The Impact of Rising Interest Rates on the Optimal Social Security Claim Age

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ABSTRACT
Individuals are eligible to claim Social Security benefits between the ages of 62 and 70. While the actuarial present value of benefits for each individual is supposed to be unchanged, regardless of the age at which they are claimed, previous research has shown that this is not true. This paper aims to analyze the impact that interest rates have on the present value of the benefits and attempts to study how different interest rate expectations affect the optimal age at which to claim Social Security. The methodology used in this paper differs from that used in previous studies; I introduce a dynamic interest rate model based on forward rates under both a deterministic and stochastic approach to calculate the Net Present Value associated with different claim ages so as to identify the optimal claim age. The results of this paper display that under certain assumptions, it is advantageous for both men and women to begin claiming retirement benefits early – between the ages of 62 and 65. Moreover, the results indicate that there are significant losses associated with delaying claims beyond the optimal claim age.

Keywords: Social Security, Net Present Value, Retirement Benefits

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Introduction

The eligibility age for claiming Social Security benefits in the United States was fixed at age 65 until 1955. However, legal reforms enacted in 1956 for women and 1961 for men enabled individuals to claim benefits, adjusted downward for early withdrawal, as early as age 62. Alternatively, individuals can progressively increase their monthly collectible by deferring their benefits until age 70. Once an individual starts claiming benefits, Social Security provides them with a fixed sum of money every month for the rest of their life, adjusted for inflation. Thus, individuals are faced with a tradeoff between claiming earlier to receive a longer stream of benefits, and delaying claims to increase their monthly collectible. This adjustment is expected to render the actuarial present value of benefits for each individual unchanged, regardless of the age at which they are claimed. However, previous research has shown that this expectation does not hold in low interest rate environments wherein it is optimal for one to delay claims to Social Security benefits (Shoven and Slavov, 2012).

In this paper, I aim to build upon the work conducted in this field by assessing the impact of delaying social security benefits during differing interest rate environments. I focus on single individuals to determine the age that maximizes the Net Present Value (NPV) of the benefits, that is, the current worth of the expected future stream of payoffs that an individual will gain from Social Security throughout his or her lifetime. I use the 1952 birth cohort of the United States population for this analysis, and the Net Present Value is calculated as of age 62 for this cohort.

Past research has often analyzed the impact of interest rates on claim age under one specific interest rate environment. However, this paper aims to identify the optimal age at which
individuals should begin accruing Social Security benefits under various interest rate scenarios. The methodology instituted here also differs in two key aspects from past studies; as opposed to a static interest rate model that has been used in the past, this analysis introduces a dynamic interest rate model that is based on forward rates. Additionally, as opposed to previous studies which use a deterministic interest rate model based on a specific date in time, this paper introduces a stochastic interest rate model based on Monte Carlo simulations to forecast future interest rates.

The results demonstrate that it is advantageous to claim benefits early during a period of rising real interest rates; the Net Present Value of claiming Social Security is maximized between the ages of 62 and 65 for both men and women under different scenarios. Moreover, we observe that if an individual delay claims beyond this optimal age, he would begin to lose more in NPV than if he had claimed his benefits at age 62. Thus, the results indicate that it is often advantageous to claim benefits at an earlier age.

The rest of this paper is organized as follows: Section 2 reviews the existing literature in this field. Section 3 describes into the applied methodology by first outlining the Social Security program and then describing the assumptions, data sources and approach used in this study. Section 4 presents the findings of the analysis and Section 5 provides a critical review of these results. Section 6 describes some future implementations of the paper and concludes.

**Review of Literature**

There has been substantive research conducted related to delaying social security benefits. Coile et al. (2002) use financial calculations and simulations of an expected utility maximization model to estimate optimal delays from Social Security payments and the gains resulting from
those delays. They show that at an interest rate of 3 percent it can be beneficial to delay claims for a period of time after retirement, and that too few men delay claiming these benefits thereby displaying sub-optimal behavior. Sun and Webb (2009) use numerical optimization techniques to illustrate that the optimal age for non-liquidity constrained single individuals lies between the ages of 67 and 70. They coin the term “Social Security Equivalent Income,” defined as the factor by which the Social Security benefits of a non-liquidity constrained retired household claiming at sub-optimal ages must be increased so that it is as well-off in expected utility terms as at the optimal combination of ages (Sun and Webb, 2009). At an interest rate of 3 percent, Sunn and Webb estimate the optimal claim age for highly risk averse single men and women to be between the ages of 67 and 70. They estimate the Social Security Equivalent Income to be 15 to 19 percent higher for individuals who claim at the optimal age, as opposed to those who claim at the earliest possible age of 62.

