prefer the default option to the implementation of new alternatives. They call this tendency “status quo bias.” Many further studies confirm these findings. Johnson et al. (2003a) show that consumers of car insurance gravitate to default coverage plans, while Johnson and Goldstein (2003) find that framing strongly influences organ donation consent rates in favor of the status quo.  

Meanwhile, as automatic enrollment plans have become more popular among employers, the potency of default options has begun to play a key role in decisions surrounding retirement savings. To explore this issue, Madrian and Shea (2001) gathered data on employee 401(k) choices in a large U.S. corporation before and after an automatic enrollment regime was introduced. Before the implementation of automatic enrollment, each employee could affirmatively opt in to the company’s 401(k) plan and choose his or her desired contribution rate. After the change in policy, each new employee was immediately enrolled in the company’s 401(k) plan at the default contribution rate and given the negative option to opt out of the plan or change the contribution rate at any time.

The results were striking. 401(k) enrollment was substantially higher for the automatically enrolled cohort than for previous cohorts. Furthermore, an overwhelming

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2 For more examples of papers examining the effect of default options outside of the retirement savings context, see Abadie and Gay (2006) and Johnson et al. (2003b)
majority of these new employees exhibited what Madrian and Shea call “default behavior.” Over 75% of the automatically enrolled cohort remained at the default contribution rate. In prior cohorts, fewer than 15% of employees chose to contribute at that particular rate. Even the default 401(k) asset allocation, comprised entirely of money market funds, was largely untouched after automatic enrollment. Once automatic enrollment was introduced, the average fraction of a participating new employee’s balance invested in money market funds grew from 8.2% to 80.5%. Madrian and Shea (2001) attribute this spike to the inherent appeal of the “path of least resistance.”

Madrian and Shea (2001) document a 48% increase in plan participation among new employees, as well as an 11% increase in total plan participation, as a result of automatic enrollment. Even those who were employed by the company before the switch to automatic enrollment seemed to be influenced by the policy. Employees in prior cohorts also gravitated towards the new default options.

Subsequent automatic enrollment research reports similar findings at different companies. Choi et al. (2001) tackle the issue using administrative data from three large U.S. firms. In each of the companies that the study examined, about 80% of new employees initially remained at the default 401(k) contribution rate and fund asset allocation. The effect of the policy change on average savings, however, was ambiguous. On one hand, Choi et al. (2001) observe a spike in plan participation. On the other hand, employees in the sample had a tendency to lean towards the low default rate. While total participation increased, new employees seemed to substitute away from higher contribution rates towards the relatively low default anchor. These two effects nearly cancel each other out, leaving the average contribution rate largely
unchanged in the sample. Choi et al. (2005b) and Beshears et al. (2005) report similar findings.

Thus, automatic enrollment increases participation rates, while default contribution rates and asset allocations become very popular as soon as they are introduced. The exact reason or method for this behavior is cause for much debate in the Behavioral Economics literature. After all, neoclassical economic models could never predict such results. The economic decision concerning one’s 401(k) plan does not change when a default is introduced, so something else must be at work here.

One explanation for this apparent choice reversal is Samuelson and Zeckhauser’s notion of “status quo bias.” They claim, simply, that people gravitate towards any alternative that is presented as “doing nothing or maintaining one’s current or previous decision” (Samuelson and Zeckhauser, p. 1). Their work is largely empirical, however, and they do not present a justification for why people behave this way.

Kahneman et al. (1991) thus seek to explain the default option phenomenon in terms of Loss Aversion, the tendency to prefer avoiding losses to acquiring gains of equal magnitude (see Kahneman and Tversky (1979, 1981)). They write, “One implication of loss aversion is that individuals have a strong tendency to stay at the status quo, because the disadvantages of leaving it loom larger than the advantages” (Kahneman et al. (1991), p. 6-7). If one would rather avoid a loss than seek a comparable profit, it will be in his or her favor not leave the status quo under sufficient uncertainty.

Madrian and Shea (2001) do not point to an inherent behavioral bias or Prospect Theory as the root cause of default behavior. Instead, they propose that employees perceive 401(k) defaults as advice. One may, upon signing with a new company, believe that the default 401(k) contribution rate is that company’s suggestion, or advised
contribution rate. Likewise, employees can perceive a default asset allocation rate to be a reflection of a financial experts’ outlook on the economy or the various offered funds.

Iyengar et al. (2004) offer yet another insight into default behavior by examining how people respond to choice. Concretely, they question the hypothesis that an increased choice set may make people worse off. Psychological and Economic theory have traditionally maintained that added choice options can only enhance well-being, providing additional opportunity to choose that which is most desired. Iyengar et al. (2004), however, argue that past a certain point, a large choice set can cause “choice overload.” When one experiences “choice overload,” she has so many choices that she prefers not to choose anything at all. At this point, she seeks to limit her choice set. In an effort to apply this concept to 401(k) decisions, Iyengar et al. (2004) found that the number of funds offered under a given 401(k) plan was negatively correlated with 401(k) plan participation. One interpretation of this result is that the complexity of the 401(k) contribution decision inspires “choice overload” in potential participants.

Beshears et al. (2005) speculate that automatic enrollment plans can boost participation rates by overcoming the “choice overload” associated with the 401(k) decision. With automatic enrollment, they argue, an employee put off by the decision’s complexity can simplify the problem. The choice set can be limited to two options, participate and accept the default or do not participate. According to the theory that Iyengar et al. (2004) propose, this potential for choice simplification should lead to greater participation and default behavior.

Beshears et al. (2005) and Choi et al. (2003, 2005a) choose to examine the 401(k) contribution decision in terms of time-inconsistency. Choi et al. (2001) present ample survey data on how employees evaluate their savings rates and how they plan to change their contribution patterns in the near future. They find that most employees do
not implement the 401(k) changes that they claim they will implement. They mean to do so, but they procrastinate instead.

The presence of procrastination exacerbates status quo bias. If a company does not automatically enroll employees in 401(k) plans, the default choice for new employees is to not contribute. A new employee may have the desire to contribute to a 401(k), but perceive deviating from the default as costly. Given the effort of research, decision-making, and likely paperwork from the Human Resources department, the employee may put off the “costly” process in favor of more immediate benefits. If a company automatically enrolls their employees in a 401(k) plan at some default contribution rate, procrastination has the same effect. In this case, however, a new employee may put off changing the contribution rate or opting out completely. The results predicted by models of procrastination (such as those presented in O’Donoghue and Rabin (1999, 2001)) are consistent with the empirical contribution rate data in Madrian and Shea (2001), Choi et al. (2001, 2005a) and Beshears et al. (2005).

Choi et al. (2003) present a model of 401(k) enrollment in order to derive optimal conditions for automatic enrollment, standard enrollment, and active decision regimes. In this context, an active decision regime is a 401(k) structure in which there is essentially no default, since new hires are required to make a 401(k) decision upon commencement of employment. The model in question has several key assumptions. Within the context of the model, employees have present-biased preferences that generate procrastination and inertia. The transaction cost varies with time and is chosen independently from a uniform distribution at the beginning of each time period.
Optimal savings rates\(^3\) are known to employees, but not to the “planners” setting the 401(k) enrollment structure and default contribution rate.

The model presented by Choi et al. (2003) implies that standard enrollment is an optimal regime when optimal contribution rates are sufficiently heterogeneous. When contribution rates are homogenous, on the other hand, automatic enrollment plans are optimal. Active decision plans are found to be optimal when procrastination effects are strong, regardless of contribution rate dispersion. In the Choi et al. (2003) model, employees’ optimal contribution rates are not affected by the presence of a default. Rather, the move from the default to the optimal contribution rate is costly, and thus present-biased preferences cause employees to not deviate from the default. This rationalization is not entirely consistent with the Samuelson and Zeckhauser (1988) notion of status quo bias. Status quo bias would imply that employees tend to disproportionately perceive the default option as an optimal contribution rate.

