Currency Wars or Efficient Spillovers?*
A General Theory of International Policy Cooperation

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Abstract

In an interconnected world, national economic policies regularly lead to large international spillover effects, which frequently trigger calls for global cooperation. This paper presents a unified framework of spillovers and international policy cooperation. We show that many contentious spillovers are actually consistent with Pareto efficiency so there is no point in policy cooperation. This holds as long as (i) countries do not exert market power, (ii) they have sufficient external policy instruments and (iii) international markets are free of imperfections. Importantly, under these three conditions, domestic market imperfections or targeting problems do not matter for the Pareto efficiency of the international allocation. We provide examples of efficient spillovers from current account intervention, monetary policy, fiscal policy, macroprudential policy/capital controls, and exchange rate management. Conversely, violations of the three conditions generically give rise to inefficiency. For each of the three, we develop guidelines for how policy cooperation can improve welfare when they are violated.

JEL Codes: F34, F41, H23

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1 Introduction

In a globally integrated economy, national economic policies lead to international spillover effects. These are frequently large and lead to considerable controversy. For example, the Brazilian finance minister Guido Mantega has used the term “currency wars” to describe the effects of US monetary easing on the country’s exchange rate (see Wheatley and Garnham, 2010). Other recent national policies that have led to international controversy include the large reserve accumulation by China and other Asian countries, the capital flow management policies by emerging market economies such as Brazil, as well as spillovers from monetary and exchange rate policy in Japan, Switzerland, the euro area and China. However, even though spillover effects may be controversial, this does not necessarily mean that they are inefficient.

The contribution of this paper is twofold. First, we show that we can narrow down the issues that give rise to inefficient spillovers to three specific categories of problems: inefficiency arises if policymakers (i) abuse market power, (ii) have imperfect policy instruments to influence external transactions, or (iii) face imperfections in international markets. This allows us to present a well-defined set of circumstances that are worth expending diplomatic efforts on. For each of the three categories of inefficiency, we provide general guidelines and examples for how cooperation can improve welfare. We also analyze when cooperation needs to involve domestic policies and when it is sufficient to coordinate only the use of external policy instruments on trade and financial flows.

Secondly, if none of these three categories of problems is present or if they have been successfully addressed, then we prove that spillovers from national economic policymaking are Pareto efficient – they simply reflect the efficient functioning of market economies, and there is no further scope for Pareto improvements via policy cooperation. This result is counter to the intuition of many commentators and policymakers who suggest that spillovers always call for cooperation. Our findings therefore help to channel the debate on international policy cooperation into the three areas where it is most likely to bear fruit.

We analyze these questions in a multi-country framework that can flexibly nest a broad class of open economy macro models and that is able to capture a wide range of domestic market imperfections and externalities. We assume that each country consists of optimizing private agents and a policymaker who has various policy instruments to affect the domestic and international transactions of the economy. When fundamentals in a given country change and/or when policymakers respond by changing their policy instruments, the actions of domestic agents are influenced, which in turn leads to general equilibrium adjustments that entail spillover effects to other economies.

We show that the spillovers from national economic policies are Pareto efficient so that there is no need for global cooperation under three sufficient conditions: (i) the policymakers in each country do not exert market power, i.e. they act as price-takers in the international market, (ii) they possess a full set of instruments to control the country’s external transactions, and (iii) there are no imperfections in international markets. Under these three conditions, we can view national policymakers as competitive agents in a well-functioning global market. This allows us to establish

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1 For a detailed review of both the theoretical and empirical literature on spillovers, see for example Kalemli-Ozcan et al. (2013).
2 For detailed discussions on these subjects, see e.g. Gallagher et al. (2012), Jeanne et al. (2012), Ostry et al. (2012), Stiglitz (2012) and Blanchard (2016).
a version of the first welfare theorem that applies to settings in which private agents are governed by policymakers in each country and interact in a complete global market. International spillover effects in such a setting constitute pecuniary externalities that are mediated through world market prices and are thus Pareto efficient. An important corollary that has the scope to greatly simplify the numerical evaluation of problems of international policy cooperation when the three conditions are satisfied is that solving the planning problem of a global economy is equivalent to the uncoordinated equilibrium.