Shoven and Slavov (2012) build upon this research by using a Net Present Value analysis to calculate the benefits from delay during low interest rate environments. Their analysis demonstrates that at zero percent interest rates, singles across all classifications benefit from delay. The average woman maximizes her NPV at age 70, and stands to gain an additional 18.4%, or $54,000 in overall value by doing so. The average male maximizes his NPV at age 69, which results in a 13.5%, or $39,000 gain in overall value. Alternatively, at a discount rate of 2.9%, the average woman maximizes her NPV at age 68 to gain an additional 4.1% in overall value while the average male maximizes his NPV at age 65, resulting in a 1.3% gain in overall value.

Thus we observe that at interest rates of 3 percent or lower, individuals can gain substantive benefits from delaying claims to Social Security. Moreover, we observe that the
optimal age for claiming benefits is at, or after, age 65 for most individuals during low interest rate environments. In previous analyses, authors’ have used a constant interest rate for the entire time horizon that is based on the interest rate environment prevalent during the time of the analysis. However, interest rates do not stay constant over time. Interest rates are influenced by the term to maturity of an underlying asset or security and thus, the rates of two similar securities that differ only in their term to maturities will be different. This relationship is displayed by a yield curve. In this paper, I attempt to account for this principle by estimating the present value of Social Security benefits under a dynamic interest rate model that alters rates depending on the time horizon. Moreover, as seen in Figure 1, which demonstrates the volatility in historical interest rates, rates fluctuate on a daily basis and the impact of market forces on interest rates makes it challenging to forecast the yield curve; the interest rate environment prevalent today is often not representative of the future rate environment. Therefore, I introduce a stochastic model based on Monte Carlo simulations to forecast future interest rates, as opposed to previous studies which use a deterministic rate model based on a specific date in time to estimate the optimum age to claim Social Security benefits.

Data and Methodology

I conduct my analysis in three stages. First, I calculate the cash flows generated by the retirement benefits for both men and women under different claim ages. I then identify the appropriate rate to discount these cash flows so as to arrive at the Net Present Value of the retirement benefits and the NPV maximizing age for each individual. To do so, I introduce a dynamic interest rate model based on a forward rate yield curve. Third, I introduce a stochastic approach based on Monte Carlo simulations to forecast future yield curves, as opposed to conducting my analysis using a yield curve based on a specific date in time.
I. Calculating Retirement Benefits

Individuals qualify for Social Security retirement benefits by accumulating retirement credits, which are awarded to a person when they pay Social Security taxes on their earnings; for example, in 2013 one credit was awarded for every quarter one earned at least $1,160. Upon accumulation of 40 retirement credits, an individual is eligible to receive retired worker benefits in the form of a life annuity, that is, a fixed sum of money paid out for the rest of his life.

The Social Security Administration calculates the benefits provided to each individual based on their lifetime earnings. Each individual’s earnings are indexed, that is, adjusted to account for changes in average wages from the year the earnings were received. The 35 highest years of indexed earnings are averaged and divided by 12 to arrive at the Average Monthly Indexed Earnings (AIME), and a nonlinear formula is applied to the AIME to arrive at the basic benefit received by individuals every month, which is referred to as the Primary Insurance Amount (PIA). For instance, retirees in the 1952 birth cohort receive 90 percent of the first $816 of AIME, 32 percent of AIME between $816 and $4,917, and 15 percent of any AIME thereafter. The PIA is estimated at the Full Retirement Age (FRA), which is age 66 for the 1952 birth cohort. However, since the benefit amount can be claimed between ages 62 and 70, the PIA is adjusted downward for early withdrawal or progressively increased for deferred collection. For the 1952 birth cohort, the benefit amount is progressively reduced by 5/9 percent each month between ages 63 and 66, and by 5/12 percent between ages 62 and 63. Conversely, these benefits increase by 2/3 percent each month for individuals in this cohort who delay claims after age 66 until age 70. Therefore, an individual who claims benefits at age 62 would receive 75% of his
PIA as a monthly annuity, whereas an individual who claims benefits at age 70 would receive 132% of his PIA as a fixed monthly annuity going forward.