Furthermore, the Choi et al. (2003) model assumes that employees are not affected by the anchor that default contribution rates provide. The model’s subjects either stay at the default contribution rate or opt out in favor of their optimal contribution rate. There is no middle ground. Behavioral economists, however, have found that study participants’ valuations and decisions are often affected by anchoring numbers. Ariely et al. (2003) present six cases where this is so. In one illuminating study, MBA students bid on various commodities after reporting the last two digits of their social security numbers. Those with above-median social security numbers, on average, bid from 57% to 107% higher than their counterparts. Ariely et al. (2003)

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\(^3\) In this section, I refer to “optimal savings rates” and “optimal contribution rates” to be consistent with the language used in Choi et al. (2003). This paper, however, adopts a different definition of welfare and thus does not assume that these rates are “optimal” in the sense that they maximize employee welfare.
suggest that further valuations, in general, are all derived from the initial anchor-biased number, although the subsequent valuations appear to follow a coherent pattern. They call this phenomenon coherent arbitrariness.

Such research suggests that default contribution rates can also serve as anchors that bias 401(k) decisions. Employees may not only elect to contribute at a default contribution rate, but they may also unknowingly choose rates that lean towards the default rate. In later sections, I will explore this issue in detail.

2.2 Behavioral Welfare Analysis

401(k) contribution decisions offer a glimpse into the larger question of how normative analysis is to be applied to behavioral models. Neoclassical welfare analysis is based on the infallibility of revealed preference. Traditional normative frameworks assume that one consistently chooses that which he most prefers from a given choice set. Choice reversals resulting from the presence of default options, anchoring bias, and time-inconsistency, however, complicate this inference. When we relax key economic assumptions, we allow non-standard decision makers to have incoherent preferences, expanded preference domains, variable lifetime preferences, and capability for consistent mistakes. If we accept that people have such non-standard preferences, then the search for optimal public policy requires new tools. Non-standard decision makers require a non-standard normative framework.

The search for this non-standard framework has sparked a discussion on the nature of welfare. Some social scientists implicitly or explicitly believe that welfare can and should be equated with observed happiness or well-being. Such researchers propose that welfare is directly measurable. Kahneman et al. (1997) advocate for the use of “experienced utility,” in normative analysis. The notion of “experienced utility” can
be traced back to Jeremy Bentham’s writing, and refers to a hedonic perception that can be reported in real time or retrospectively. The authors propose using “experienced utility” as a basis for behavioral welfare analysis, while accepting that choices are made by maximizing “decision utility.”


Such studies assume that data on self-reported happiness is a valid proxy for well-being, or even human welfare. Frank (2003) argues that there exist consistent, valid, and reliable measures of well-being. Koszegi and Rabin (2008) argue that choice data is not sufficient for non-standard welfare analysis, and should be supplemented with well-being measurements. Such views suggest that preference reversals arise from the disparity between decision and experience utility, and that behavioral welfare analysis should be founded on direct measurement of the latter.

Other economists have shed doubt on the direct measurement of happiness as a basis for normative analysis. Loewenstein and Ubel (2008) challenge the normative validity of self-reported happiness data on the grounds that it is severely biased by emotional adaptation. They reference the time tradeoff studies of Smith et al. (2006), which show that sick patients who report “normal” or “average” happiness are often willing to give up many years of their life expectancy to achieve a certain health outcome. The way in which patients emotionally adapt to their environment causes
them to report a similar level of happiness before and after the onset of illness. This by no means, Loewenstein and Ubel argue, is grounds to claim that these people are as well off as they were with their health.

Bernheim (2008) expresses a similar concern, arguing that people may well choose the midpoint of any survey scale to denote a typical or average state. With no absolute scale on which to measure happiness, study participants arbitrarily and inconsistently decide what each response signifies and how feelings, emotions, and circumstances should be aggregated. Furthermore, certain contexts may bias self-reported happiness even further. If one were asked, for example, to report their happiness at work in front of their boss, the resulting data would likely have an upward bias.

Ensuring that answers are anonymous does not bridge the gap between self-reported happiness and internal or true happiness. Bernheim (2008) shows that this gap is, in fact, empirically insurmountable. Suppose an environment maps to a vector of sensations, which are then aggregated to form internal happiness. Furthermore, this internal happiness function will then be an argument in the mapping from an environment to a report of happiness. Without further specification, one cannot uniquely determine internal happiness given reported happiness.

Alternatively, some behavioral normative frameworks inherit the neoclassical notion of welfare as satisfaction of objectives. Such frameworks do not assume that one can measure welfare directly, but rather that choices reveal peoples’ objectives and that welfare can be inferred based on these objectives. To account for behavioral effects and preference reversals, however, one must either allow for a broadened preference domain or allow for multiple and potentially competing objectives.
By broadening the preference domain, one accepts that factors other than the nature of the chosen object influence well being. These factors may include the environment that the choice is made in, the choices of others, or the composition of the choice set. With such a framework, decisions still maximize one coherent, objective function. Gul and Pesendorfer (2001) illustrate this approach with a dynamic choice model that factors in utility losses resulting from temptation. The choice set itself, in this case, factors into well being since the presence of temptation makes people worse off. Nevertheless, the model is a result of consistent objectives that govern choice.

Koszegi and Rabin (2008) object to such a framework, as they convincingly argue that choice patterns cannot uniquely identify any objective function given a broadened preference domain. In other words, we may find that choice behavior is consistent with the utility function presented in Gul and Pesendorfer (2001), but we cannot know that it is the only utility function for which this is true. Though many objective functions can provide a useful rationalization or story in conjunction with a broadened preference domain, we cannot determine a unique mapping from choices to objectives upon which to structure normative analysis.

Another alternative strategy allows for multiple competing objective functions or mistakes to govern choice behavior. Choice models that take the form of multi-self games fall into this category, as do choice models that seek to represent a faulty decision process. Bernheim (2008) points out that this strategy is also subject to an insurmountable identification problem. He points out that the two interpretations of the \( \beta, \delta \) model in Laibson et al. (1998) and Asheim (2008) respectively yield different welfare implications. Furthermore, there is neither a way to identify which formulation should serve as the basis for normative analysis, nor a reason why there may not be many more equally valid reformulations of the model.
In short, the behavioral welfare economics literature has, for some time now, been in pursuit of objective functions that are able to model and predict non-standard choice. Unfortunately, as discussed in Bernheim (2008), there is simply no unique inverse mapping from non-standard choice observations to any of these behavioral rationalizations. So long as welfare is defined in terms of revealed and satisfied objectives, arbitrary assumptions must be accepted in order to perform normative policy analysis that accounts for non-standard choice.

With this in mind, Bernheim and Rangel (2009) present an alternative normative framework that defines welfare directly in terms of choice. The result is a generalized welfare criterion that respects choice directly without any need for rationalizing underlying objectives or preferences. All that it requires is information concerning choices and the respective environments in which they are made. Moreover, the results of this generalized welfare criterion approach those of the standard welfare criterion as behavioral anomalies and biases become small. This is intuitively appealing, as this implies that the welfare implications of the framework will mirror those of the standard framework in the absence of non-standard choices.

At the heart of this normative framework is the unambiguous choice relation, denoted by \( P^* \). This relation specifies that \( x \) is unambiguously chosen over \( y \) (which is written \( xP^*y \)) if and only if \( y \) is never chosen when \( x \) is available. Bernheim and Rangel (2009) show that the \( P^* \) relation is always acyclic and is the most discerning criterion that never overrules choice under several permissible assumptions. In this framework, the unambiguous choice relation replaces the standard revealed preference relation seen in standard welfare analysis.

Bernheim and Rangel (2009) define a generalized choice situation, \( G \), as a paring between a constraint set, \( X \), and an ancillary condition, \( d \), such that \( G = (X, d) \). The
constraint set is the collection of all possible choice elements that are available to the individual. $X$ is thus a subset of $\mathbb{N}$, which can be defined as the complete collection of all possible choice objects. Ancillary conditions are features of the environment that are not a part of the choice set, but can nevertheless influence the choice that is made. A question framed in terms of gains and the same question framed in terms of losses, for example, represent two different ancillary conditions.

Another attractive feature of this framework is that while choice is respected, social planners or policy makers have the freedom to restrict the welfare-relevant choice domain in order to achieve more discerning welfare results. This can be done for any number of reasons. Certainly, paternalism can lead a policy maker to restrict certain choice situations that lead to undesirable choices. Neurological insights, such as in the case of addiction (see Bernheim and Rangel (2004, 2005)), can also provide reason to remove certain choice situations from the set of welfare relevant choice situations. Likewise, it is often best if the welfare relevant choice domain is restricted to informed choices. In any case, the framework allows for flexibility in the careful pruning of generalized choice situations that lead to mistakes or poor decisions.