We obtain our results by observing that, under quite general conditions, we can condense the welfare function of each country into a reduced-form welfare function $V(\cdot)$ that only depends on the country’s international transactions. Condition (i) ensures that there are no monopolistic distortions; condition (ii) guarantees that each domestic policymaker can actually implement her desired external allocation; condition (iii) ensures that the marginal rates of substitution of all countries are equated in equilibrium. As a result, we can apply the first welfare theorem at the level of national policymakers, interpreting $V(\cdot)$ as the utility functions of competitive agents in a complete market. Our model is general enough to allow for a wide range of domestic market imperfections, including price stickiness, financial constraints, incentive/selection constraints, missing or imperfect domestic markets, and imperfect domestic policy instruments.

Our framework offers clear guidelines for how policy cooperation can improve welfare when one or several of the conditions are violated: (i) Cooperation must ensure that countries refrain from monopolistic behavior and act with “benign neglect” towards international variables. (ii) If countries have imperfect external instruments, cooperation aims to expand the set of instruments or to use the existing set more efficiently, for example by making countries with better instruments assist those with worse instruments. (iii) If imperfections in international markets lead to inefficiency, global coordination is necessary since the imperfections are outside of the domain of individual national policymakers. We provide examples that characterize the scope for coordination in each of the three cases.

We analyze a number of examples of international spillovers effects that have been contentious in recent policy debates. Applying our framework for efficiency analysis, it turns out that none of them inherently leads to inefficiency. This underlines that the fundamental driving forces behind many spillovers effects are likely efficient, and that inefficiency arises from violations of one of the three conditions (i) to (iii).

Our first example, chosen because of its stark simplicity, analyzes the general equilibrium effects that arise when a country experiences endowment shocks. Optimizing agents smooth such shocks in international capital markets, importing more in periods in which their endowments are relatively scarce and exporting more when they are abundant. From the perspective of the rest of the world, the resulting equilibrium adjustments in prices and quantities traded represent international spillover effects.

Our next example analyzes a simple motive for policy intervention by considering an economy in which exports generate positive learning externalities. A policymaker finds it optimal to sub-

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3We can also categorize examples of successful international cooperation along these lines: For example, the WTO rules out monopolistic behavior; Basel III provides policymakers with new counter-cyclical capital buffers (better instruments), recent IMF proposals allow countries to use capital controls (new instruments; see Ostry et al, 2011), and swap lines between advanced economy central banks provide new forms of liquidity insurance (more complete markets).
dize exports or engage in equivalent current account interventions, e.g. by accumulating foreign reserves. This increases savings abroad and tends to push down the world interest rate, but – subject to conditions (i) to (iii) – such spillover effects are efficient.

Our third example describes an economy that suffers from a shortage of aggregate demand that cannot be corrected using domestic monetary policy because of a binding zero-lower-bound on the nominal interest rate. A national policymaker finds it optimal to impose controls on capital inflows or subsidize outflows in order to mitigate the demand shortage. This tends to push down the world interest rate. However, under the three conditions discussed earlier, this behavior leads, again, to a Pareto efficient global equilibrium.

The next example analyzes the spillovers from a domestic policy: if an economy experiences a shock that makes it desirable to increase fiscal spending, the country will import more in the period of higher spending, which tends to push up the world interest rate. Again, equilibrium is efficient under the three discussed conditions. In the context of fiscal policy, it is of particular importance that policymakers act competitively, e.g. that they do not distort spending in favor of domestic industries, even though part of the spending will spill over to other countries. During the Great Recession, the policymakers of the G-7 engaged in coordinated fiscal stimulus to internalize such spillovers.

A fifth example investigates the spillovers of macroprudential policies or capital controls that are imposed to curb excessive borrowing. If a country imposes such financial regulations to internalize potential fire-sale externalities, it will reduce borrowing and imports at the time the regulation binds, but potentially mitigate the sharp current account reversal that occur if excessive borrowing leads to crisis. These spillover effects tend to reduce the volatility of the world interest rate.

Our final example analyzes an economy in which some domestic agents face incomplete risk markets and cannot insure against exchange rate fluctuations. Intervening in the economy’s capital account to stabilize the exchange rate can serve as a second-best insurance policy. Again, the outcome is Pareto efficient under the three conditions identified above. This illustrates that our results on global Pareto efficiency continue to hold even if a national planner intervenes in the current account to pursue purely domestic distributive objectives or to implement domestic political preferences.

