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Abstract

To the extent of my knowledge, this paper is the first attempt to subject relative income under political economy analysis. The paper will introduce a utility function that includes relative income which will drive our models. According to the model, income distribution will coincide with wage distribution under specific assumptions. An intervention in income distribution may help agents to consume leisure but may effect the performance of the economy. When voters are facing the voting choice between equal and unequal policy, there are several parameters that may affect the voting outcome. For every unequal policy, there is an equal policy that is preferred by majority of voters and welfare improving and vice versa for every equal policy. However, not all policy that pass the vote is welfare improving and not all welfare improving policy will pass the vote. Then, we analyze the effect of income tax. Tax decreases the gap between the riches and the poorest, resulting in higher relative standing for almost all agents. The tax scheme we analyze will make all voters who receive redistribution and some net tax payers better off. Income tax that keeps economic efficiency and decreases income gap is welfare-improving.

1 Introduction and Motivation

Suppose you can choose one from two different communities to live. In community A, you will earn annual income of 35,000 USD while everyone else earns 40,000 USD. If you choose community B, you will earn annual income of 30,000 USD while everyone else earns 25,000 USD. Everything else in the two communities is equal. Would you choose to live in community A or community B? According to classical economic assumption the more the better, rational economic agents will undoubtedly choose community A as he will earn 5,000 USD more than community B. However, it is also obvious why someone would rather live in community B. where you have less absolute income than A but higher relative income compare to other people. The hypothetical question has appeared in one or another version in many surveys. The empirical answers to the question are mixed. Some people only care about absolute income and some
only care about relative income while for most of us it depends on how much we have to forgo to increase our relative standing. From empirical evidence, we do not only care about absolute income but also how much we have relative to others.

Relative standing plays an important role in economic decision in every level from private consumption to international stage. In the fairy tale, Snowwhite’s stepmother cares so much about being the fairest that she attempts to murder Snowwhite. Empirically, one’s own attractiveness is found to be highly positional (Solnick and Hemenway 1997). In a broader context, houses are also sensitive to relative standing. At Stanford, any student who lives in 1,000 square feet university housing can consider his room spacious, comparing to other rooms available. However, in neighboring Palo Alo, 1,000 square feet apartment is considered small, comparing to houses where most successful professionals live. In an international level, peace is relative. Skirmishes along borderline with a neighbor country is enough to prompt major troops mobilization in some countries. However, in some parts of the world, such conflict is considered nothing of great concern. These are just a few examples how relative standing can affect economic decision.

An immediate question following the importance of relative standing is why we concern about relative standing. For Stanford students whose grades depend on how well we perform relative to other students in the same class, this question is already answered. There are always payoffs for outdoing others. In
any competition, the winner is the one who can outdo other competitors. Perfectly competitive market ensures only the firms with lowest cost structures will survive until another firm with lower cost structure enters the market. In this case, the firms’ survival is the payoff for being able to produce cheaper products than its competitors. Another obvious example is job market. Only the best applicant in an application pool will get the position. We are trying to outdo others simply because we care about the payoffs that will go to the winner.

Apart from relative standing in performance, the rationale behind relative consumption is more puzzling. How can my consumption affect your consumption? Classical economic assumption states that we only care about how much we consume. One explanation is that relative consumption is a signal of relative performance. Those who perform better will earn more resource and thus are able to consume more. The practice of dressing up in nice business attires when meeting with potential clients is a good example of how we use consumption to signal our relative ability. A more successful businessman will be able to afford more expensive business attire. Another reason behind our concern about relative consumption is in our biology. In his 2007 book, Frank attributes the concern as a result of evolution process. The chance of our genes’ survival depends on how relatively many our offspring we have compared to our competitors which, in turns, depends on how much resource we are able to consume compared to others. In a sense, relative consumption signals our abilities which are signs of desirable genetic traits and allows our offspring to outbreed our
Pursuing higher relative standing can incur inefficiency. Hopkins and Kornienko showed that the position of an agent in positional consumption coincides with her position in income distribution (2004). Thus, the effort spent to improve one’s relative standing only lands one in the same place. The dominant strategy is to spend as much resource and effort as possible to improve one’s relative standing. If others do not spend as much resource on improving their relative standing, an agent can advance in standing. If others are actively engaging in improving their status, the agent has to engage in such spending lest he will fall behind. The game resembles Prisoners’ Dilemma. As every agent plays their dominant strategy, they became worse off collectively. The resource wasted on relative standing race can be consumed in other meaningful ways.

For a more concrete scenario, imagine a group of students studying for an exam that will be graded on curve. If none of them studies, the result of the exam will be determined by their abilities alone. Suppose they can study to improve their performance and they have equal “marginal return of study”. If all of them spend 10 hours studying, the result will be the same as when none of them studies. Instead of studying for 10 hours, all of them might as well not study. The time they wasted studying can be consumed for leisure. However, anyone of them can gain an edge by studying. Eventually, they have less leisure time and their grades do not change. Such condition is the Prisoners’ Dilemma of relative standing.
Income is the one of many attributes where positional concern is prevalent. In other words, an agent’s level of conspicuous or positional consumption is determined by her income. While one’s own attractiveness and intelligence are highly positional and inborn, agents can improve those attributes. For example, by consuming cosmetics and plastic surgery, an agent can improve her attractiveness and consuming higher education can improve an agent’s intelligence; thus, such consumptions can be viewed as positional consumptions. Of course, innate attributes such as attractiveness and intelligence are endowed from birth. However, given the same level of endowment of such attributes, the one with more income can invest more into improving such attributes. In addition, income is a signal of economic ability. Economically successful individuals generally earn more than unsuccessful ones. In short, income does not only determine our levels of consumption but also signal our abilities. Thus, relative income represents both relative consumption and relative performance.

Income distribution is closely related to political institution. Many of government’s policies are direct or indirect income redistribution such as taxes, subsidies and development projects. It is not surprising for voters to vote for politicians whose policies will give the voters more net income. However, when voters are concerned about relative income as well as absolute income, coming up with a policy to win a vote is more complicated. For example, it is no longer sufficient to raise income for all voters. If voters are concerned more about status, they may feel indifferent towards the new policy that does not change
relative income. This paper will explore in greater details how relative income may affect voting choices when voters are concerned about relative income and analyze the effect from the voting outcome on the economy.