In the following sections, I will explore this generalized welfare criterion in greater depth. In doing so, I will apply the Bernheim and Rangel (2009) normative framework to 401(k) enrollment decisions and discuss potential restrictions for the welfare relevant domain of contribution choices.
3. The Model

The choice model developed in this section solves for the 401(k) contribution rate of an employee in the presence of a default option. This choice model differs from the 401(k) choice model presented in Choi et al. (2003) in several important ways. First and foremost, this model incorporates anchoring effects as well default behavior. I make the assumption that even those who opt out of the default contribution rate are influenced by the presence and value of the default. For any number of reasons, such as subconscious anchoring or the perception of a suggestion, individuals “lean” towards the default even if they do not choose it. Choi et al. (2003) make no such suggestion, implying that individuals either stick to the default or opt out in favor of their unbiased, optimal contribution choice.

While Choi et al. (2003) focus on time-inconsistency, the model presented in this paper abstracts from such irregularities. It solves for a reduced form of the contribution rate trajectory, which provides a powerful summary statistic for an individual’s 401(k) choices. The model, itself, is purely descriptive. Nevertheless, its interpretation is consistent with insights from the behavioral literature. I will focus on this interpretation in later sections.

3.1 401(k) Contribution Choice in Response to a Default Option

Suppose $x_i$ is the contribution rate of individual $i$. This value is the observed percentage of income that a given individual contributes to his or her 401(k) retirement account. Assume that internal and external anchors govern decision making, and define $k_i$ to be one’s internal anchor. The value of $k_i$ is not necessarily an optimal contribution
rate, but rather the starting point for each 401(k) decision, specific to each individual\(^4\). In the population, internal anchors are assumed to take on a normal distribution, with mean \(\mu\) and variance \(\sigma^2\). External anchors are elements of an individual’s environment that influence choice. In the model, default contribution rate options are endogenous external anchors. The variable \(d\) represents the default contribution rate. This value is positive in automatic enrollment structures, and is equal to zero in a standard enrollment regime.

Now, assume that the choice of \(x_i\) is determined by a stochastic process such that

\[
\{ x_i | d \} = \begin{cases} 
  c_i(k,d,\theta,\gamma) & \text{with probability } \lambda(d,c_i,\alpha,\beta) \\
  d & \text{with probability } 1 - \lambda(d,c_i,\alpha,\beta)
\end{cases}
\]

(1)

where

\[
c_i(k,d,\theta) = k_i + \gamma \left[ \frac{1}{1 + e^{-\theta(d-k_i)}} - \frac{1}{2} \right]
\]

(2)

and

\[
\lambda(d,c_i,\beta) = \frac{|d - c_i(k,d,\theta,\gamma)|}{\beta + |d - c_i(k,d,\theta,\gamma)|}
\]

(3)

We define \(c_i(k,d,\theta)\) as the “latent” contribution rate choice. Thus, one’s latent contribution rate choice reflects the internal anchor adjusted by an external anchoring term that pulls the contribution rate towards the default. This sort of choice behavior is consistent with the findings of Ariely et al. (2003).

In this context, \(\theta\) and \(\gamma\) act as a “lean parameters,” in the latent choice function. Their values determine the extent to which the contribution rate of individual \(i\) leans

\(^4\) While \(k_i\) can be interpreted as the counterfactual contribution rate choice in the absence of an external anchor, I refrain from labeling it an optimal contribution rate. The word “optimal” yields welfare implications that are not intended at this stage. In later sections, welfare relationships will be constructed by applying the Bernheim and Rangel (2009) generalized normative framework.
towards the default. If $\theta$ and $\gamma$ are large, for instance, an employee may be more inclined to choose a contribution rate close to the default. Thus, the term $\gamma \left[ \frac{1}{1 + e^{-\theta(d-k_i)}} - \frac{1}{2} \right]$ accounts for the anchoring effect, determining how much pull the default rate has. This anchoring effect is assumed to be proportional to the distance between the default contribution rate and an individual’s internal anchor.

If $k_i$ is equal to the default rate, then the equation simplifies to

$$x_i = k_i = d$$

(4)

If we use the default as a starting point for the contribution rate decision, it cannot effectively pull the value of $x$ any closer to itself.

Also note

$$\lim_{(d-k_i) \to \infty} \gamma \left[ \frac{1}{1 + e^{-\theta(d-k_i)}} - \frac{1}{2} \right] = \frac{\gamma}{2}$$

(5)

This implies that, within the scope of the model, a social planner cannot generate any choice he or she desires by setting the appropriate default. The largest effect on an internal anchor that a default rate can have, in this specification, is $\gamma/2$ percentage points.

We can rationalize the latent choice function by assuming that each employee maximizes his or her decision utility

$$U_i(x,d) = -[x - (k_i + \gamma \left[ \frac{1}{1 + e^{-\theta(d-k_i)}} - \frac{1}{2} \right])^2$$

(6)

with respect to $x$. I refer to $U_i(x,d)$ as the decision utility in order to avoid any link between this choice function and welfare implications. This function generates a choice, but does not necessarily reflect how well off an employee is having made that particular choice and not another. This terminology is adopted from Kahneman et al. (1997), who
imply that people make choices based on decision utility, but welfare and policy analysis should be based on experienced utility. While this paper makes use of an alternative normative framework, I adopt this distinction when building the positive model of 401(k) contribution choice.

In the model, employees do not always opt out of the default and choose to contribute at a rate equal to $c_i(k, d, \theta, \gamma)$. Each employee chooses to remain at the default with probability $\lambda(d, c_i, \beta)$. This probability is proportional to the distance between the default contribution rate and the latent choice. When the default is close to the latent choice, an employee is more likely to accept the default. When the default is very far, on the other hand, from the latent choice, he or she opts out and chooses $c_i(k, d, \theta, \gamma)$ with near certainty.

The function $\lambda(d, c_i, \beta)$ depends on one free parameter that has not yet been introduced: $\beta^5$. The parameter $\beta$ determines how quickly the probability of opting out approaches one as the distance between $d$ and $c_i(k, d, \theta, \gamma)$ approaches infinity.

### 3.2 The Parameters of the Model

The 401(k) contribution choice model has five free parameters, which are summarized in Table 1. If the values of these five parameters are known, the model is fully specified and generates a distribution of contribution rates for any given default rate. In the following section, I introduce maximum likelihood estimation in order to estimate the model’s parameters.

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5 Initially, I explored a specification of the model with an additional parameter, $\alpha$, such that $\lambda(d, c_i, \alpha, \beta) = \alpha + (1 - \alpha) \frac{|d - c_i(k, d, \theta, \gamma)|}{\beta + |d - c_i(k, d, \theta, \gamma)|}$. The data, however, consistently pushed the parameter estimate to the boundary point $\alpha = 0$, so this sixth parameter was omitted from the model’s final specification.
Table 1: The Model’s Parameters

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<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>The lean parameter that determines the extent to which external anchors affect contribution rates relative to the distance between the internal anchor and the default option.</td>
<td>$\theta \in \mathbb{R}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The lean parameter that determines the extent to which external anchors can influence a contribution choice. $\gamma/2$ percentage points is the largest value by which an individual’s latent choice can differ from his or her internal anchor. If $\gamma = 0$ then default options do not produce anchoring effects.</td>
<td>$\gamma \geq 0$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>A parameter that determines the relationship between the distance of a latent choice from the default contribution rate and the probability of opting out of the default option.</td>
<td>$\beta \geq 0$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>The population mean of internal anchors.</td>
<td>$\mu \in \mathbb{R}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The population standard deviation of internal anchors.</td>
<td>$\sigma \geq 0$</td>
</tr>
</tbody>
</table>

3.3 Maximum Likelihood Estimation of the Model’s Parameters

Maximum likelihood estimation provides parameter estimates by solving for those parameter values that maximize the likelihood of having observed the empirical data. Suppose a 401(k) choice, $x$, is an integer choice such that $x \in \{0, 1, 2, \ldots, M\}$, where $M$ is the maximum allowed contribution rate. The probability of observing $x$ is denoted by the function $L_x(d, \theta, \gamma, \beta, \mu, \sigma)$. Let $\Phi$ represent the cumulative density function associated with the underlying normal distribution of $k$, and let $k^*$ be the unique value of $k$ that solves the equation $c_x(k, d, \theta) = x$. The probability of observing $x$ if $x \neq d$ is written
With this, we can now express $L_x(d, \theta, \gamma, \beta, \mu, \sigma)$ as

$$L_x(d, \theta, \gamma, \beta, \mu, \sigma) = \begin{cases} l_{x=d}(d, \theta, \gamma, \beta, \mu, \sigma) & \text{for } x \neq d \\ \Phi(k_{x+0.5}^*(d, \theta, \gamma), \mu, \sigma) - \Phi(k_{x-0.5}^*(d, \theta, \gamma), \mu, \sigma) \lambda(d, x, \beta) & \text{for } x = 0 \\ [1 - \Phi(k_{M+0.5}^*(d, \theta, \gamma), \mu, \sigma)] \lambda(d, M, \beta) & \text{for } x = M \end{cases} \tag{8}$$