In each of the described examples, spillover effects are Pareto efficient as long as the three benchmark conditions for efficiency are met. Any arguments about the desirability of global policy coordination therefore needs to be made on the basis of deviations from these conditions, not on the basis of observing that policies generate spillover effects in a globalized world.

The final part of our paper examines the three areas of international policy coordination that are highlighted by our efficiency conditions: (i) When policymakers act monopolistically, coordination should aim to restrict monopolistic behavior so as to maximize gains from trade. The basic idea has been well understood at least since the rebuttal of mercantilism by Smith (1776). We add to this literature by providing general conditions for the direction of monopolistic intervention that...
help to distinguish monopolistic intervention from intervention to correct domestic market imperfections. However, we also show that there are ample circumstances under which it is difficult to distinguish monopolistic from corrective intervention. Furthermore, we show that a national policymaker would never use domestic policies for monopolistic reasons unless she faces restrictions on her external policy instruments.

(ii) A country’s external policy instruments are imperfect if policymakers do not have sufficient instruments to target the country’s external transactions in the desired manner. This includes external policy instruments that are missing, costly to impose, too coarse, subject to fiscal considerations, or commitment problems, etc. In all these cases, domestic policymakers do not have sufficient control over the external allocations chosen by private agents. International policy cooperation can generically improve outcomes. Broadly speaking, cooperation takes the form of countries with better instruments or better-targeted instruments assisting those without. For example, if a country experiences externalities from capital inflows but has no instrument to control them, welfare is improved if other countries control their capital outflows. Importantly, efficiency condition (ii) only requires the set of external policy instruments to be perfect – it does not matter if domestic policy instruments are incomplete or restricted.

Policy cooperation under incomplete instruments has a rich intellectual tradition, going back to the targets and instruments approach of Tinbergen (1952) and Theil (1968). They observed in a reduced-form setting without private agents that incomplete instruments may give rise to a role for economic policy cooperation. Our contribution to this literature is to embed the Tinbergen-Theil approach into a general equilibrium framework in which optimizing individual agents interact in a market setting. This leads to a number of novel findings. First, we show that many of the spillover effects that would suggest a role for cooperation in the Tinbergen-Theil framework actually constitute efficient pecuniary externalities. Once the optimizing behavior of private agents is taken into account, a wide range of spillovers can be considered as efficient. Secondly, monopoly power and incomplete markets create independent roles for cooperation even if policy instruments are complete – a fact that was not considered by Tinbergen and Theil.

(iii) When the international market is subject to imperfections, the first welfare theorem no longer applies and a global planner can generally improve outcomes. Borrowing from Leo Tolstoy’s quote on unhappy families, each imperfect market is imperfect in its own way. In the given paper, we limit our attention to two examples of international market imperfections. We refer to the rich literature on market imperfections in general equilibrium models with individual optimizing agents. By reinterpreting national economic policymakers as individual agents who interact in the international market, the inefficiency results of this literature and the lessons for the desir-

Keynesian framework [Corsetti et al. (2011)] provide a detailed summary of the implications for monetary policymaking in open economies.

In the more recent literature, Jeanné (2014) provides an interesting example where the coordination of macroprudential policies is warranted because of missing policy instruments.

Recent applications in which international market imperfections create a case for cooperation include Bengui (2013) who analyzes the need for coordination on liquidity policies when global markets for liquidity are incomplete, and Jeanné (2014) who analyzes a world economy in which agents are restricted to trading bonds denominated in the currency of a single country.

This literature includes, for example, Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986) who discuss the implications of incomplete markets for the efficiency of equilibrium, or Farhi and Werning (2016) for a general treatment of the effects of price stickiness.
ability of intervention can be applied to a setting of national planners to make a case for global cooperation.

From a methodological perspective, our paper offers several useful results for the literature on international macroeconomics. First, it identifies an efficient benchmark as well as general conditions for how to construct models in which non-cooperative equilibria are efficient or generically inefficient. This is also useful in the numerical implementation of quantitative models: when the first welfare theorem holds, solving for the non-cooperative equilibrium of competitive national planners is equivalent to solving for globally efficient allocations. Likewise, our results on separability imply that domestic and external allocations can be solved in two separate steps, mitigating the curse of dimensionality in numerical applications.