To estimate the Net Present Value of the benefits accrued from Social Security, an individual’s stream of monthly cash flows received as the Primary Insurance Amount is projected until his or her death. This analysis assumes that a person earns $60,000 on average in any given year, adjusted for inflation, and that a person works for 35 years before the age of 62. To calculate the life expectancies for both men and women, I use data provided by the Social Security Administration. The Social Security Administration provides cohort life tables every ten years to estimate life expectancies for a period of time. Thus, for this analysis, the life expectancies are taken from the 1950 cohort life table. To calculate life expectancies, the administration develops a mortality rate at each age, such as age 62, which is equal to the probability of a group member dying before reaching the subsequent year, age 63, conditional on surviving to age 62. It then applies this mortality rate to a given population size from a specific cohort to estimate the life expectancy at each year for that sample of the population. This analysis uses the life expectancies at age 62, which are 19.8 and 22.6 for men and women respectively.

II. Selecting the Appropriate Discount Rate

Each of these future monthly payments is then discounted by a predetermined interest rate, known as the “discount” rate. The rationale behind discounting these future payoffs is that one could invest that sum today at the current discount rate to attain the same projected value in the future. Hence, discounting the aforementioned future cash flows to age 62 at a predetermined discount rate enables us to estimate the present value of these benefits at age 62. This is
illustrated in the formula below where “CF” represents cash flows and “k” represents the discount rate.

\[
NPV = CF_0 + \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t} = \sum_{t=0}^{n} \frac{CF_t}{(1+k)^t}
\]

In this paper, I use U.S. treasury bonds, adjusted for inflation, as a proxy for real interest rates. The Federal Reserve provides daily estimates of the U.S. treasury bonds with 6 month, 1 year, 3 year, 5 year, 7 year, 10 year, 20 year and 30 year maturities, which are then displayed as a yield curve. The curve for November 1, 2013 is to represent the current interest rate environment and is henceforth referred to as “today’s” yield curve. This yield curve is outlined in Figure 2. Using today’s yield curve, we can determine what interest rates will be in the future by calculating what are referred to as “forward” rates, which allow buyers and sellers to determine future interest rates today. Forward rates can be calculated using the subsequent formula where “f” represents the forward rate, “k” represents the current interest rate and “t” represents the timeframe in reference:

\[
_f f_{t+1} = \frac{(1+k_{t+1})^{y+1}}{(1+k)^y} - 1
\]

For instance, \( f_{3} f_{4} \) represents the one-year interest rate between years 3 and 4, which can be obtained from dividing a current four year adjusted yield by a current three year adjusted yield. The forward yield curve derived from the yield curve of November 1, 2013 is displayed in Figure 3. Using forward rates, we can lock in a fixed rate of return that a person can earn on an asset between two distinct time periods in the future. For instance, using \( f_{3} f_{4} \), an individual can lock in a specific rate of interest that can be earned between years 3 and 4. Thus, I generate a
dynamic curve based on forward rates with which I can discount different future payments made
by the Social Security Administration to an individual. Since both the payment amount and
timeframe of the Social Security benefits are fixed, the forward rate curve can determine the
appropriate future discount rates by which to discount these Social Security benefits.

Additionally, previous analyses are based on a deterministic model that implies that the
future interest environment will be similar to today’s rate environment. However, given the
inherent volatility embedded in interest rates, there is a large amount of uncertainty surrounding
the shape of the yield curve going forward. The yield curve changes on a daily basis and there is
no assurance that historical rates will be representative of future interest rates. Therefore to
model this uncertainty, I introduce a stochastic interest rate model that uses Monte Carlo
simulations to estimate the NPV maximizing age under 1,000 different interest rate scenarios.