The particular 401(k) contribution choice data that this paper works with is grouped in bins, so define $B$ bins ($b=1, b=2, \ldots, b=B$). Each bin has an upper bound, $H_b$, and a lower bound, $L_b$. The lower bound for the lowest bin is $-\infty$ while the upper bound for the highest bin is $\infty$. The rest of the lower and upper bounds enclose integer valued contribution rates. The probability that an observation falls into a particular bin can now be written

$$L_B(d, \theta, \gamma, \beta, \mu, \sigma) = \sum_{x=L_b}^{H_b} L_x(d, \theta, \gamma, \beta, \mu, \sigma) \tag{9}$$

Suppose that we observe data for a set of $M$ defaults ($d_1, d_2, \ldots, d_M$), and for each default $m$, there are $N_m$ observations. Let $N^b_m$ denote the number of observations for a default $m$ that fall into the bin $b$. The log likelihood of the sample is written as

$$\log L(\theta, \gamma, \beta, \mu, \sigma) = \sum_{m=1}^{M} \sum_{b=1}^{B} N^b_m \log L_B(\theta, \gamma, \beta, \mu, \sigma) \tag{10}$$

The log likelihood function depends on the five parameters of the model, such that the vector of parameters that maximizes the function is the vector of maximum likelihood parameter estimates.
In order to perform hypothesis testing on these parameter estimates, we write the Hessian matrix of the negative log likelihood function. In practice, the second order partial derivatives are finite difference approximations. The standard errors vector is obtained in solving the algorithm

\[ SE = \sqrt{\text{Diag}(\text{Hessian}^{-1})} \]  

where Hessian is the Hessian Matrix of the negative log likelihood function.

3.4 The Data

Figure 1: A Summary of the Data Sample

Source: Beshears et al. (2005)
The parameterization of the 401(k) choice model makes use of the data that Beshears et al. (2005) provide for “Company A,” a medium-sized U.S. chemicals company (see Figure 1). The sample includes contribution rates of two employee cohorts, each with 15-24 months of tenure, grouped into seven bins. The first cohort was automatically enrolled at a default rate of 3%, while the second cohort joined the company after the default contribution rate was raised to 6%. The sample size of the first cohort is 167 while the sample size of the second cohort is 47.

4. Applying The Choice-Based Normative Framework

To provide a solid foundation for policy analysis, any non-standard model of choice needs to be accompanied by a generalized normative framework that encompasses such a model. This paper adopts the choice-theoretic framework proposed by Bernheim and Rangel (2009), and applies it to the results of the parameterized 401(k) choice model.

The model outlined in the preceding section predicts the proportion of the cohort population\(^6\) making a particular choice in response to a given default. We then define welfare directly in terms of choice, assuming that people are better off having that which they would themselves choose. The default contribution rate that one faces in making this choice characterizes the exogenous ancillary condition that the choice depends on.

Recall \(\mathcal{X}\) is the set of all choice objects, and a generalized choice situation, \(G = (X, d)\), is defined by a constraint set \(X \subseteq \mathcal{X}\) and an ancillary condition \(d\). \(\Gamma^*\) denotes the set

\(^6\) Throughout the paper I refer to the “cohort population,” rather than the “population” to acknowledge that the observed sample cohort may not be representative of the greater population of employees in the U.S.
of generalized choice situations in the context of a specific positive model, such as the default choice model in this paper. A choice correspondence $C: \Gamma^* \Rightarrow \mathbb{N}$, with $C(X,d) \subseteq X$ for all $(X,d) \in \Gamma^*$, is dictated by the model.

The direct application of a choice based framework onto this choice correspondence yields no discerning welfare relationships. If we define welfare in terms of choice and deem all 401(k) choices welfare-relevant, then any 401(k) choice made by an individual is optimal. We cannot, in this case, define any welfare relationships between choices made under different default rates.

In order to generate discerning welfare relationships, we must limit the choices that we consider welfare relevant. We “prune” $\Gamma^*$ and define a welfare-relevant choice domain, $\Gamma \subseteq \Gamma^*$. Ideally, uninformed choices should be excluded from the choice set. In the presence of a default option, one valid worry is that an employee will blindly accept the default without taking the time to make an informed decision. If we observe an employee opting out of the default, on the other hand, it is fairly safe to assume that the employee is making a calculated decision. These are grounds for restricting the model’s welfare relevant choice domain to “opt-out choices,” in which an employee opts out of the default option and chooses his or her latent contribution rate choice. Therefore $\Gamma$, in the case of the 401(k) choice model, will be comprised of all choices such that $\{x_i \ | \ d\} = c_i(k, d, \theta, \gamma)$. Note that the “opt-out” choice can equal the default contribution rate if $c_i(k, d, \theta, \gamma) = d$. I assume that those choosing the default option are making a welfare relevant choice only if their latent contribution rate is equal to the default option.

The choice based welfare analysis in this paper consists of two experiments. The first experiment determines what proportion of the cohort population is made
unambiguously better off or unambiguously worse off, for a change in default contribution rate from 0% to 3%. This change mirrors the common shift from standard enrollment to automatic enrollment with a conservative default contribution rate. The second experiment determines the proportion of the cohort population that is made unambiguously better off or unambiguously worse off by an increase in the default contribution rate from 3% to 6%. This scenario mimics an employer deciding on a more aggressive default, as advocated by Choi et al. (2001).

5. Parameterization Results

Several refinements of an exhaustive grid search in MATLAB generated the following parameter estimates (see Appendix for the associated MATLAB code).

Table 2: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>0.94</td>
<td>0.1343</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>4.20</td>
<td>0.9966</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.13</td>
<td>0.1488</td>
</tr>
<tr>
<td>( \mu )</td>
<td>8.3</td>
<td>0.3646</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>5.0</td>
<td>0.3232</td>
</tr>
</tbody>
</table>

The estimates for both \( \theta \) and \( \gamma \) are statistically significant at the one percent level. When the model is parameterized in this way, an employee’s latent choice is given as

\[
c_i(k, d, \theta) = k_i + 4.2 \left[ \frac{1}{1 + e^{-0.95(d-k_i)}} - \frac{1}{2} \right]
\]  

(12)

It follows that the maximum amount by which one’s latent choice can differ from one’s internal anchor is 2.1 percentage points. To illustrate these results, Figure 2 provides a
graph of the latent choice as a function of the default rate for an individual with an internal anchor of eight percent.

**Figure 2: Latent Choice of an Average Employee as a Function of the Default Option**

Recall that an employee either chooses his or her latent contribution rate choice or fails to opt out of the default. The parameterization results imply that one chooses to opt out of the default rate with probability

\[ \lambda(d, c_i) = \frac{|d - c_i|}{0.13 + |d - c_i|} \]  \hspace{1cm} (13)

This implies, for example, that if one’s latent choice differs from the default rate by two percentage points, he or she will opt out of the default with a probability of 0.939 (or alternatively, accept the default with a probability of 0.061). Figure 3 shows the relationship between the probability of opting out and the difference between the default rate and one’s latent choice.
Figure 3: The Probability of Opting Out of the Default

6. Welfare Comparison Results

When a new policy changes the default contribution rate, employee behavior falls into four distinct categories. The first category, call them “default lovers,” chose the default before the policy change and continue to contribute at the default rate after the policy change. The second category, “default haters,” did not choose the default before the plan change and do not choose the default afterwards. The third category, “opt-ins,” did not choose the default rate under the old plan, but choose the default when facing the new enrollment scheme. The final category, “opt-outs,” chose the default before the policy change, but opt out of the default after the policy change.