The remainder of the paper is structured as follows: Section 2 describes several examples of spillovers that have led to controversy in recent years. Sections 3 and 4 introduce our general model setup and examine its welfare properties, stating the general conditions under which spillovers are efficient. Sections 5 to 7 examine the case for international policy coordination to rule out monopolistic behavior and to address imperfections in external policy instruments and in international markets.

2 Examples of Spillovers

This section investigates several tangible examples of international spillovers, spanning from a simple example of a real shock to several policy interventions by domestic policymakers that are designed to improve domestic welfare but have international spillover effects on other countries. In each of these cases, we characterize spillovers by describing how a change in fundamentals shifts a country’s net demand curve for international transactions. In general equilibrium, this leads to spillovers, i.e. changes in the prices and/or quantities at which countries transact with each other. A reader who is mainly interested in the technical contribution of the paper can jump over this section and go directly to section 3. Appendix B complements the current section by deriving the main efficiency and inefficiency results of the paper in a simple two-period setup building on section 2.2.

2.1 Real Spillovers

We start with a very simple example that captures one of the main elements of our analysis. Assume a two-period world economy in which there is a single consumption good and intertemporal trade. Consider a country $i$ that is inhabited by a representative agent who chooses how to allocate consumption across the two periods

$$\max_{c_0^i, c_1^i, m_0^i, m_1^i} U^i = u(c_0^i) + u(c_1^i) \quad \text{s.t.} \quad c_0^i = y_0^i + m_0^i$$
$$c_1^i = y_1^i + m_1^i$$
$$m_0^i + m_1^i / R \leq 0$$
where \( c^i_t \) and \( y^i_t \) denote consumption and output, \( m^i_t \geq 0 \) denotes net imports (or, if negative, exports) of the consumption good or equivalently capital inflows (outflows), and \( R \) is the relative intertemporal price of consumption goods in period 0 vs. period 1 or, equivalently, the gross world interest rate. The period utility functions are given by \( u(c) = c^{1-\theta} / (1 - \theta) \). The optimization problem is subject to two period budget constraints that capture the domestic budget constraints for a given level of imports, plus a dynamic budget constraint that reflects the intertemporal external budget constraint.

**Reduced-Form Welfare Functions** We reformulate this setup in vector notation, which will later allow us to underline how it nests into our general framework. Let us call the column vectors \( m^i = (m^i_0, m^i_1)^T \) and \( x^i = (c^i_0, c^i_1) \) the external and domestic allocation of country \( i \), and let us define the international price vector \( Q = (1, 1/R) \). Then we can denote the reduced-form welfare of the representative agent for a given external allocation \( m^i \) by the function

\[
V^i_m(m^i) = u(y^i_0 + m^i_0) + u(y^i_1 + m^i_1)
\]

The representative agent in country \( i \) solves the optimization problem

\[
\max_{m^i} V^i_m(m^i) \quad \text{s.t.} \quad Q \cdot m^i \leq 0
\]

Assigning shadow price \( \lambda^i \), this yields the optimality condition

\[
V^i_m = \lambda^i Q^T \quad \text{or, equivalently,} \quad \frac{u'(c^i_0)}{u'(c^i_1)} = R
\]

where \( V^i_m = \partial V^i / \partial m^i = (u'(c^i_0), u'(c^i_1))^T \) denotes the vector of marginal utilities of net imports. The first equation describes optimality in vector notation: the marginal utility of each type of imports equals its market price times the (scalar) shadow price of wealth. The second equation is obtained by substituting out the shadow price: private agents equate their intertemporal marginal rates of substitution (MRS) to the common world interest rate. The optimality condition to together with the external budget constraint can be solved for

\[
m^i_0 = \frac{y^i_1 R^{-\frac{1}{\theta}} - y^i_0}{1 + R^{\frac{\theta-1}{\theta}}} = -\frac{m^i_1}{R}
\]

In short, optimal consumption smoothing implies greater net imports the higher the (properly discounted) gap between output in periods 0 and 1.