In this paper, we will analyze the effect of relative income concern on voting choice and the consequences on welfare. I am not aware of any previous literature that attempts to subject positional concern under political economy analysis. This paper is intended to be a further development on existing literatures on relative income and status concern by taking the application in a new direction. We will discuss some relevant literatures in the next section. Then, we will introduce the utility function that drives our analysis based on previous models used by past literature. We will discuss how the function is an improvement from previous model and how well it represents actual preferences. We will also prove that income distribution will coincide with wage distribution under specific assumptions. After that, we will use the function to analyze voting outcomes in two particular scenarios. First scenario is when agents are facing choices between equal and unequal income distribution policy. The other scenario is the incidence of income taxes. We will study what policy will win the vote and the consequences of such policy on welfare. In addition we will do comparative statics of the results when parameters change. Finally, we will conclude the findings and discuss possible applications of the model.
2 Literature Review

In this section, we will review some of the literatures that studies relevant subjects to our topics.

There are many empirical studies on the effect of relative standing. Richard Easterlin (1974, 1995) studied the relationship between real income growth and happiness. While the income growth has shown increasing trend, happiness has remained largely constant. Therefore, happiness must be determined by relative income rather than absolute. Solnick and Hemenway (1998) surveyed the degree of positional concern on various consumptions such as education, attractiveness, income, etc. They found some goods are more positional than the other. Examples of the areas where relative standings are prevalent are attractiveness and praise. Johansson-Stenman, Carlsson and Daruvala (2002) found that individuals are inequality averse and concerned with relative income. They conducted a survey asking hypothetical questions about the society that participants preferred their grandchildren to live in. They suggested that marginal social utility of income may be negative. Alpizar, Carlsson and Johansson-Stenman (2004) confirmed that people are concerned with relative income and consumptions while such degree of concern depends on the goods. In addition, they found that relative consumption is also important for vacation and insurance which are largely considered to be absolute, and absolute consumption is important for cars and houses which are thought to be positional.
Robert Frank argued in his book (2007) that positional concern led to positional consumption which created welfare loss. Resources that were spent in positional consumption could be consumed in non-positional goods which could give greater utility. He also explained consumption with regard to context and the rationale behind our concern about status. Clark, Frijters and Shields (2008) gave an explanation to inconsistency between Easterlin’s findings and strong correlation between individual’s level of income and happiness. They stated that the findings are consistent when utility function included relative income. They explored how income can be compared to other’s and one’s own income in the past and suggested that utility function may include absolute income, relative income, leisure and other socio-economic factors.

There are a number of literatures that studies the effect of status competition by using mathematical models. Giacomo Corneo and Olivier Jeanne has contributed to the area. Their papers in 1996 and 2001 show that when agents care about their relative wealth, equality will lead to higher economic growth. Since they used different model from Hopkins and Kornienko, they have different result. In this case, equality gives more incentive for agents to compete for status which is determined by wealth. Therefore, agents will invest more in order to gain more wealth and higher status. Corneo and Jeanne provide a different view on the effect of status on economic growth.

Hopkins and Kornienko co-authored many papers that model agents as status seeking and analyze the effect. Their model exerts that agents gain status
by consuming positional goods. The more positional goods an agent consumes comparing to other agents will give her higher status. However, these goods have no economic value nor give them any utility other than their utility from status. In their 2004 paper, they proved that the position of an agent in positional consumption coincides with her position in income distribution. When every agent consumes positional goods, the effect on their position is cancelled out by other agents’ consumption. The game of status turns out to be a form of Prisoner’s Dilemma. The agents’ best response whether others consume positional goods or not is to consume positional goods. As a result, agents have less to spend on normal consumption and thus become worse off. In the end, agents’ status does not change as a result of their positional consumption but they are all worse off since they have less money to spend on normal goods.

In 2006 paper, Hopkins and Kornienko showed that more social equality gives more incentives to consume positional consumption; thus, lead to lower economic growth. The later paper shows the effect of change in income distribution on individual’s utility. Creating more inequality can lead to higher economic growth since it decreases the status gained from positional consumption. Change in income distribution that will benefit every agent has to increase both the amount of income and income dispersion. In their papers, they recommend proper income tax and subsidy to enhance economic growth. Hopkins and Tatiana’s models assume that agents gain status by consuming positional or “conspicuous” consumption. Their utility model assumes that agents earn
utility from normal goods times their status which is determined by their percentile of their conspicuous consumption level. This is analogous to auction model where agents earn the utility from the goods multiplied by probability that they win the item. Their model only takes care of cardinal ranking while agents may care about ordinal ranking as well. Hopkins and Kornienko’s model will serve as a basis for our model.

3 Model

3.1 General Setting

Suppose that there is an economy with a number of agents who can directly vote to enforce a policy that affects income distribution. The voting rule is as follows: given the status quo of income distribution, a new pattern of distribution will be proposed. The proposal require at least $\alpha$ fraction of total votes to be implemented. We can think of $\alpha$ as the “critical mass” of the economy. Assuming that agents will vote to enforce a policy that maximize their expected utility, at least $\alpha$ of total voters have to prefer the proposal over the status quo to enforce the proposal. When $\alpha$ is lesser than $\frac{1}{2}$, there is no stable outcome. When lesser than half of voters prefer the proposal and vote to pass it, the rest will vote to overturn it and the voting will keep going back and forth. We will reach the stable outcome only when $\alpha$ is bounded by $\frac{1}{2}$ and 1.

Let an agent $i$’s utility function be

$$u_i = y_i + \rho \cdot \frac{y_i - y_{\min}}{y_{\max} - y_{\min}}$$

(1)
where

\( y_i \) is agent i’s income.

\( y_{\text{min}} \) is the minimum income of the economy.

\( y_{\text{max}} \) is the maximum income of the economy.

\( \rho \) is a non-negative constant.

Let \( g \) stands for \( y_{\text{max}} - y_{\text{min}} \) for convenience.

We will assume that agents have perfect information regarding \( y_i, y_{\text{min}}, \) and \( y_{\text{max}} \) under status quo and the proposed distribution. The term \( \rho \cdot \frac{y_i - y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} \) denotes our status or rank function which is determined by income. Hopkins and Kornienko proved that status by consuming conspicuous consumption coincide with the status in income distribution in the equilibrium. In Corneo and Jeanne’s model, status is determined by wealth. In reality, an agent’s wealth is also determined by his total income up to present. Therefore, it makes sense to set the status function to be solely dependent on income.