III. Introducing a Stochastic Model

The Monte Carlo simulations are performed using the means and standard deviations of
historical interest rates. I assume that interest rates follow a lognormal distribution, which
implies that interest rates follow a geometric Brownian motion. $i$ denotes one of the eight
maturities of U.S. treasury bonds listed by the Fed: 6 month, 1 year, 3 year, 5 year, 7 year, 10
year, 20 year and 30 year maturities. $\mu_i$ denotes the mean interest rate for a particular maturity
and time period. For instance, $\mu_3$ represents the mean real interest rate of a U.S. treasury that has
a 3 year maturity. Similarly, $\sigma_i$ represents the standard deviation for a particular maturity and
time period. The Monte Carlo simulation is done in three stages. First, $\mu_i$ and $\sigma_i$ are obtained for
a particular time period. Next, an independent standard-normal random variable $z$ is generated
for each simulation, $k$, using Excel’s random number generator. For this analysis, $k$ ranges from
1 to 1,000. This number is combined with $\mu_i$ and $\sigma_i$ to generate a yield for each maturity, $y_i$, that has the properties of the underlying historical rates.

$$y_k = \mu_i + \sigma_i z_k$$  \hspace{1cm} (2)

Finally the rate for each maturity is simulated using the transformation:

$$Y_k = \exp(y_k) - 1$$  \hspace{1cm} (3)

From each of the simulated yield curve, the corresponding forward yield curve is calculated. The forward rate curves for each period are used to calculate the Net Present Value of the benefits under different claim ages and therefore determine the NPV maximizing age for claiming Social Security benefits. I then calculate the overall mean and standard deviation of the optimal claim age across all simulations.

I break down my analysis into three segments. First, a base case is set up to calculate the NPV maximizing ages for men and women under constant interest rates; two alternative rates of 0.1 percent and 3 percent were used in the base case. Second, I use the dynamic interest rate model to estimate the NPV maximizing age during the current interest rate environment in the United States. I extend this analysis to examine my results under different interest rate environments and use representative yields curve from each of the past three decades to do so. Third, I use the stochastic approach based on Monte Carlo simulations to estimate the optimum age at which to claim Social Security benefits. I conduct this analysis for today’s interest rate environment and then extend the analysis examine my results under different interest rate environments seen in each of the past three decades.
Results

I. Base Case

For the base case of the analysis, I estimate the Net Present Value for both, men and women at age 62 using two constant real rates: 0.1 percent and 3 percent. At an interest rate of 0.1 percent, males benefit from delay and maximize their NPV at age 68. As observed in Figures 4 and 5, which display the gains from delaying under the base case, men are able to increase their gains from delay with each subsequent year until age 68; they can gain an additional 9.9 percent in NPV by claiming at age 68, relative to if they would have claimed their benefits at age 62. Similarly, women maximize their NPV at age 69 and stand to gain an additional 13.8 percent by delaying claims beyond age 62. Alternatively, at an interest rate of 3 percent, men maximize their NPV at age 65. However, as seen in Figure 5, the gains from delay drop to 1.47 percent at age 65, and decrease in value after age 65. On the other hand, women maximize their NPV at age 67, and can gain 3.3 percent in NPV by doing so.

The base case uses a static interest rate model to perform the analysis, a methodology that is similar to that used in the historic literature on this topic. As opposed to a static interest rate model that has been used in the past, I now analyze the results using a dynamic interest rate model that is based on forward rates.

II. Dynamic Interest Rate Model under a Deterministic Approach

A. Using today’s yield curve and forward rates

Under a dynamic interest rate model, the gains from delaying beyond age 62 decrease substantially compared to the base case. Females maximize their NPV at age 64 and gain only 0.3 percent in NPV by doing so. Moreover, if a female delays gains past age 64, she begins to
suffer losses on her NPV and would have been better off receiving benefits from age 62. The losses suffered from delaying claims to benefit are even more apparent for males. Men maximize their NPV at age 62 and thereby, do not gain from delay; moreover, the losses suffered by men due to delay increase with each year beyond age 62 until age 70. The aforementioned gains and losses for both, men and women can be observed in Figure 6.

B. Using historical yield curves and forward rates

To extend this analysis and examine my results under different interest rate environments, I use representative yields curve from each of the past three decades. Thus, I select one each from 2004, 1994 and 1984. Each curve is selected for the first day in the month of January.