**Spillovers of Endowment Shocks** An endowment shock creates international spillover effects by shifting the demand curves of the representative agent (for given international prices) by

\[
\left. \frac{dm^i}{dy^i_0} \right|_Q = \left( \begin{array}{c} -s \\ Rs \end{array} \right) \quad \text{where} \quad s = \frac{1}{1 + R^{\frac{\theta-1}{\theta}}}
\]

A positive period 0 endowment shock \( dy^i_0 \) will lead to a negative shift in demand for imports/capital inflows at date 0 and vice versa at date 1. In the general equilibrium of the world economy, this
will translate into smaller date 0 net inflows and (unless country \( i \) is a small open economy) into a lower world interest rate, as we will describe in further detail in the ensuing section. These quantity and price adjustments represent one of the simplest examples of global spillover effects. The spillovers affect the welfare of other countries disparately depending on their financial position: borrowing countries benefit from the lower world interest rate, whereas lending countries will be hurt.

2.2 Spillovers of Current Account Intervention

We extend our previous example from section 2.1 by including a policymaker and assuming a very simple motive for policy intervention: learning-by-exporting externalities. Specifically, we assume that period 1 output is a function of aggregate period 0 net exports \( y^1_i = y^1_i (\mathit{\mathbf{M}}^0_i) \) that is continuous and increasing \( y^1_i (\mathit{\mathbf{M}}^0_i) > 0 \), capturing that higher aggregate net exports increase growth. We use upper-case letters \( \mathit{\mathbf{M}}_i \) to distinguish aggregate imports from individual-level imports \( m_i \) since a representative agent takes aggregate allocations as given. In equilibrium, however, aggregate allocations equal individual allocations \( \mathit{\mathbf{M}}_i = m_i \). Using this notation, the reduced-form welfare function of country \( i \) given a pair of individual and aggregate external allocations \( (m_i, \mathit{\mathbf{M}}_i) \) is

\[
V^i (m_i, \mathit{\mathbf{M}}_i) = u (y^0_i + m^0_i) + u (y^1_i (\mathit{\mathbf{M}}^0_i) + m^1_i)
\]

For simplicity we assume that period utility is \( u (c) = \log c \) in the current example.

A policymaker who can regulate the external transactions of private agents and internalizes that \( m_i = \mathit{\mathbf{M}}_i \) will solve the optimization problem

\[
\max_{\mathit{\mathbf{M}}_i} V^i (\mathit{\mathbf{M}}_i, \mathit{\mathbf{M}}^i) \quad \text{s.t.} \quad Q \cdot \mathit{\mathbf{M}}_i \leq 0
\]  

(3)

Assigning shadow price \( \Lambda^i \), the optimality condition is

\[
V^i_m + V^i_M = \Lambda^i Q^T \quad \text{or, equivalently,} \quad \frac{u'(c^0_i)}{u'(c^1_i)} = R + y^1_i (\mathit{\mathbf{M}}^0_i) 
\]

(4)

The first equation in (4) states that the policymaker equates the sum of the private and uninternalized social marginal utility of imports to the world market price. In the second equation, this is re-written in terms of the MRS of consumers – recall that \( y^1_i > 0 \) so the planner in a developing country increases the MRS of private agents, encouraging them to export in period 1 in order to benefit from the learning externalities.

Let us express the planner’s intervention in terms of tax instruments \( \tau^i = (\tau^0_i, \tau^1_i) \) on the external transactions of private agents \( m^i \). Tax revenue is rebated as a lump-sum transfer \( T^i \) so the external budget constraint of private agents takes the form \( \frac{Q}{1 - \tau^i} \cdot m^i \leq T^i \) and the optimality condition of private agents is \( (1 - \tau^i) V^i_m = \lambda^i Q \) (where all vector multiplications and divisions are

\footnote{We do not want to take a stance on the practical relevance of such externalities in this paper. Our main objective here is to start with one of the simplest possible settings in which there is scope for policy intervention. For a strand of literature that postulates that such learning effects have been an important driver for countries that engaged in current account intervention, esp. in East Asia, see for example [Rodrik (2008)] and [Korinek and Serven (2016)].}
element-by-element unless indicated by the inner product operator, e.g. in $Q \cdot m_i^t$). The optimal tax vector equates this private optimality condition to the planner’s optimality condition (1):

$$
\tau^i = - \left( \frac{V^i_M}{V^i_m} \right)^T = \left( y^i_1 \cdot \frac{u^i(c^i_1)}{u^i(c^i_0)}, 0 \right)
$$  \hspace{1cm} (5)

The planner subsidizes period 0 exports $\tau^i_0 > 0$ (or, equivalently, taxes imports) and sets $\tau^i_1 \equiv 0$ in period 1 since there are no further externalities. The planner could also use quantity interventions and set period 0 net exports to the optimal level $M^i_0$ to internalize the growth externalities. In economies with closed capital accounts, this can equivalently be achieved by accumulating $-M^i_0$ in foreign reserves.