3.1.1 Analysis of General Setting

The status function measures how much an agent is richer than the poorest agent compared to how much the richest agent is richer than the poorest. When income is uniformly distributed, the status function follows Hopkins and Kornienko’s model that based agent’s status on the portion of population that has less income than his. The function is adapted from a status function \( \frac{y_i}{y_{\text{ref}}} \) where \( y_{\text{ref}} \) is a reference income. The reference income is replaced by the difference between the maximum income and the minimum income and \( y_i \) is replaced by
$y_i - y_{min}$. In essence, the agents are comparing how better off they are than the worst off to how better off the richest agent is than the worst off. This emphasizes on agents’ relative cardinal income. In reality, agents do not know how many people earn less than they do nor what percentile they are in the income distribution while the information on how much the richest and the poorest in an economy are earning is relatively better known. Thus, such cardinal ranking approach resembles how agents perceive their status better than ordinal ranking. We can use the conventional $0 = 0$ when all agents earn the same income. However, it is unlikely in reality that all agents will earn the same income and view themselves at lowest status. We will deal with this case when it arises in our analysis.

The constant $\rho$ captures two major components of utility from status. First, $\rho$ captures the weight an agent put in status compare to her absolute income. Realistically, such weight may differ from agent to agent. For simplicity, we assume that it is the same for every agents in this model. Second, it also helps to scale $\frac{y_i - y_{min}}{y_{max} - y_{min}}$ to the same magnitude as $y_i$ since $y_i$ is non-negative with no upper bound where $\frac{y_i - y_{min}}{y_{max} - y_{min}}$ is bounded between 0 and 1. The need for $\rho$ to scale status function to the same magnitude as income is problematic in itself. This may skew the agents’ true preference over status and income. The magnitude of income may also vary greatly from the lowest to the highest earners. For example, the lowest earner may earn 25,000 USD while the richest may earn 1 billion USD. However, we can think of such scaling as how agents
translate his status into income. For example, being half as better off as the richest agents compared to the poorest may give utility equals to having 50,000 USD more income. In our model, such two components are lumped together in form of $\rho$ and assumed to be equal for all agents for simplicity.

In reality, we expect $\rho$ to be a function of agent $i$’s income and the difference between the maximum and the minimum income. As agents are richer, they may care less about their relative income while poorer agents may concern more about their status. Since rich agents are abundant in absolute income which already place them in higher status, they may not be concerned about status as much as poorer agents. From another perspective, the rich has more resource to participate in positional arms race and may be more engaged in such battle.

The difference between the maximum and minimum income denoted by $g$ also influences how important status is for agents. For example, earning 100,000 USD when the richest earn 200,000 USD and the poorest earn 0 should not be equivalent to earning 100,000 USD when the richest earn 100,001 USD and the poorest earn 99,999 USD. Despite earning equal median income, it is obvious that relative well-being are different in the two hypothetical distribution. In our model, we omit such detail for simplicity. We assume that $\rho$ remains constant for the range of $g$ in our model and all agents have the same $\rho$.

The marginal utility of income in this model is constant and equal to $1 + \frac{\epsilon}{g}$. While conventional understanding of income states that marginal utility of income is diminishing, such model incorporating such marginal utility is too math-
ematically complicated to handle. Our model exhibits constant marginal utility while we will address how diminishing marginal utility may affect the results. The marginal utility of income is increasing in $\rho$ and decreasing in $g$. When agents weigh relative standing more than absolute income and income equivalent of absolute income increases, $\rho$ increases. As relative standing becomes more important to agents, they gain more marginal utility from income since each additional income advances them to higher relative standing. On the contrary, as $g$, how much the richest are better off than the poorest increases, each additional income does not advance the agents as much. Thus, the marginal utility decreases in $g$.

3.1.2 Proposal’s “Fairness”

We defined a fair proposal as a proposal that if agent i and agent j have the same amount of income before the proposal is implemented, then their income will be equal after the proposal is implemented. We shall focus on only fair proposal in our analysis. The fair proposal will ensure that agents with equal income will undergo the same treatment and that we will treat agents differently only if their incomes are different. Such criteria is important as it allows us to treat the proposal as a linear map of agents’ incomes from one income distribution to another. Notice that the income after the proposal is implemented is not necessarily equal to the income before the proposal is implemented and agents with equal income after the implementation are not necessarily have equal income before the implementation.
3.1.3 Economic Efficiency

Assuming that income is a function of wage and time spent working, $y_i = w_i \cdot t$ and utility function is increasing in income. Suppose wages are exogenously determined by factors omitted in our model such as education, experience, ability, race, etc. Thus, we will hold wage constant for each agent. Agents are homogenous and indifferent about leisure. Suppose further that wages are the only source of income. We will show that the income distribution will coincide with wage distribution in the ‘natural’ equilibrium where the government has no policy regarding wages or income distribution.

First, we will show that each agent will work as long as they can. Since $U_i$ is increasing in $y_i$ and $y_i = w_i \cdot t_i$, holding wage as an exogenous parameter, agents can improve their utility by increase the time they spend working. Naturally, we cannot spend infinite time working. Let $t_{\text{max}}$ denoted the maximum time agents can work as determined by legal or physical reason. Agent can choose any $t_i \in [0, t_{\text{max}}]$. As utility is increasing in income, all agents will choose $t_i = t_{\text{max}}$ to maximize his utility. Next, it follows that incomes are determined by wages alone. Since all agents choose $t_i = t_{\text{max}}$ to maximize utility, the difference in their income is a result of their wages. Holding $t_i = t_{\text{max}}$ for all $i$ and no other source of income, income function $y_i = w_i \cdot t_{\text{max}}$ is linear and increasing in wages. Thus, $w_i > w_j$ implies $y_i > y_j$. We conclude that income distribution coincides with wage distribution. In other words, $F_w(w_i) = F_y(y(w_i))$ where $F_w$ and $F_y$ are cummulative distribution function of wage and income distribution.
respectively. We will call this equilibrium, “efficient equilibrium” as the economy is running at its full potential since all agents are spending $t_{\text{max}}$ time working.