In 1984 the United States experienced a high real interest rate environment and consequently we observe a relatively high forward curve, with rates ranging between 4 percent and 9 percent depending on maturity, as seen in Figure 7. During this period, both males and females maximize their NPV at age 62 and suffer significant losses in NPV by delaying claims to their Social Security benefits. Females, on average, suffer a 2.9 percent loss in NPV for every year that they delay by, and can experience up to a 23.1 percent loss in total NPV by delaying claims until age 70 (Figure 8). Concurrently, men can face a 3.2 percent loss in NPV for each year that they delay claims by, and a sizeable 25.7 percent loss in NPV for claiming as late as age 70.

In January 1994, we observe a steady interest rate environment with an upward sloping real interest rate curve. Forward rates remained close to 1 percent for short term maturities and 4.5 percent for long term securities (Figure 9). As observed in 1984, both men and women
maximize the NPV received from Social Security benefits at age 62 during this period (Figure 10). However, the losses incurred from delaying are lower than those incurred by individuals in the high interest rate environment of 1984; women face an average loss of 1.1 percent in NPV for every year that they delay beyond age 62, with a maximum loss of 9.2 percent when claiming as late as age 70. Men face an average loss of 1.6 percent for every year that they delay beyond age 62, with a maximum loss of 13.1 percent when claiming at age 70.

In 2004, the United States experienced a low real interest rate environment. Forward rates remained close to 0 percent for short term maturities and 3 percent for long term securities (Figure 11). In this interest rate environment, both men and women can increase their NPV’s by delaying their claims beyond age 62. Women maximize their claims at age 64 and can gain an additional 1.0 percent in NPV by doing so. Concurrently, men maximize their claims at age 64 and can gain an additional 0.1 percent in NPV by doing so (Figure 12). However, delaying claims beyond the optimal age adversely affects both demographics, as observed in Figure 8.

Therefore we observe that our results are consistent across different time periods. During periods of high interest rates, such as in 1984 and 1994, we observe that it is optimal to claim at age 62. Additionally, during low interest rate periods, such as in the current interest rate environment or in 2004, there can be benefits that arise from delaying claims to one’s Social Security benefits. However, even in such low rate environments it is beneficial to delay claims by one or two years and the gain in NPV from delaying is marginal. Moreover, if an individual delays claims beyond this optimal age, he would begin to lose more in NPV than if he had claimed his benefits at age 62. These results display that it is often advantageous to claim at an earlier age and these results are summarized in Figure 13. In contrasting the two approaches, we observe that it is beneficial to delay claims under a static model, which does not adjust for future
interest rate expectations. However under a dynamic model that adjusts for future interest rate expectations, it is beneficial to claim Social Security at an early age. Moreover, it is important to note that accounting for future interest rate expectations has more of an impact on the results than the actual interest rate term structure; regardless of the time period selected, it is optimal to claim at an earlier age than that suggested by the static interest rate model.

**III. Dynamic Interest Rate Model under a Stochastic Approach**

**A. Using today’s yield curve and forward rates**

As opposed to conducting the analysis using a yield curve based on a specific date in time, I introduce a stochastic approach based on Monte Carlo simulations to forecast 1,000 future yield curves from which I calculate the NPV maximizing age. The inputs for this stochastic model, $\mu_i$ and $\sigma_i$, are obtained from the monthly yield curves between the years 2010 and 2014. We observe that the mean claim age is 62.86 for men with a standard deviation of 0.39 years and 63.20 for women, with a standard deviation of 0.62 years. These results are similar to the results obtained from a non-stochastic model and indicate that it is advantageous for both men and women to claim their retirement benefits early.

**B. Using historical yield curves and forward rates**

I extend this analysis and examine my results under different interest rate environments. To do so, I obtain the $\mu_i$ and $\sigma_i$ statistics for the monthly yield curves in each of the past three decades: 1980 – 1989, 1990 – 1999, 2000 – 2009. For the period 1980 – 1989, we observe that the mean claim age is 63.45 for men with a standard deviation of 1.49 years and 63.95 for women with a standard deviation of 1.87 years. For the successive decade (1990 – 1999) the median claim age is 63.03 for men with a standard deviation of 0.52 years and 63.84 for women
with a standard deviation of 1.09 years. Lastly, for the decade 2000 – 2009, the mean claim for men is 64.03 with a standard deviation of 2.02 years and 64.56 for women with a standard deviation of 2.32 years.