I assume that if an employee chooses the default option, he or she will not opt out when the default moves closer to his or her internal anchor. Such an assumption is consistent with the nonzero parameter estimate for $\beta$. For example, if an individual with
an internal anchor equal to 1% chooses the default rate when the default option is 6%, I assume that he will also choose a default option of 5%.

Thus for any given type of individual, defined by an internal anchor, we can use the model of 401(k) choice to calculate the proportion of individuals in each of the four response categories. For example, take individuals with an internal anchor of zero when the default rate rises from 3% to 6%. The model predicts that when the default rate is 3%, 96% of such individuals opt out and choose a contribution rate of 2% and the remaining 4% choose the default. After the default rate rises, 98% of the individuals in the group choose to contribute 2% and only 2% of the individuals choose the higher default rate.

As a result, 2% of such individuals are “default lovers,” 96% are “default haters,” 2% are “opt-ins,” and none are “opt outs” based on the predicted responses to the default change. The welfare relevant choice domain of those with an internal anchor of zero includes all choices that are made when opting out of the default (i.e. \{0,1,2\}). We cannot discern welfare relations between these choices, but assuming monotonicity, 0, 1, and 2 are all unambiguously preferred to 3 (0P*3, 1P*3, 2P*3). Likewise, 3 is unambiguously preferred to 6 (3P*6).

These choice based welfare relationships imply that all “default lovers” and “opt-ins” with internal anchors equal to zero are worse off as a result of the policy change. Both responses result in a choice of 6% after the policy change, which is not a choice in the welfare relevant choice domain. Default haters always make choices within the welfare relevant choice domain, such that the generalized normative framework produces an ambiguous welfare relationship between their choices before and after the policy change.
I carry out these calculations for all internal anchors. Moreover, since our parameterization has defined the distribution of internal anchors in the cohort population, the proportion of the total cohort population that is characterized by each internal anchor is known. Aggregating over all internal anchors provides us with the total cohort proportion made better off and worse off by a policy change, as well as the proportion of those for whom no discerning welfare relations can be determined. The two following tables summarize these results.

**Table 3: Response to a Change in Default Rate from 0% to 3%**

<table>
<thead>
<tr>
<th>Category</th>
<th>Percent of Cohort Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default Lovers</td>
<td>9.5</td>
</tr>
<tr>
<td>Default Haters</td>
<td>68.5</td>
</tr>
<tr>
<td>Opt-Ins</td>
<td>17</td>
</tr>
<tr>
<td>Opt-Outs</td>
<td>5</td>
</tr>
<tr>
<td>Those Made Better off</td>
<td>2.4</td>
</tr>
<tr>
<td>Those Made Worse off</td>
<td>2.1</td>
</tr>
<tr>
<td>Those For Whom No Discerning Welfare Statements Can Be Made</td>
<td>95.5</td>
</tr>
</tbody>
</table>

**Table 4: Response to a Change in Default Rate from 3% to 6%**

<table>
<thead>
<tr>
<th>Category</th>
<th>Percent of Cohort Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default Lovers</td>
<td>15.5</td>
</tr>
<tr>
<td>Default Haters</td>
<td>52</td>
</tr>
<tr>
<td>Opt-Ins</td>
<td>22.5</td>
</tr>
<tr>
<td>Opt-Outs</td>
<td>10</td>
</tr>
<tr>
<td>Those Made Better off</td>
<td>2.7</td>
</tr>
<tr>
<td>Those Made Worse off</td>
<td>2.3</td>
</tr>
<tr>
<td>Those For Whom No Discerning Welfare Statements Can Be Made</td>
<td>95</td>
</tr>
</tbody>
</table>

If the welfare relevant choice domain includes only latent contribution choices, a change in the default rate from 0% to 3% results in 2.7% of the cohort population better
off, 2.3% of the cohort population worse off, and 95.5% of the cohort population for whom the welfare results are ambiguous. Likewise, an increase in default contribution rate from 3% to 6% results in 2.7% of the cohort population better off, 2.3% worse off, and 95% for whom the welfare results are ambiguous.

7. Analysis of the Model

The nonzero value of the gamma parameter estimate is consistent with the notion that anchoring bias does, in fact, affect 401(k) contribution rate decision-making. Until now, the Behavioral Economics literature has assumed that the primary effect of default 401(k) contribution rate changes manifests itself through status quo bias. That is, people have a tendency to disproportionally choose a default option, and changes in the default option can lead to choice reversals when people choose the default in both cases. The findings in this paper confirm these suspicions, but also suggest that anchoring bias may have an economically significant effect on decision-making in the context of 401(k) contributions.

The presence of anchoring bias implies that defaults can lead to choice reversals even when the default is not chosen. Even people who systematically do not choose the default are affected by the value that the default takes on. The “power of suggestion,” as discussed in Madrian and Shea (2001), may have a partial effect on consumer saving, rather than simply a binary effect. If a company’s default rate is set at 3%, for example, an employee with an internal anchor of 10% may view the firm’s relatively low default rate as a suggestion, or implicit advice, and choose to contribute 7%. Prior models of 401(k) choice in the presence of a default do not account for such behavior.
Another, perhaps surprising, result is the relatively high mean of internal anchoring rates in the cohort population implied by the model and empirical data. The median observed contribution rate for both samples falls in the 6% bin. The median and mean of the internal anchor distribution are estimated at 8.3%, a significantly higher rate. This estimate suggests that low external anchors, or defaults, may be dragging down employee contribution rates. This possibility is noteworthy if the goal of a particular policy is strictly to increase employee savings. Standard enrollment, for instance, provides a default contribution rate of 0%. The model suggests that this anchor negatively influences contribution rates across the board. Moreover, automatic enrollment plans will not be successful in raising saving rates unless default options are aggressive enough to set high anchors.

The paper’s maximum likelihood estimation results are consistent with the intuitive notion that the probability of accepting a default is inversely proportional to the difference between one’s latent choice and the default. If this were not the case, $\beta$ would take on a value of 0. The fact that the $\beta$ estimate is nonzero implies that the number of people who exhibit default behavior is a function of the default contribution rate, and is endogenous.

The model’s specification does not explicitly name procrastination, but procrastination effects are implicit in the reduced form. In the model’s interpretation and comparative statics, one can think of procrastination as an exogenous determinant of the $\lambda$ function, manifested in the parameter $\beta$. This is to say that if a change in policy increases the likelihood of procrastination (e.g. the 401(k) choice decision is made more complex), we would expect this exogenous shock to raise the value of $\beta$. The model
would thus predict that the change in policy would result in a greater propensity to accept the default.

The model estimation in this paper depends upon a relatively small and particular sample. More precise parameter estimates could be obtained with more observations. Unfortunately, due to the relatively recent rise in automatic enrollment plans, there is currently little data that researchers can use to compare contribution rate distributions under various default regimes. In time, if 401(k) trends continue, more information will become available, and economists will be able to better gauge the effects of behavioral anomalies arising from defaults.

As a result of the current lack in data availability, the parameter estimates for the model presented in this paper rely on data from only one cohort within one company. In order to extrapolate the parameterization of the model to American employees in general, it is necessary to assume that the mid-sized chemicals company from which these observations come from is a typical, or average, U.S. company. Testing the validity of this assumption, however, is beyond the scope of this paper. The model’s parameterization is valid for the observed demographic, and cross-company data is needed to secure a more general parameterization.

With regard to the U.S. chemicals company sample, this paper adopts from Beshears et al. (2005) the implicit assumption that there are no significant differences between the employees that faced a 3% default and the employees that faced a 6% default. This assumption allows for the direct comparison necessary for the model’s parameterization. Since the two samples come from the same cohort at the same firm, this is likely to be a reasonable assumption if we abstract from changes in macroeconomic conditions and other company policies.

This paper adopts and applies a generalized welfare criterion to the positive 401(k) choice model, restricting the welfare relevant choice domain to choices that opt out of a default. Under this restriction, we find that the introduction of an automatic enrollment regime with a conservative default rate does not produce the large increase in well being that the Behavioral Economics literature has previously implied. Likewise, we find that raising the default rate from 3% to 6% has mixed welfare effects.