Suppose there are taxes. Let $y_i > y_j$ be incomes before tax and $y'_i$ and $y'_j$ be incomes after tax. $y'_i = w_i \cdot t_i - T_i$ and $y'_j = w_j \cdot t_j - T_j$ where $T_i$ and $T_j$ are taxes for agent $i$ and agent $j$ and $t_i = t_j = t_{\text{max}}$. Since agent $i$ and agent $j$’s incomes before tax are different, they pay different amount of tax. Notice that tax can be negative, which means that they are receiving aid or redistribution. Suppose $y'_i < y'_j$. This implies $U(y'_i) < U(y'_j)$. For agent $i$, it now does not make sense for him to choose $t_i = t_{\text{max}}$. He will earn more utility if he work fewer hours and earn the same income as agent $j$. Even though agent $i$ is indifferent towards leisure, working fewer hours makes him better off than working $t_{\text{max}}$. From a macro level, agent $i$’s fewer working hours leads to lower GDP, as a consequence, we arrive at an “inefficient” equilibrium. Note that we determine “efficiency” by economic output only and “inefficient” equilibrium may be welfare improving if agents are concerned about leisure but they are unable to consume leisure before tax because they care much more about status than leisure. Notice that agent $j$ still choose to work $t_{\text{max}}$ and are unable to consume leisure unless he can lower his income to fall into different tax bracket which may give him higher income after tax.

From our model, policies regarding wages, working time and income may enable agent to consume leisure. Since income distribution coincides with wage distribution, policy that affects wage distribution will affect income distribu-
tion. While it may not directly impact leisure consumption, increasing wage may enable some agents who are not concerned as much about status to work fewer hours and consume leisure. Capping maximum working time will enable agents to consume leisure by decreasing $t_{max}$. Income tax will also enable agents to consume more leisure as demonstrated above. However, keep in mind that leisure may cost economic efficiency. To maintain economic efficiency, the income distribution before the policy is implemented has to coincide with the income distribution after the implementation.

In next section, we will examine the voting choice between equal and unequal income distribution.

### 3.2 Equality or unequaility?

#### 3.2.1 Settings

Now we will consider when agents face policy choices between unequal income distribution and equal income distribution. We will work with the two extremes. The unequal policy will make agents' income uniformly distributed between the minimum income $y_{min}$ and the maximum income $y_{max}$. While the equal policy will make all agents’ income equals to $y_e$. We do not fix the total income of the population in this model. Normally, total income at a given time should be equal to the total GDP of the economy. Different income distribution policies implies change fundamental economic structure, yielding different level of GDP.
Next we will address how we will deal with status function when all agents earn the same amount of income. Under equality policy, every agents’ status is the same and determined by an exogenous constant $c$. This can be viewed as an effect of a “perception” of the well-being of the agents. Since every agents earns the same income, they can view themselves as being the first, the last or somewhere in between. In this model, $c$ will take value between 0 and 1, inclusive. We assume that $c$ is exogenous and equal for all agents.

### 3.2.2 Analysis

Under equality policy, every agents has the same income and rank. Thus,

$$u_i = u_e$$ \hspace{2cm} \text{for all agent } i \hspace{2cm} (2)$$

$$u_e = y_e + \rho \cdot c \hspace{2cm} (3)$$

This implies that everyone is earning the same utility under equal policy while in reality agents who prefer unequal policy may feel less happy under the policy that they do not prefer. This is omitted for simplicity. This model assumes that agents only care about their income with no concern about ideology.

Under unequal policy, agents earn different incomes. Their rank is determined by their position in income distribution.

$$u_i = y_i + \rho \cdot \left(\frac{y_i - y_{\text{min}}}{y}\right) \hspace{2cm} (4)$$

There exists an agent whose income $y^*$ under unequal policy which makes him indifferent between equal and unequal policy. Note that

$$y^* \text{ such that } u_e = u_i(y^*) \hspace{2cm} (5)$$
\[ y_e + \rho \cdot c = y^* + \rho \cdot \left( \frac{y^*-y_{min}}{g} \right) \] 
\[ y^* = \left( \frac{1}{1+\rho} \right) \left( y_e + \rho \cdot c + \frac{\rho}{g} y_{min} \right) \]

Suppose agents start in equal policy but have information about their income if unequal policy is implemented. We want to change parameters to make at least \( \alpha \) fraction of voters prefer the unequal policy. \( u_i \) is increasing in \( y_i \). Therefore, in the proposed unequal policy, any \( y_i \) such that \( y_i > y^* \) implies \( u_i > u_e \). Since \( y^* \) is the income such that agents are indifferent between the two policy, any income greater than \( y^* \) will give agents more utility than \( u_e \). We need \( \alpha \) portion of voters to prefer unequal policy to equal policy. Therefore, we need to fulfill the condition:

\[ \frac{y_{max} - y^*}{g} \geq \alpha \]

On left hand side, \( \frac{y_{max} - y^*}{g} \) denotes the proportion of agents whose expected income under proposed unequal policy is greater than \( y^* \) compare to total voters. The portion has to be greater than \( \alpha \) to enforce unequal policy.

\[ y_{max} - y^* \geq \alpha g \]  

Substitute \( y^* \) from above and solve for an inequality.

\[ y_{max} - \alpha g + \rho \cdot (1 - \alpha - c) \geq y_e \]

There exists \( y_c^* \), an income level under equal policy, such that

\[ y_{max} - \alpha g + \rho \cdot (1 - \alpha - c) = y_c^* \]

Alternatively, the equation can be rewritten as

\[ (1 - \alpha) y_{max} + \alpha y_{min} + \rho \cdot (1 - \alpha - c) = y_c^* \]

The variables on the left hand side are from the proposed unequal policy. We
can use this equation to determine whether the proposed unequal policy will win the vote against the status quo equal policy. $y_e^*$ is the maximum $y_e$ such that at least $\alpha$ of all voters will prefer unequal policy than the equal policy. Any $y_e > y_e^*$ will obstruct some specific unequal policies. In other words, if the status quo is equal policy, such $y_e^*$ is the minimum income requirement under equal policy in order to remain in effect.

### 3.2.3 Comparative Statics

Now we will analyze the effect on $y_e^*$ when other parameters change.