We therefore observe that the NPV maximizing ages for both men and women across different time horizons, lies between age 62 and 65. These results are consistent with those measured using a non-stochastic model which thereby reaffirms that it is often advantageous to claim at an earlier age. These results are summarized in Figure 14. We observe that the findings under the stochastic approach are similar to that under the deterministic approach, which reasserts that accounting for future interest rate expectations has more of an impact on the results than the actual interest rate term structure.

**Critical Review of Analysis**

The findings presented in the paper display the important role that interest rates play in determining the optimal age for claiming Social Security benefits. In very low, stable interest rate environments, wherein rates remain constant at a rate close to zero percent, individuals can increase their NPV by 10 to 14 percent by delaying claims till age 67 or later. However, during periods of rising interest rates, in which longer maturity securities have higher rates than shorter term securities, it is beneficial for one to claim benefits as early as possible. Moreover, in certain cases in which it is beneficial to delay, these benefits are often transformed into losses incurred beyond the optimal age. Thus, this analysis exhibits the impact that interest rates have on the present value of benefits, and the results indicate that the optimal age for claiming benefits often lies between the ages of 62 and 64.
Although this analysis attempts to account for the uncertainty associated with interest rates, it must be noted that the results of this analysis could drastically vary with deviations away from the forecasted scenarios. The dynamic nature of interest rates makes them difficult to predict, and the uncertainty associated with interest rates will always remain a concern in an analysis that attempts to forecast rates to estimate the appropriate discount rate.

It is also important to note that this analysis is developed for single retirees and cannot be generalized for the entire population. Benefits accruing to other classifications, which extend, but are not limited to, disabled individuals, widowers and married couples, fall outside the scope of this study. Moreover, it is assumed that the single retirees do not have preexisting debt or other interest bearing liabilities outstanding. Such liabilities could alter the optimal behavior of these individuals. In one scenario, the ability of a woman to pay off liabilities that bear significant rates of interest earlier might be more valuable than delaying claims until age 64.

Another important consideration is the assessment of life expectancies of individuals. Although we are able to assess the life expectancy of the average male and average female, it is very difficult to apply these life expectancies on an individual level; although males have an average life expectancy of approximately 82 years, an individual male with deteriorating health conditions might only have 5 years to live after the age of 62. In an alternative scenario, an extremely healthy male could live for more than 30 years after age 62. Thus, this paper does not account for the uncertainty in life expectancy and while the results are consistent on the aggregate level, they are subject to sharper variations on the individual level.

Additionally, this analysis assumes that individuals stop working at the age of 62 and does not factor in any additional income received after they stop working. Working beyond the
age of 62 will alter the NPV of an individual’s lifetime earnings, which will thereby affect his or her optimal claim age. Finally, this analysis assumes that all individuals seek to maximize their NPV. However, individuals may seek to maximize their overall welfare instead, which would incorporate factors that extend simply beyond monetary gains.

It is essential to bear the aforementioned factors in mind when developing this analysis in the future. Two aspects that I would focus on would be targeted at alleviating some of the aforementioned factors. First, I would develop sub-categorizations for the life expectancies for both, males and females. These sub-categorizations, which can be based criteria such as health status and race would enable individuals to interpret the results more accurately. Second, I would extend this analysis beyond single retirees and focus on developing similar results for another classification of the population, such as married workers.

Conclusion

This paper uses a Net Present Value approach to estimate the NPV maximizing age for retirees who will claim Social Security benefits. The results of this paper demonstrate that it is advantageous to claim benefits early under both a deterministic and stochastic rate model. The research presented in this paper, and its future potential can be immensely beneficial in improving the Social Security program. The government can restructure the system to make it more responsive to fluctuations in the interest rate so as to reduce the disparity between early and deferred collection of benefits. Second, the government can use this analysis to assist in educating the population about the rationale behind the current discrepancies that exist among different claiming ages. Finally, when forecasting the payments required for future retirees, the government can enhance their analysis by factoring in differing life expectancies based on
factors extending, but not limited to, race and health status so as to ensure that it is using a more accurate approach in estimating its future obligations.
Bibliography


Figure 1.