**Policy Spillovers of Current Account Intervention**  The policymaker’s intervention reduces net imports and creates an international spillover by shifting the demand curves of the representative agent by

$$
\left. \frac{dm^i}{d\tau^i_0} \right|_Q = \begin{pmatrix} -s \\ Rs \end{pmatrix}, \quad \text{where} \quad s = \frac{y^i_0 + y^i_1 / R}{(2 - \tau^i_0)^2}
$$  \hspace{1cm} (6)

Greater intervention $d\tau^i_0$ to internalize learning-by-exporting externalities leads to a negative shift in import demand at date 0 and vice versa at date 1. As a result, in the global equilibrium, other countries will import more and the world interest rates will decline. Again, the welfare of other countries increases or decreases depending on whether they are borrowers or lenders.

### 2.3 Spillovers of Current Account Intervention at the ZLB

This section studies the multilateral implications of export promotion policies that counter aggregate demand externalities at the zero lower bound (ZLB) on nominal interest rates in the spirit of Krugman [1998] and Eggertsson and Woodford [2003]. We extend the example in section 2.1 with a New Keynesian structure for production in period 0 and a utility function of

$$
U^i = u(c^i_0) - d(\ell^i_0) + u(c^i_1)
$$

where $\ell^i_0$ is labor supplied in period 0. We assume a continuum $z \in [0, 1]$ of monopolistic intermediate goods producers in the first period $t = 0$ who each hire labor at the market wage $w^i_0$ to produce an intermediate good of variety $z$ according to the linear function $y^i_0 z = \ell^i_0 z$, where labor market clearing requires $\int \ell^i_0 dz = \ell^i_0$. The intermediate goods are combined using a CES production function

$$
y^i_1 = \left( \int_0^1 y^i_0 (\frac{z}{\varepsilon}) \frac{dz}{\varepsilon} \right)^{\frac{1}{\varepsilon - 1}}
$$

where the elasticity of substitution satisfies $\varepsilon > 1$. We assume that the monopoly wedge arising from monopolistic competition is corrected by a proportional subsidy $\frac{1}{\varepsilon - 1}$ that is financed by a lump-sum tax on producers. Intermediate firms are owned by the representative agent, so their...
period 0 wage income and profits equal final output, which in turn equals labor supply \( \ell_0^i + \pi_0^i = y_0^i = \ell_0^i \). The labor supply of the representative agent is determined by the optimality condition

\[
d^i \left( \ell_0^i \right) = w_0^i u'(c_0^i)
\]

We assume that the nominal price of one unit of final consumption good follows an exogenous path \( P^i = (1, \Pi^i) \) that is credibly enforced by the central bank (see e.g. Korinek and Simsek, 2016, for further motivation). This assumption precludes the central bank from committing to monetary expansion/inflation in period 1 in order to stimulate output in period 0.

The ZLB constraint \( i^i \geq 0 \), which is commonly motivated by the existence of cash that delivers zero net returns as an alternative savings vehicle, together with the Euler equation imposes a ceiling on aggregate period 0 consumption,

\[
u' \left( C_0^i \right) \geq \frac{u' \left( C_1^i \right)}{\Pi^i} (7)
\]

Intuitively, a binding ZLB implies that consumption is too expensive in period 0 compared to period 1, limiting aggregate demand in period 0 to the level indicated by the constraint.

**Equilibrium in the Domestic Economy**

Given the external transactions \( M^i \), there are two possible equilibria in the domestic economy. If the ZLB constraint is slack, output is at its efficient level determined by the optimality condition \( u' \left( C_0^i \right) = d^i \left( L_0^i \right) \), which we call potential output \( Y_0^i* \). This is feasible if inequality (7) is satisfied for the efficient level of output, i.e. if \( u' \left( Y_0^i* + M_0^i \right) \geq u' \left( Y_1^i + M_1^i \right) /\Pi^i \). This will be the case in the laissez-faire equilibrium whenever the world interest rate is sufficiently high.