Increase in $\alpha$ results in decrease in $y_e^*$. This implies it is easier to maintain equal policy. As higher proportion of population is required to enforce a policy, it is more difficult for unequal policy to pass since more people have to prefer new policy than current policy. To pass unequal policy, we need more voters to earn more income or better status in unequal policy. As a result, minimum income requirement for equal policy drops.

Increase in $c$ also decreases $y_e^*$. When agents perceive themselves in higher status under equal policy, they gain more utility from status and more agents will be happier to be under equal policy than under unequal policy. Therefore, it is more difficult to enforce unequal policy and $y_e^*$ decreases.
The effect of $\rho$ depends on the term $1 - \alpha - c$.

$$\rho \uparrow \begin{cases} 
    y^*_e \downarrow & \text{if } c > 1 - \alpha \\
    y^*_e \uparrow & \text{if } c < 1 - \alpha \\
    \text{no effect on } y^*_e & \text{if } c = 1 - \alpha
\end{cases}$$

When perceived status under equal policy is higher than a threshold, as agents care more about their status, more agents will prefer equal over unequal policy. This is because when perceived status is higher, more agents will immediately earn higher status under equal policy. When the perceived status is lower, more agents will immediately earn lower status under equal policy and thus have more incentive to vote for unequal policy. To further this analysis, $\alpha$ is bounded between $\frac{1}{2}$ and 1. $1 - \alpha$ takes value in the range $[0, \frac{1}{2}]$. Therefore, for any $c > \frac{1}{2}$, increase in $\rho$ will definitely decrease $y^*_e$. When $c \geq \frac{1}{2}$, at least half of all agents immediately perceive themselves as having higher status.

Now we will further analyze the effect of the proposed income distribution under unequal policy on $y^*_e$. First we will analyze the effect of income gap $g$ while we fix $\bar{y}$, average income under unequal policy.

Let

$$y'_{max} = y_{max} + \frac{\epsilon}{2} \quad (13)$$

$$y'_{min} = y_{min} - \frac{\epsilon}{2} \quad (14)$$

$$g' = g + \epsilon \quad (15)$$

We substitute these parameters in previous equation.

$$y'_{max} - \alpha \cdot g' + \rho \cdot (1 - \alpha - c) = y^*_{e,\epsilon} \quad (16)$$

$$y_{max} - \alpha \cdot g + \rho \cdot (1 - \alpha - c) - \epsilon \cdot (\alpha - \frac{1}{2}) = y^*_{e,\epsilon} \quad (17)$$
\[ y_c^* - \epsilon \cdot (\alpha - \frac{1}{2}) = y_{c,e} \] (18)

As the gap increases, \( \epsilon \) is greater and \( y_{c,e}^* \) decreases. When the income distribution becomes more dispersed and an agent’s income does not change, he immediately earn lower status and thus lower utility. Thus, \( y_{c,e}^* \) is less than \( y_c^* \).

Next we will analyze change in \( \overline{y} \) while we fix \( g \).

Let
\[ y_i' = y_i + \epsilon \text{ for all agent } i \] (19)
\[ \overline{y}' = \overline{y} + \epsilon \] (20)
\[ y_{max}' = \alpha \cdot g + \rho \cdot (1 - \alpha - c) = y_{c,e}^* \] (21)
\[ y_c^* + \epsilon = y_{c,e}^* \] (22)

When the whole income distribution shifts up by \( \epsilon \), all agents immediately earn higher income while getting the same status. Therefore, more agents will prefer such capitalist policy more than communist policy. Thus, \( y_{c,e}^* \) has to increase by at least \( \epsilon \) to keep up with such shift.

From alternative formula of \( y_{c,e}^* \):
\[ y_{max} - \alpha g + \rho \cdot (1 - \alpha - c) = y_{c,e}^* \] We can further analyze the effect of increase in \( y_{max} \) and \( y_{min} \). Increase in either \( y_{max} \) and \( y_{min} \) implies that the minimum income requirement for equal policy, \( y_{c,e}^* \), has to increase to keep up. However, \( y_{min} \) has greater effect on \( y_{c,e}^* \). This is because when \( y_{max} \) increases, the mean income \( \overline{y} \) increases as well as \( g \), the gap between the richest and the poorest. Increase in \( \overline{y} \) leads to higher \( y_{c,e}^* \) while increase in \( g \) leads to lower \( y_{c,e}^* \). As a result, they cancel the effect of each other. The effect of \( \overline{y} \) is stronger, resulting in greater \( y_{c,e}^* \). However, when \( y_{min} \) increases, both increase
and decrease g makes \( y^*_e \) increase. As the lowest income increases, it is more difficult to maintain the equal policy.

### 3.2.4 Welfare Analysis

Now we will analyze total welfare under each policy. Total welfare can be captured by the average welfare. Since utility function is linear, the average utility is the utility of the average income. This will make the calculation easier.

\[
\begin{align*}
\overline{U}_e &= u_e = y_e + \rho \cdot c \\
\overline{U}_i &= u(y_i) = y_i + \rho \cdot \frac{y_i - y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} \\
\overline{U}_i &= \frac{y_{\text{max}} + y_{\text{min}}}{2} + \frac{\rho}{2}
\end{align*}
\]

For a given unequal policy with certain \( y_{\text{min}}, y_{\text{max}} \) and g, there exist a particular \( y^*_w \) such that the total welfare under equal policy is equal to the total welfare under unequal policy.

\[
\begin{align*}
U_e(y^*_w) &= U_i \\
u_e(y^*_w) &= \overline{U}_i \\
y^*_w + \rho \cdot c &= \frac{y_{\text{max}} + y_{\text{min}}}{2} + \frac{\rho}{2} \\
y^*_w &= \frac{y_{\text{max}} + y_{\text{min}}}{2} + \rho \cdot (\frac{1}{2} - c) \\
or equivalently,
\end{align*}
\]

\[
y^*_w = y_{\text{max}} - \frac{1}{2} g + \rho \cdot (\frac{1}{2} - c)
\]

From the equation, \( y_e > y^*_w \) implies \( U_e > U_i \). Also, \( y^*_w \) decreases in c. When c increases, voters gain more utility from status and require less absolute income to maintain the same level of utility.