Fluctuating Historical Interest Rates in the United States, 1959 – 2008

Note: The nominal interest rate is the rate on five year US government bonds. The inflation rate was estimated by calculating the annual growth in the Consumer Price Index. The real interest rate is the difference between the nominal interest rate and the inflation rate.

Figure 2.

Yield Curve for Real Interest Rates as of November 1, 2013

Source: Federal Reserve

Figure 3.

Yield Curve for Forward Rates derived from yield curve of November 1, 2013

Note: Forward rates can be calculated using the subsequent formula where “f” represents the forward rate, “k” represents the current interest rate and “t” represents the timeframe in reference. Refer to Equation (1).
Figure 4.
Gains (Losses) from Delay under the Base Case using a constant interest rate of 0.1 percent

Note: Maximum Gains (Men): 9.8% | Maximum Gains (Women): 13.9%

Figure 5.
Gains (Losses) from Delay under the Base Case using a constant interest rate of 3 percent

Note: Maximum Gains (Men): 1.5 | Maximum Gains (Women): 3.3%
Figure 6.

Gains (Losses) from Delay using a Dynamic Interest Rate model under a Deterministic Approach | Today’s Rates

Note: Maximum Gains from Delay (Men): 0.0% | Maximum Gains (Women): 0.25%
Figure 7.
Yield Curve for Forward Rates derived from yield curve of January 1, 1984

Figure 8.
Gains (Losses) from Delay using a Dynamic Interest Rate model under a Deterministic Approach (1984)

Note: Maximum Gains from Delay (Men): 0.0% | Maximum Gains (Women): 0.0%
Figure 9.
Yield Curve for Forward Rates derived from yield curve of January 1, 1994

Figure 8.
Gains (Losses) from Delay using a Dynamic Interest Rate model under a Deterministic Approach (1994)

Note: Maximum Gains from Delay (Men): 0.0% | Maximum Gains (Women): 0.0%
Figure 11.
Yield Curve for Forward Rates derived from yield curve of January 1, 2004

![Yield Curve for Forward Rates](image)

Figure 12.
Gains (Losses) from Delay using a Dynamic Interest Rate model under a Deterministic Approach (2004)

![Gains (Losses) from Delay](image)

Note: Maximum Gains from Delay (Men): 0.1% | Maximum Gains (Women): 1.0%
Figure 13.
NPV Maximizing Ages for Single Men and Women under both, Constant and Dynamic Interest Rate Models using a Deterministic Approach

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>Gender</th>
<th>Males (years)</th>
<th>Females (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant Rates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1%</td>
<td></td>
<td>68</td>
<td>69</td>
</tr>
<tr>
<td>3.0%</td>
<td></td>
<td>65</td>
<td>67</td>
</tr>
<tr>
<td><strong>Dynamic Rates</strong></td>
<td><strong>Deterministic Approach</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Today’s Rates</td>
<td></td>
<td>62</td>
<td>64</td>
</tr>
<tr>
<td>1984</td>
<td></td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>1994</td>
<td></td>
<td>62</td>
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</tr>
<tr>
<td>2004</td>
<td></td>
<td>64</td>
<td>64</td>
</tr>
</tbody>
</table>
Figure 14.
NPV Maximizing Ages for Single Men and Women under Dynamic Interest Rate Models using a Stochastic Approach

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Age Statistic</th>
<th>Males (years)</th>
<th>Females (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Today</strong></td>
<td>Mean</td>
<td>62.86</td>
<td>63.20</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.39</td>
<td>0.62</td>
</tr>
<tr>
<td><strong>1980 – 1989</strong></td>
<td>Mean</td>
<td>63.45</td>
<td>63.95</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>1.49</td>
<td>1.87</td>
</tr>
<tr>
<td><strong>1990 – 1999</strong></td>
<td>Mean</td>
<td>63.03</td>
<td>63.84</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.52</td>
<td>1.09</td>
</tr>
<tr>
<td><strong>2000 – 2009</strong></td>
<td>Mean</td>
<td>64.03</td>
<td>2.02</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>64.56</td>
<td>2.32</td>
</tr>
</tbody>
</table>