When we break down the cohort population into four groups (default lovers, default haters, opt-ins, and opt-outs) we see why this is the case. When automatic enrollment is introduced, the default is moved towards the center of the distribution of internal anchor rates. As a result, a large part of the distribution has a greater chance of accepting the default under a 3% default scheme, as opposed to the standard enrollment scheme. This paper’s application of the choice based normative framework implies that those people who choose a high contribution rate when facing a 0% default, but choose the default when the default is 3%, are made worse off by introduction of automatic enrollment. This includes a number of “opt-ins.” The change in default contribution rate from 3% to 6% also moves the default closer to the center of the internal anchor rate distribution, causing that policy to suffer from a similar problem.

On the other hand, those who have an internal anchor rate near 3% benefit when the default increases from 0% to 3%. Those who have an internal anchor rate near 6% benefit when the default increases from 3% to 6%. Such people benefit from having a default near their internal anchor, since default behavior does not lead them to make different choices than they otherwise would. “Default lovers” are made unambiguously better off whenever the default moves closer to their internal anchor rates. These cases
account for most of the people who are made better off under the two policy shifts that we examine.

We draw no discerning welfare conclusions for an extremely large part of the cohort population. This does not imply that the normative framework is not powerful. Given the parameterization of the choice model, we find that most peoples’ choices were not affected by the policy changes. Employees with internal anchors sufficiently far from both defaults tend to not change their behavior in response to the two experiments, acting as “default haters.” This trend accounts for most of the cases in which welfare relations were ambiguous.

While this paper chooses to restrict the welfare relevant choice domain to choices made when opting out of the default, this is by no means the only or the best way to restrict the welfare relevant choice domain. While it is common to assume that default choices are suboptimal when they result in choice reversals, one can think of many reasons to object to such a restriction. Suppose, for instance, firms set default contribution rates with the help of financial experts to provide benign advice to employees. If this were the case, it would not make sense to assume that employees taking such advice would not be making welfare relevant choices. On the contrary, financially illiterate employees may be much better off taking the advice of financial planners than setting contribution rates based off of their internal anchors.

In any case, restrictions to the welfare relevant choice domain need to be made with policy goals in mind. If the goal, for instance, is to enroll as many people in 401(k) plans as possible, then the generalized normative framework can be applied with the welfare relevant choice domain restricted to choices of nonzero contribution rates. If the goal is to encourage employees to fully take advantage of employer matching, then the welfare relevant choice domain can be restricted to only include choices greater than or
equal to the employer match rate. The choice based normative framework can judge the welfare effects of policy shifts in terms of any such goals within the context of the 401(k) choice model.

9. Conclusion

This paper proposes a model of 401(k) contribution rate choice and applies a generalized normative framework to assess welfare implications of default contribution rate changes. The parameters of the model are estimated using maximum likelihood estimation, and the resulting parameter estimates are consistent with the hypothesis that default contribution rates influence contribution choices through anchoring bias, as well as status quo bias.

To generate discerning welfare relationships, the welfare relevant choice domain of the model is restricted to choices that equal an employee’s latent contribution rate choice. Under this restriction, I find that the change from a 0% default to a 3% default makes 2.4% of the cohort population better off, and 2.1% of the population worse off. This result calls into question the common assumption that the introduction of automatic enrollment increases overall employee welfare by increasing participation. I also find that an increase of the default from 3% to 6% makes 2.7% of the cohort population unambiguously better off and 2.3% of the population unambiguously worse off. Alternative restrictions to the welfare relevant choice domain, however, will yield different results. With that in mind, this paper encourages policy makers to be explicit in their welfare assumptions, and to properly align such assumptions with policy goals.

A lot can still be learned from further estimations and reformulations of the model. This paper draws its results from a limited pool of data. When more data on
401(k) enrollment is available, the 401(k) choice model can be tested and estimated more thoroughly using cross-company data. More data sources would also give behavioral researchers a better idea of how various demographics and cohorts respond differently to default options. Such information is crucial for policy makers that target the well being of specific demographics.

We must also acknowledge that default options are not the only external anchors driving 401(k) decision-making. For some, the employer match rate may also be an important conscious or unconscious benchmark. At this time, relatively little research has tied changes in the employer match rate to changes in employee welfare.

Furthermore, retirement savings are still just one example of the influence that default options have on peoples’ actions and well-being. Default options can also play a key role in health policy, energy policy, advertising, and any other walk of life in which behavioral biases are observed. The better we understand how we respond to the status quo, the better we can structure policy to encourage informed choices.
10. Appendix

10.1 MATLAB Code

GridSearch.m

clear all;

%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
%The following code estimates the parameters by way of a grid
%search simulation, returning the parameter estimates that yield
%the greatest log-likelihood.
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

tic;
maxlikelihood = ll(.05,.2,.1,.1,2,.5);
optTheta = 0;
optAlpha = 0;
optBeta = 0;
optMu = 0;
optSigma = 0;
optGamma = 0;

for theta=0:.25:4
    for alpha=0:.1:.9
        for beta=.1:.25:2.1
            for mu=6:9
                for sigma=4.5:.2:5.5
                    for gamma=0:.25:4.25
                        likelihood = ll(theta,gamma,alpha,beta,mu,sigma);
                        if likelihood > maxlikelihood
                            maxlikelihood = likelihood;
                            optTheta = theta;
                            optAlpha = alpha;
                            optBeta = beta;
                            optMu = mu;
                            optSigma = sigma;
                            optGamma = gamma;
                        end
                    end
                end
            end
        end
    end
end

maxlikelihood
optTheta %The parameter estimate for theta.
optGamma %The parameter estimate for gamma.
optAlpha \%The parameter estimate for alpha.
optBeta \%The parameter estimate for beta.
optMu \%The parameter estimate for mu.
optSigma \%The parameter estimate for sigma.
toc;

RefinedGridSearch.m

%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
% The following code refines the simulation, iterating through
% potential parameter estimates near those estimates produced by
% the first simulation.
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
tic;
maxlikelihood = ll(.05,.2,.1,.1,.5);
optTheta = 0;
optAlpha = 0;
optBeta = 0;
optMu = 0;
optSigma = 0;
optGamma = 0;
for theta=.95:.02:5
    for alpha=0
        for beta=.10:.02:.25
            for mu=8:.1:8.4
                for sigma=4.7:.1:5.3
                    for gamma=4:.02:4.2
                        likelihood =
                            ll(theta,gamma,alpha,beta,mu,sigma);
                        if likelihood > maxlikelihood
                            maxlikelihood = likelihood;
                            optTheta = theta;
                            optAlpha = alpha;
                            optBeta = beta;
                            optMu = mu;
                            optSigma = sigma;
                            optGamma = gamma;
                        end
                    end
                end
            end
        end
    end
end
maxlikelihood
optTheta \%The refined parameter estimate of theta.
optGamma \%The refined parameter estimate of gamma.
optAlpha %The refined parameter estimate of alpha.
opBeta %The refined parameter estimate of beta.
opMu %The refined parameter estimate of mu.
opSigma %The refined parameter estimate of sigma.
toc;

tic;
maxlikelihood = ll(.05,.2,.1,.1,2,.5);
opTheta = 0;
opAlpha = 0;
opBeta = 0;
opMu = 0;
opSigma = 0;
opGamma = 0;
for theta=.91:.01:.98
    for alpha=0
        for beta=.1:.01:.18
            for mu=8:.1:8.5
                for sigma=4.9:.1:5.1
                    for gamma=4.14:.01:4.22
                        likelihood =
                            ll(theta,gamma,alpha,beta,mu,sigma);
                        if likelihood > maxlikelihood
                            maxlikelihood = likelihood;
opTheta = theta;
opAlpha = alpha;
opBeta = beta;
opMu = mu;
opSigma = sigma;
opGamma = gamma;
                        end
                    end
                end
            end
        end
    end
end
maxlikelihood
optTheta %The refined parameter estimate of theta.
opGamma %The refined parameter estimate of gamma.
opAlpha %The refined parameter estimate of alpha.
opBeta %The refined parameter estimate of beta.
opMu %The refined parameter estimate of mu.
opSigma %The refined parameter estimate of sigma.
toc;
function ll = LogLikelihood(theta, gamma, alpha, beta, mu, sigma)

d = 3;