Compare to \( y^*_e \)
\[ y_e^* = y_{max} - \alpha \cdot g + \rho \cdot (1 - \alpha - c) \]
\[ y_w^* = y_e^* \quad \text{if} \quad \alpha = \frac{1}{2} \]
\[ y_w^* > y_e^* \quad \text{if} \quad \alpha > \frac{1}{2} \]
\[ y_w^* < y_e^* \quad \text{if} \quad \alpha < \frac{1}{2} \]

Since \( \frac{1}{2} \geq \alpha \geq 1 \), \( y_w^* \) that is lesser than \( y_e^* \) does not exist.

We arrive at a profound result. Suppose the status quo is equal policy. The fact that an unequal policy does not pass the vote does not imply that it is not welfare improving. Consider

\[ y_w^* - y_e^* = (\alpha - \frac{1}{2}) \cdot (g + \rho) \]  
(31)

The difference is always positive. It is possible for \( y_e \) of incumbent equal policy to fall between \( y_e^* \) and \( y_w^* \). The difference increases with all parameters: \( \alpha, g, \rho \). Increase in \( \alpha \) and \( g \) decreases \( y_e^* \) but does not affect \( y_w^* \). Therefore, the difference increases. Increase in \( \rho \) means that voters concern more about status. This amplifies the effect of status. To compensate for voters who gain more utility from status under unequal policy than under equal policy, the absolute income needs to increase.

In this section, we analyze the voting choice between equal and unequal policy. With status quo implementing equal policy, We derive \( y_e^* \), the minimum income requirement for equal policy to remain in effect. The effect of \( \rho \) on \( y_e^* \) depends on \( c \), the status under equal policy and \( \alpha \), the fraction of total vote required to pass the policy. Increase in \( g \) results in decrease \( y_e^* \) while increase in \( \overline{y} \) leads to increase in \( y_e^* \). However, the effect of \( \overline{y} \) is greater than the effect of
We also derive $y^*_w$, the income level under equal policy that will generate as much welfare as a specific unequal policy. We find that $y^*_e < y^*_w$ and maintaining equal policy may not lead to higher welfare.

Consider a similar scenario. Suppose the status quo is now implementing unequal policy and we want to implement equal policy. $y^*_e$ now represents minimum income requirement for equal policy to pass the vote. We derive a formula for $y^*_e$

$$y^*_e = y_{\text{max}} - (1 - \alpha)g + \rho \cdot (\alpha - c)$$ (32)

Alternatively,

$$y^*_e = \alpha y_{\text{max}} + (1 - \alpha)y_{\text{min}} + \rho \cdot (\alpha - c)$$ (33)

The result is similar to the case that equal policy is the status quo. However, we get $y^*_e > y^*_w$. In other words, every equal policy that pass the vote when specific unequal policy is the status quo is welfare-improving. One explanation is that in this case, the equal policy has to make at least $\alpha$ fraction of total voters prefer the equal policy while in case the equal policy is status quo, it is sufficient to make $1 - \alpha$ of total voters better off.

### 3.3 Income Tax

#### 3.3.1 Settings

In this section, we assume that income before tax is uniformly distributed. Each agent earns wage according to a wage distribution by exogenous factors. Due to their concern about relative standing, all agents work as much time as
they can to maximize their utility by earning as much income as possible. As a result, their income distribution coincides with their wage distribution. We keep the same fairness criteria in redistribution. Namely, the agents who earn equal income before the proposal will earn equal income after the proposal. The general setting of the two sections is the same.

In reality, there are many ways to redistribute income. Agents are considering whether to implement an income tax proposal. The proposal is as follows. To make $\alpha$ portion of agents better off, the proposal will give extra income to them according to how much income they earn. Since we are focusing on income redistribution. We assume that there is no significant change in the economic system. We will introduce constraint such that agents’ time inputs and GDP will remain the same. Because $\alpha$ portion of voters will recieve redistribution, the proposal need to finance such spending. Needing only $\alpha$ to pass the vote, the proposal will make the rest $1 - \alpha$ portion of the agents pay “tax”. The tax revenue must be equal to the redistribution spending. In reality, the proposal can incur debt or surplus. However, we will only allow balanced budget. There are many ways to pick such group of $\alpha$ portion of agents. We will focus on the case when such group is the lowest $\alpha$ portion of income distribution.

There are a few miscellaneous details that need to be discussed. In this section, for a given variable $x$, $x'$ denotes such variable after the redistribution. For example, $y_i$ is agent $i$'s income before the redistribution and $y'_i$ is agent $i$’s income after redistribution. In addition, since our utility function is linear, the
average utility is the utility of an agent with average income. This is true for all distribution. We will focus on the average income and average utility as a representative of a group of agents.

3.3.2 Analysis

We will begin with uniform distribution of income and we want to redistribute income to the lowest $\alpha$ of the distribution by taxing the upper $1 - \alpha$ portion. We will show below that such scheme will pass the vote since every agent in $\alpha$ portion has strictly greater utility after the redistribution. We are looking for agents whose $y^*$ such that $F(y^*) = \alpha$. In other words, they are at the borderline between receiving and paying tax. Since income is uniformly distributed, we get

\begin{equation}
\frac{y^*-y_{min}}{y_{min}} = \alpha
\end{equation}

\begin{equation}
y^* = y_{min} + \alpha g.
\end{equation}

For agent $i$ whose income is $y_i$ such that $y_{min} \leq y_i \leq y^*$, they will receive redistribution or negative tax equals to $\frac{y^*-y_i}{y^*-y_{min}} \cdot D$. Their income after “tax” $y'_i$ is equal to $y_i + \frac{y^*-y_i}{y^*-y_{min}} \cdot D$. While for agent $j$ whose income is $y_j$ such that $y^* < y_j \leq y_{max}$, they will pay tax equals to $\frac{\alpha D}{1-\alpha} \cdot \frac{y_j-y'}{y_{max}-y}$. Their income after tax is $y'_j = y_j - \frac{\alpha D}{1-\alpha} \cdot \frac{y_j-y'}{y_{max}-y}$. We can think of this as two different tax brackets. When taking public goods consumption into account, the lower bracket is the net recipient of redistribution while the higher bracket is the net payers who finance the public goods provision.
There are some restrictions on D to prevent economic inefficiency. Consider the after tax income functions for both brackets.