Conversely, the ZLB constraint is binding if this inequality is violated, i.e. if the relative price of today’s goods in terms of tomorrow’s goods, captured by the domestic interest rate, cannot fall any further. In that case, the domestic interest rate is \( i^i = 0 \) and, given the New Keynesian supply side, we can solve for the demand-determined level of period 0 output by imposing an equality in equation (7) and substituting the period budget constraints \( C_1^i = Y_1^i + M_1^i \) to obtain

\[
Y_0^i = \left( u' \right)^{-1} \left[ u' \left( Y_1^i + M_1^i \right) /\Pi^i \right] - M_0^i (8)
\]

The wage \( w_0^i \) adjusts so that domestic agents supply the required amount of labor \( L_0^i = Y_0^i \). Equation (8) highlights that any additional unit of net imports \( M_0^i \) (or, equivalently, any net capital inflow) reduces domestic output one-for-one when output is demand-determined. This situation captures the essential characteristic of a liquidity trap.

Denoting the two aggregate objects \( w_0^i \left( M^i \right) \) and \( L_0^i \left( M^i \right) \) as functions of the external transactions \( M^i \), we can express the reduced-form welfare function of the country

\[
V^i \left( m^i, M^i \right) = u \left( w_0^i \left( M^i \right) \ell_0^i + \left[ 1 - w_0^i \left( M^i \right) \right] L_0^i \left( M^i \right) + m_0^i \right) - d \left( \ell_0^i \right) + u \left( y_1^i + m_1^i \right)
\]

\[10\]It is well known in the New Keynesian literature that the problems associated with the zero lower bound could be avoided if the monetary authority was able to commit to a higher inflation rate. See e.g. Eggertsson and Woodford (2003).
This formulation highlights that individual agents recognize the effect of their external transactions on their individual consumption but not on aggregate demand and employment.

**Optimal Policy** A planner who maximizes the welfare of the representative agent will internalize these aggregate demand effects by solving the usual problem \( (3) \). Assuming a symmetric allocation, the externalities of net capital flows \( M_i \) are

\[
V_i^M = \left[ u' \left( C_i^0 \right) - d' \left( L_i^0 \right) \right] L_i^0 \left( M_i^0 \right) = \left[ u' \left( C_i^0 \right) - d' \left( L_i^0 \right) \right] \left( \frac{-1}{u'' \left( c_i^1 \right) / \Pi_i} \right)
\]

which satisfy \( V_{M0}^i < 0 \) and \( V_{M1}^i > 0 \) whenever the economy is in a liquidity trap. The brackets \( [u' \left( C_i^0 \right) - d' \left( L_i^0 \right)] \) in this expression capture the labor wedge, i.e. the net benefit of an additional unit of output. Date 0 imports increase the aggregate demand shortfall whereas date 1 imports relax it since they create a future consumption boom, which – via the Euler equation – stimulates today’s consumption.\(^{11}\) The planner thus finds it optimal to tax period 0 net inflows and subsidize period 1 net inflows at rates \( \tau_i^t = -V_i^M / V_i^M \).

Note that the capital account interventions of a planner in this setting are second-best policies since the first-best policy would be to restore domestic price flexibility to abolish the ZLB constraint. The planner solves the optimal trade-off between foregoing profitable opportunities for intertemporal trading with foreigners and wasting profitable production opportunities because of the ZLB.

**Spillovers of Export Promotion at the ZLB** The spillovers of the described policy consist of greater net exports in period 0 and greater net imports in period 1, which can be characterized analogously to expression \( (6) \). Unless the country is a small economy, the intervention will lead to a lower world interest rate.

### 2.4 Spillovers of Fiscal Shocks

We consider another extension of our first example on real spillovers in which there is a role for fiscal policy.\(^ {12} \) Assume that agents value not only private consumption but also government spending \( G_i \) according to the utility function

\[
U_i^t \left( x_i^t \right) = u \left( c_i^0 \right) + u \left( c_i^1 \right) + \alpha u \left( G_i^0 \right) + u \left( G_i^1 \right)
\]

\(^{11}\)In a time-consistent setting for capital account interventions, the planner would not be able to commit to future policy actions. The intervention during a liquidity trap would still be given by the same expression \( V_{M,t} = d' \left( L_i^t \right) - u' \left( c_i^t \right) \), but after the liquidity trap has passed the planner would find \( V_{M,t+1} = 0 \) and no further intervention would occur.