bin1 = Lx(0, d, theta, gamma, alpha, beta, mu, sigma);
bin2 = Lx(1, d, theta, gamma, alpha, beta, mu, sigma) + Lx(2, d, theta, gamma, alpha, beta, mu, sigma);
bin3 = Lx(3, d, theta, gamma, alpha, beta, mu, sigma);
bin4 = Lx(4, d, theta, gamma, alpha, beta, mu, sigma) + Lx(5, d, theta, gamma, alpha, beta, mu, sigma);
bin5 = Lx(6, d, theta, gamma, alpha, beta, mu, sigma);
bin6 = Lx(7, d, theta, gamma, alpha, beta, mu, sigma) + Lx(8, d, theta, gamma, alpha, beta, mu, sigma) + Lx(9, d, theta, gamma, alpha, beta, mu, sigma) + Lx(10, d, theta, gamma, alpha, beta, mu, sigma);
bin7 = Lx(11, d, theta, gamma, alpha, beta, mu, sigma) + Lx(12, d, theta, gamma, alpha, beta, mu, sigma) + Lx(13, d, theta, gamma, alpha, beta, mu, sigma) + Lx(14, d, theta, gamma, alpha, beta, mu, sigma) + Lx(15, d, theta, gamma, alpha, beta, mu, sigma);

binLikelihood = [log(bin1) log(bin2) log(bin3) log(bin4) log(bin5) log(bin6) log(bin7)];
propForD3 = [.04; .01; .28; .03; .24; .18; .23];
propForD6 = [.09; .06; .04; .03; .49; .17; .13];
n = 167; % Sample size of those who faced a 3% default rate

likelihoodForD3 = binLikelihood * (n * propForD3);

d = 6;
n=47; % Sample size of those observed who faced a 6% default rate

bin1 = Lx(0, d, theta, gamma, alpha, beta, mu, sigma);
bin2 = Lx(1, d, theta, gamma, alpha, beta, mu, sigma) + Lx(2, d, theta, gamma, alpha, beta, mu, sigma);
bin3 = Lx(3, d, theta, gamma, alpha, beta, mu, sigma);
bin4 = Lx(4, d, theta, gamma, alpha, beta, mu, sigma) + Lx(5, d, theta, gamma, alpha, beta, mu, sigma);
bin5 = Lx(6, d, theta, gamma, alpha, beta, mu, sigma);
bin6 = Lx(7, d, theta, gamma, alpha, beta, mu, sigma) + Lx(8, d, theta, gamma, alpha, beta, mu, sigma) + Lx(9, d, theta, gamma, alpha, beta, mu, sigma) + Lx(10, d, theta, gamma, alpha, beta, mu, sigma);
bin7 = Lx(11, d, theta, gamma, alpha, beta, mu, sigma) + Lx(12, d, theta, gamma, alpha, beta, mu, sigma) + Lx(13, d, theta, gamma, alpha, beta, mu, sigma) + Lx(14, d, theta, gamma, alpha, beta, mu, sigma) + Lx(15, d, theta, gamma, alpha, beta, mu, sigma);

binLikelihood = [log(bin1) log(bin2) log(bin3) log(bin4)
log(bin5) log(bin6) log(bin7)];

likelihoodForD6 = binLikelihood * (n * propForD6);
% The total log likelihood of the entire distribution is the sum of the log likelihood values for each default.

ll = (likelihoodForD3 + likelihoodForD6);

Lx.m

% The function Lx solves for the likelihood of a particular observation assuming a normal distribution of internal anchoring rates in the population. In this case, the maximum contribution rate is 15 percent, but this need not be so.

function l = Lx(x, d, theta, gamma, alpha, beta, mu, sigma)

% The following code corresponds to Equation (8) in the paper.

l = 0;
if x > 0 && x~=d && x < 15
likelihood = (normcdf(KfromX(x+.5,d,theta, gamma), mu, sigma) - normcdf(KfromX(x-.5,d,theta,gamma), mu, sigma)) * lambda(d,theta,gamma,KfromX(x,d,theta,gamma),alpha,beta);
if likelihood < 0
    l = 0;
else
    l = likelihood;
end
elseif x==0
    l = normcdf(.5, mu, sigma) * lambda(d, theta, gamma, KfromX(0,d,theta, gamma), alpha, beta);
if x==d
    chanceOfDefault = 0;
    for i=0:15
        prob = 0;
        if i~=0 && i~=d && i~=15
            prob = (normcdf(KfromX(i+.5,d,theta, gamma), mu, sigma) - normcdf(KfromX(i-.5,d,theta, gamma), mu, sigma))*lambda(d,theta,gamma,KfromX(i,d,theta, gamma),alpha,beta);
        end
    end
end
elseif i==0 && i==d
    prob = normcdf(.5, mu, sigma) * lambda(d, theta, gamma, KfromX(0,d,theta,gamma),alpha,beta);
elseif i==15 && i==d
    prob = (1 - normcdf(KfromX(14.5,d,theta,gamma)))
         * lambda(d,theta,gamma,KfromX(15,d,theta,gamma)
            ,alpha,beta);
end
prob = prob * ((1 - lambda(d,theta,gamma,KfromX(i,d,theta,gamma)
               ,alpha,beta)) /
               (lambda(d,theta,gamma,KfromX(i,d,theta,gamma),
               alpha,beta));
chanceOfDefault = chanceOfDefault + prob;
end
l = (normcdf(KfromX(x+.5,d,theta,gamma), mu, sigma)) +
    chanceOfDefault;
else x==15
    l = (1 - normcdf(KfromX(14.5,d,theta,gamma), mu, sigma)) *
        lambda(d,theta,gamma,KfromX(15,d,theta,gamma),alpha,beta);
if x==d
    chanceOfDefault = 0;
for i=0:15
    prob = 0;
    if i==0 && i==d && i==15
        prob = (normcdf(KfromX(i+.5,d,theta,gamma), mu, sigma) - normcdf(KfromX(i-.5,d,theta,gamma),
            mu, sigma)) *
            lambda(d,theta,gamma,KfromX(i,d,theta,gamma),
            alpha,beta);
    elseif i==0 && i==d
        prob = normcdf(.5, mu, sigma) * lambda(d, theta, gamma, KfromX(0,d,theta,gamma), alpha, beta);
    elseif i==15 && i==d
        prob = (1 - normcdf(KfromX(14.5,d,theta,gamma))) *
            lambda(d,theta,gamma,KfromX(15,d,theta,gamma),alpha,beta);
end
    prob = prob * ((1 lambda(d,theta,gamma,
            KfromX(i,d,theta,gamma),
            alpha,beta))) /
            (lambda(d,theta,gamma,KfromX(i,d,theta,gamma),
               alpha,beta));
    chanceOfDefault = chanceOfDefault + prob;
end
l = (1 - normcdf(KfromX(14.5,d,theta,gamma), mu, sigma))
    + chanceOfDefault;
elseif x==d
    chanceOfDefault = 0;
    for i=0:15
        prob = 0;
        if i~=0 && i~=d && i~=15
            prob = (normcdf(KfromX(i+.5,d,theta,gamma), mu, sigma) - normcdf(KfromX(i-.5,d,theta,gamma), mu, sigma)) * lambda(d,theta,gamma, KfromX(i,d,theta,gamma),alpha,beta);
        elseif i==0 && i~=d
            prob = normcdf(.5, mu, sigma) * lambda(d, theta, gamma, KfromX(0,d,theta,gamma), alpha, beta);
        elseif i==15 && i~=d
            prob = (1 - normcdf(KfromX(14.5,d,theta,gamma))) * lambda(d,theta,gamma,KfromX(15,d,theta,gamma),alpha,beta);
        end
        if i ~= d
            prob = prob * ((1 - lambda(d,theta,gamma, KfromX(i,d,theta,gamma),alpha,beta)) / (lambda(d,theta,gamma,KfromX(i,d,theta,gamma),alpha,beta)));
            chanceOfDefault = chanceOfDefault + prob;
        end
    end
    l = (normcdf(KfromX(d+.5,d,theta,gamma), mu, sigma) - normcdf(KfromX(d-.5,d,theta,gamma),mu, sigma)) + chanceOfDefault;
end

C.m

%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
%The function c solves for the latent contribution rate, given
%values for the default (d), the lean parameters (theta and
%gamma), and the internal anchor (k).
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

function c = C(d, theta, gamma, k)
% c = k + (d - k)*gamma;
    c_i = k + gamma*((1 /(1 + exp(-theta*(d-k)))) - (1/2));
    if ((c_i - d)*(k - d)) < 0
        c = d;
    else c = c_i;
end
The function KfromX solves for an internal anchor (k) given a latent contribution choice.

```matlab
function kfromx = KfromX(x, d, theta, gamma)

if x == d
    kfromx = d;
    return
end

if (gamma*theta) < 4
    k_i = x;
    for j=1:20;
        k_n = k_i - (k_i + gamma*(((1/(1 + exp(-theta*(d-k_i)))) - (1/2)) - x)/(1 + (gamma*(theta^2)*(d-k_i)*exp(-theta*(d-k_i)))^2));
        if k_n == k_i
            break
        end
        k_i = k_n;
    end
    kfromx = k_i;
    return
else
    a = d;
end
```

The variable 'a' defines one bound on the binary search. It is the value of k that minimizes (k-d) for all k not equal to d.