\[ y'_i = y_i + \frac{y^* - y_i}{y^* - y_{\text{min}}} \cdot D \quad \text{for} \quad y_{\text{min}} \leq y_i \leq y^* \]  
(36)

\[ y'_j = y_j - \frac{\alpha D}{1-\alpha} \cdot \frac{y_i - y^*}{y_{\text{max}} - y^*} \quad \text{for} \quad y^* < y_j \leq y_{\text{max}} \]  
(37)

The functions can be rewritten as

\[ y'_i = (1 - \frac{D}{\alpha g})y_i + \frac{y^* D}{\alpha g} \]  
(38)

\[ y'_j = (1 - \frac{\alpha D}{(1-\alpha)^2 g})y_i + \frac{\alpha D}{(1-\alpha)^2 g}y^* \]  
(39)

In our model, we want both \( y'_i \) and \( y'_j \) to increase in \( y_i \). Notice that the redistribution is perfectly balanced. We want to keep the economy efficient to avoid agents taking time off work to fall into lower bracket and incurring public debt since our model is one period. Therefore, we arrive at two constraints for D

\[ 1 - \frac{D}{\alpha g} > 0 \]  
(40)

\[ 1 - \frac{\alpha D}{(1-\alpha)^2 g} > 0 \]  
(41)

Thus,

\[ D < \alpha g \]  
(42)

\[ D < \frac{(1-\alpha)^2}{\alpha} g \]  
(43)

The latter constraint is more restrictive because \( \alpha > \frac{(1-\alpha)^2}{\alpha} \) for all \( \alpha \in \left[ \frac{1}{2}, 1 \right] \).

Hence, we will keep it as the only constraint

We will begin by calculating utility before and after tax of agent i whose \( y_i \) is in the net recipient bracket when tax is implemented. In other words, \( y_{\text{min}} \leq y_i \leq y^* \)

\[ u_i = y_i + \rho \cdot \frac{y_i - y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} \]  
(44)
\[ u'_i = y'_i + \rho \cdot \frac{y'_i - y'_{\text{min}}}{y'_{\text{max}} - y'_{\text{min}}} \]  
(45)

\[ u'_i = y_i + \frac{y'_{\text{max}} - y_i}{y'_{\text{max}} - y_{\text{min}}} \cdot D + \rho \cdot \frac{y_i + \frac{y'_{\text{max}} - y_i}{y'_{\text{max}} - y_{\text{min}}} \cdot D - y_{\text{min}} - D}{y_{\text{max}} - y_{\text{min}} - \frac{D}{1-\alpha}} \]  
(46)

\[ \Delta u_i = u'_i - u_i = \frac{y'_{\text{max}} - y_i}{\alpha g} \cdot D + \rho \cdot \frac{D}{g - \frac{D}{1-\alpha}} \cdot \frac{y_i - y_{\text{min}}}{g} \cdot \frac{1}{\alpha(1-\alpha)} \]  
(47)

From such formula, we conclude that all agent \( i \) whose incomes fall in the lower bracket will prefer the tax scheme because \( \Delta u_i \) is positive for all \( y_i \) in the bracket. Besides earning more income, the gap between \( y_{\text{max}} \) and \( y_{\text{min}} \) decreases so they also gain more utility from status. The term \( g - \frac{D}{1-\alpha} \) is positive from our constraint on \( D \).

Next, we repeat the process for agent \( j \) whose income \( y_j \) is in the net payer bracket or \( y^* < y_j \leq y_{\text{max}} \).

\[ u_j = y_j + \rho \cdot \frac{y_j - y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} \]  
(48)

\[ u'_j = y_j - \frac{\alpha D}{1-\alpha} \cdot \frac{y_i - y^*}{y_{\text{max}} - y^*} + \rho \cdot \frac{y_j - \frac{\alpha D}{1-\alpha} \cdot \frac{y_i - y^*}{y_{\text{max}} - y^*} - y_{\text{min}} - D}{y_{\text{max}} - y_{\text{min}} - \frac{D}{1-\alpha}} \]  
(49)

\[ \Delta u_j = -\frac{\alpha D}{1-\alpha} \cdot \frac{y_i - y^*}{y_{\text{max}} - y^*} + \rho \cdot \frac{D}{g - \frac{D}{1-\alpha}} \cdot \frac{y_{\text{max}} - y_i}{g} \cdot \frac{2\alpha - 1}{(1-\alpha)^2} \]  
(50)

When \( D \) increases, it is obvious that \( \Delta u_i \) will increase and \( \Delta u_j \) will decrease because each voter in \( \alpha \) fraction will receive more redistribution and each voter in \( 1 - \alpha \) will pay more tax. Increase in \( \rho \) leads to increase in both \( \Delta u_i \) and \( \Delta u_j \).

This is because the gap between the richest and the poorest decreases after tax and every voter gains higher status and earns more utility.

We will now show that there are some agents in the net payer bracket who prefer the tax scheme. \( \Delta u_j \) is decreasing in \( y_j \) and continuous. \( \Delta u_j(y^*) \) is positive while \( \Delta u_j(y_{\text{max}}) \) is negative. Thus, there exist \( y_0 \) in this bracket such
that $\Delta u_j(y_0) = 0$. We further calculate for $y_0$.

\[ y_0 = y_{\max} - \frac{\alpha(1-\alpha)}{\alpha + \frac{\alpha}{\gamma - \frac{\alpha}{\gamma}}(2\alpha - 1)} \cdot g \]  
\[ (51) \]

\[ y_0 - y^* = (1 - \alpha)g \cdot \left[1 - \frac{\alpha}{\alpha + \frac{\alpha}{\gamma - \frac{\alpha}{\gamma}}(2\alpha - 1)}\right] \]  
\[ (52) \]

\[ \frac{y_0 - y^*}{g} = (1 - \alpha)[1 - \frac{\alpha}{\alpha + \frac{\alpha}{\gamma - \frac{\alpha}{\gamma}}(2\alpha - 1)}] \]  
\[ (53) \]

From above equations, there is a fraction of voters in the net payer bracket that prefer the tax scheme. $\frac{y_0 - y^*}{g}$ denotes such fraction. This may sound counterintuitive. The explanation is that even though these voters have lower income after tax, they gain more utility from the decreasing gap between them and the richer voters. The gain offsets the loss from absolute income and they earn more utility.

### 3.3.3 Welfare Analysis

Now we will analyze the change in overall welfare if tax is implemented. Since $\Delta u_i$ and $\Delta u_j$ are linear, the average of change in welfare in each tax bracket is equal to the change in utility of the agent with average income. Let $\overline{\Delta u_\alpha}$ and $\overline{\Delta u_{1-\alpha}}$ be the average difference in welfare in the net recipient bracket and the net payer bracket, respectively.

First, we will calculate the average income before tax of both brackets and substitue them in the formulae.

For net recipient bracket,

\[ \overline{y}_\alpha = \frac{y^* + y_{\min}}{2} \]  
\[ (54) \]

\[ \overline{y}_\alpha = y_{\min} + \frac{\alpha g}{2} \]  
\[ (55) \]

\[ \overline{\Delta u_\alpha} = \frac{D}{2} + \rho \cdot \frac{D}{g} \cdot \frac{\alpha}{\gamma - \frac{\alpha}{\gamma}} \cdot \frac{1}{2(1-\alpha)} \]  
\[ (56) \]
For net payer bracket,
\[ y_{1-\alpha} = \frac{y_{\text{max}} + y^*}{2} \]  
\[ y_{1-\alpha} = \bar{y} + \frac{\alpha g}{2} \]  
\[ \Delta u_{1-\alpha} = -\frac{\alpha D}{2(1-\alpha)} + \rho \cdot \frac{D}{g - \frac{D}{1-\alpha}} \cdot \frac{2\alpha - 1}{2(1-\alpha)} \cdot \frac{1}{2} \]  

Finally, we will calculate the difference in total utility as weighted average of the two brackets
\[ \Delta U = \alpha \Delta u_\alpha + (1 - \alpha) \Delta u_{1-\alpha} \]  
\[ \Delta U = \rho \cdot \frac{D}{g - \frac{D}{1-\alpha}} \cdot \frac{2\alpha^2 - 2\alpha + 1}{(1-\alpha)} \cdot \frac{1}{2} \]  

From the model, this redistribution is welfare-improving. \( \Delta U \) is always positive and increasing in \( \rho \) and \( D \). After taxation, the gap between the richest and the poorest decreases. In general, agents are in a better relative standing and increase in \( \rho \) will amplify the gain in status. Increase in \( D \) will give more absolute income to voters in \( \alpha \) fraction and decrease the gap, making all agents better off. Notice that \( D \) is bounded above and the upper bound is decreasing in \( \alpha \). This implies that when more voters are net recipients, it is more difficult to finance public goods and the overall quantity decreases. The tax burden on the net payer also increases in \( \alpha \). Total GDP remains constant but the average welfare improves. This shows that the welfare that the \( \alpha \) portion gained from receiving redistribution \( D \) outweigh the welfare that the \( 1 - \alpha \) portion loses from paying tax. This is also due to the fact that there are more agents in \( \alpha \) portion than the other portion.
4 Conclusion

To the extent of my knowledge, this paper is the first attempt to put relative income under political economy analysis. According to the model, income distribution will coincide with wage distribution under specific assumptions. Without any intervention, agents are unable to consume leisure. An intervention in income distribution may help agents to consume leisure but may effect the performance of the economy. Next, we examine the voting choice between equal and unequal policy. For every unequal policy, there is an equal policy that is preferred by majority of voters and welfare improving and vice versa for every equal policy. However, not all policy that pass the vote is welfare improving and not all welfare improving policy will pass the vote. Then, we analyze the effect of income tax. The tax scheme is under constraint to preserve economic efficiency. Tax decreases the gap between the riches and the poorest, resulting in higher relative standing for almost all agents. Voters who are net recipients are better off in the incident of tax while some voters who are net payers are better off in the incidence of tax because the utility gain from relative income is greater than the utility loss from absolute income. Income tax that keeps economic efficiency and decreases income gap is welfare-improving.

The model can be applied in many ways. The main idea of equal and unequal policy is that majority rule may not yield welfare-improving policy. Equal policy may represent socialist policy and unequal policy may represent capitalist policy. From the model, there are socialist policy that are prefered to capitalist policy...
and vice versa. The findings may help explain the transition between socialist economy to capitalist economy and vice versa. Notice that ideology does not affect voters preference in our model while it may affect people’s utility in reality. The transition will occur when the majority of voters are better off while it may leave some voters worse off.

The equal and unequal policy choice can help explain resource allocation during the Cold War. Communist countries that were dependent on the USSR were facing treats to be converted to capitalism while capitalist countries were facing communism threat. Both the USA and the USSR may send economic aid to these countries to try to maximize the number of countries on their sides. Such aid may be determined by the model. If $y_e$ in a communist country was too low, the USSR had to send in economic aid to aid $y_e$ above $y^*_e$. The capitalist counterpart was more complicated since both mean income and income dispersion had to be taken into account. If expected $y_e$ is high enough and mean income is low, a capitalist country might convert to communism. The Marshall plan after World War II to aid European countries can be an example of an attempt to keep mean income under capitalism higher than $y_e$.

The findings about income redistribution may help policy maker make better redistribution policy. Income tax creates greater overall utility by decreasing the gap between the rich and the poor and redistribute the wealth to the majority of voters. When voters are concerned more about status, income tax has greater effect on increase utility. We see that some net payers are better off under
tax. Therefore, it is possible to raise more tax from net payers to finance other projects or to redistribute more to the net recipients. In multiple period model, extra tax from net payers may be used towards capital investment to raise overall productivity which will raise income in the next period.

The model is not without limitations. Our utility function only takes into account absolute income and relative income and we treat agents as if they are memoryless and past income or utility does not affect agent’s utility. In reality, well-being over time and leisure may influence utility as well. A complete utility function should take in to account absolute income over time, relative income over time and leisure over time. Perfect information regarding income distribution may not be available to voters. Another issue is who the agents are comparing themselves to? People who are socially close to one another are more likely to compete for status among themselves. The presence of other agents in the same economy who are socially distant may not affect the perception of relative standing. In addition, we assume that agents’ incomes after policy implementation is determined. In reality, there are uncertainties about the income after implementation. The presence of such risks may alter voting outcome. There are many empirical studies that can be done to improve our model. In short, this model is just a beginning in this relatively unexplored area and there are many improvements that can be made on the model both theoretically and empirically.
5 References