The variable b defines the other bound of the binary search. It is the k value that serves as an upper bound for the region of the choice function that is not invertible.
if x > d
    b = x + (gamma/2);
else
    b = x - (gamma/2);
end

% When gamma*theta > 4, the choice function does not generate a unique inverse, so I approximate k using the bisection method when x is in this region.

if (abs(x - d)) >= (gamma/2)
    k_i = x;
    for j=1:20;
        k_n = k_i - (k_i + gamma*((1 /(1 + exp(-theta*(d-k_i)))) - (1/2)) - x)/(1 + (gamma*(theta^2)*(d-k_i)*exp(-theta*(d-k_i)))/((1 + exp(-theta*(d-k_i))))^2));
        if k_n == k_i
            break
        end
        k_i = k_n;
    end
    kfromx = k_i;
    return
else
    k_i = bisectionMethod(x,d,theta,gamma,a,b);
end
kfromx = k_i;

bisectionMethod.m

function bisectionMethod = bM(x,d,theta,gamma,a,b)

% This function approximates the root of the choice equation by using the bisection method.

if x >= d
    for i=0:20
        midpoint = (a+b)/2;
        choiceFunction = midpoint + gamma*((1 /(1 + exp(-theta*(d-midpoint)))) - (1/2)) - x;
        if choiceFunction < 0
            a = midpoint;
        else
            b = midpoint;
        end
    end
    kfromx = midpoint;
end
if x < d
    for i=0:20
        midpoint = (a+b)/2;
        choiceFunction = midpoint + gamma*((1/(1 + exp(-theta*(d-midpoint)))) - (1/2)) - x;
        if choiceFunction < 0
            b = midpoint;
        else
            a = midpoint;
        end
    end
    bisectionMethod = (a+b)/2;
end
end

% The following function solves for the probability of opting out of the default. This method is analogous to Equation (3).
function l = lambda(d, theta, gamma, k, alpha, beta)

l = alpha + (1 - alpha)*abs(d - C(d, theta, gamma, k))/(beta + abs(d - C(d, theta, gamma, k)));

% The following function approximates the standard errors of the MLE parameter estimates by inverting the Hessian Matrix of the negative log likelihood function. I obtain the Hessian Matrix by approximating second order partial derivatives with finite differences.
% Note that the method ll.m solves for the log likelihood, not the negative log likelihood. Calculations here are adjusted accordingly.

h = .0001; %approximation step size
k = .0001; %approximation step size
theta = .94;
gamma = 4.1;
alpha = 0;
beta = .13;
mu = 8.3;
sigma = 5;

% Second order partial derivative approximation by finite differences.

fThetaSq = (-ll(theta + h,gamma,alpha,beta,mu,sigma) +
(2*ll(theta,gamma,alpha,beta,mu,sigma)) - ll(theta - h,
gamma,alpha,beta,mu,sigma))/(h^2);
fGammaSq = (-ll(theta,gamma + h,alpha,beta,mu,sigma) +
(2*ll(theta,gamma,alpha,beta,mu,sigma)) - ll(theta, gamma -
h,alpha,beta,mu,sigma))/(h^2);
fBetaSq = (-ll(theta,gamma,alpha,beta + h,mu,sigma)+
(2*ll(theta,gamma,alpha,beta,mu,sigma)) - ll(theta,
gamma,alpha,beta - h,mu,sigma))/(h^2);
fMuSq = (-ll(theta,gamma,alpha,beta + h,mu,sigma)+
(2*ll(theta,gamma,alpha,beta,mu,sigma)) - ll(theta,
gamma,alpha,beta,mu + h,sigma))/(h^2);
fSigmaSq = (-ll(theta,gamma,alpha,beta,mu,sigma + h)+
(2*ll(theta,gamma,alpha,beta,mu,sigma)) - ll(theta,
gamma,alpha,beta,mu,sigma - h))/(h^2);

fThetaGamma = (-ll(theta + h,gamma + k,alpha,beta,mu,sigma) +
ll(theta + h, gamma - k, alpha, beta, mu, sigma) + ll(theta - h,
gamma + k, alpha, beta, mu, sigma) - ll(theta - h, gamma -
k, alpha, beta, mu, sigma))/(4*h*k);
fThetaBeta = (-ll(theta + h,gamma,alpha,beta + k,mu,sigma) +
ll(theta + h, gamma, alpha, beta - k,mu, sigma) + ll(theta - h,
gamma, alpha, beta + k,mu, sigma) - ll(theta - h, gamma, alpha, beta -
k,mu, sigma))/(4*h*k);
fThetaMu = (-ll(theta + h,gamma,alpha,beta,mu + k,sigma) +
ll(theta + h, gamma, alpha, beta, mu - k,sigma) + ll(theta - h,
gamma, alpha, beta, mu + k,sigma) - ll(theta - h, gamma, alpha, beta -
k,sigma))/(4*h*k);
fThetaSigma = (-ll(theta + h,gamma,alpha,beta,mu,sigma + k) +
ll(theta + h, gamma, alpha, beta, mu,sigma - k) + ll(theta - h,
gamma, alpha, beta, mu, sigma + k) - ll(theta - h, gamma, alpha, beta, mu, sigma - k))/(4*h*k);
fGammaBeta = (-ll(theta,gamma + h,alpha,beta + k,mu,sigma) +
ll(theta, gamma + h, alpha, beta + k,mu, sigma) + ll(theta, gamma,
alpha, beta + k,mu, sigma) - ll(theta, gamma, alpha, beta +
k,mu, sigma))/(4*h*k);
- h, alpha, beta + k, mu, sigma) - ll(theta, gamma - h, alpha, beta - k, mu, sigma))/(4*h*k);
fGammaMu = (-ll(theta, gamma + h, alpha, beta, mu + k, sigma) + ll(theta, gamma + h, alpha, beta, mu - k, sigma) + ll(theta, gamma - h, alpha, beta, mu + k, sigma) - ll(theta, gamma - h, alpha, beta, mu - k, sigma))/(4*h*k);
fGammaSigma = (-ll(theta, gamma + h, alpha, beta, mu, sigma + k) + ll(theta, gamma + h, alpha, beta, mu, sigma - k) + ll(theta, gamma - h, alpha, beta, mu, sigma + k) - ll(theta, gamma - h, alpha, beta, mu, sigma - k))/(4*h*k);
fBetaMu = (-ll(theta, gamma, alpha, beta + h, mu + k, sigma) + ll(theta, gamma, alpha, beta + h, mu - k, sigma) + ll(theta, gamma, alpha - h, mu + k, sigma) - ll(theta, gamma, alpha - h, mu - k, sigma))/(4*h*k);
fBetaSigma = (-ll(theta, gamma, alpha, beta + h, mu, sigma + k) + ll(theta, gamma, alpha, beta + h, mu, sigma - k) + ll(theta, gamma, alpha - h, mu, sigma + k) - ll(theta, gamma, alpha - h, mu, sigma - k))/(4*h*k);
fMuSigma = (-ll(theta, gamma, alpha, beta, mu + h, sigma + k) + ll(theta, gamma, alpha, beta, mu + h, sigma - k) + ll(theta, gamma, alpha, beta, mu - h, sigma + k) - ll(theta, gamma, alpha, beta, mu - h, sigma - k))/(4*h*k);

% The Hessian Matrix

Hessian = [fThetaSq fThetaGamma fThetaBeta fThetaMu fThetaSigma fThetaGamma fGammaSq fGammaBeta fGammaMu fGammaSigma fThetaBeta fGammaBeta fBetaSq fBetaMu fBetaSigma fThetaMu fGammaMu fBetaMu fMuSq fMuSigma fMuSigma fSigmaSq]

% The following algorithm solves for the vector of standard errors
Stderr = sqrt(diag(inv(Hessian)))
References