\(^{12}\)In the aftermath of the Great Recession, for example, coordination of fiscal stimulus has been at the center of discussions in the international economic policy arena (see e.g. the G-20 Leaders’ Declaration, Nov. 2008; Spilimbergo et al., 2008). One of the important considerations in this context is for policymakers to refrain from exerting market power, i.e. to hold back stimulus because part of it spills over to other countries, as we will discuss in more detail in sections \( 5 \) and \( 6 \).
For simplicity, we use the same period utility \( u(\cdot) \) for both types of spending. We will vary the parameter \( \alpha \) to induce shocks to the preference for fiscal spending. We assume that government raises the revenue necessary for its spending via lump-sum taxes in the amounts of \( G_i^0 \) and \( G_i^1 \), giving rise to the period budget constraint \( c_i^t + G_i^t \leq y_i^t + m_i^t \). A domestic policymaker who takes international prices as given sets period 0 fiscal spending such that

\[
    u' \left( C_0^i \right) = \alpha u' \left( G_0^i \right)
\]

or, equivalently, \( C_0^i = \alpha^{-\frac{1}{\theta}} G_0^i \) and sets \( C_1^i = G_1^i \) for period 1. Using the short-hand notation \( A = 1 + \alpha^{\frac{1}{\theta}} \), the Euler equation of private agents implies

\[
    M_0^i = \frac{AY_1^i/R^\frac{1}{\theta} - 2Y_0}{AR^{1-\frac{1}{\theta}} + 2}
\]

and consumption and government spending satisfy \( C_0^i = \alpha^{-\frac{1}{\theta}} G_0^i = (Y_1^i + M_0^i)/A \) and \( C_1^i = G_1^i = (Y_1^i - RM_0^i)/2 \).

### Spillovers of Fiscal Shocks

A shock to the preference for period 0 fiscal spending can be captured by \( d\alpha \) or, equivalently, by \( dA = \frac{1}{\theta} \alpha^{\frac{1}{\theta}} d\alpha \) and creates international spillover effects by shifting the demand curves of the economy by

\[
    \left. \frac{dM_i^t}{dA} \right|_Q = \left( \begin{array}{c} S \\ - S/R \end{array} \right)
\]

where

\[
    S = \frac{2R^{-\frac{1}{\theta}} (Y_1^i + Y_0^i R)}{\left( AR^{1-\frac{1}{\theta}} + 2 \right)^2} > 0
\]

In the equilibrium of the world economy, a greater preference for fiscal spending in period 0 induces the economy to import more in period 0 and export more in period 1. Unless economy \( i \) is small, this translates into a higher world interest rate with the typical global spillover effects.

Note that the setup in this example can also be interpreted from the perspective of intratemporal trade if the utility function represents private and government consumption on two different types of goods \( k = 0, 1 \) in a given time period. The described fiscal shock then captures that it may be desirable to reallocate spending towards certain types of goods, for example domestic goods. The resulting spillovers consist of raising the relative price of type \( k = 0 \) goods, which improve or deteriorate the country’s terms of trade.

### 2.5 Spillovers of Macroprudential Policy

This example illustrates the spillover effects of macroprudential policy interventions or capital controls that are designed to mitigate fire-sale externalities in an economy with financial frictions. We build on the framework of Jeanne and Korinek (2010) to motivate the policy but analyze the resulting spillovers. We do so by adding a third time period to our earlier examples so that \( t = 0, 1, 2 \) and assuming a quasilinear utility function

\[
    U^i = u \left( c_0^i \right) + u \left( c_1^i \right) + c_2^i
\]

\[\text{\footnotesize \textsuperscript{13}}\]

Since Ricardian equivalence holds in the described example, it is irrelevant whether private agents or the government or both engage in external transactions – for simplicity, our formulation here assumes that only private agents do.
where \( u(c_i) = \log c_i \). We collect the prices of discount bonds in the world economy in the row vector \( Q = (1, 1/R_1, 1/(R_1R_2)) \) and the net imports in each time period in the column vector \( m^i = (m^i_0, m^i_1, m^i_2)^T \). The consumer faces the usual period budget constraints \( c^i_t = y^i_t + m^i_t \) for \( t = 0, 1, 2 \) as well as the usual intertemporal budget \( Q \cdot m^i \leq 0 \).

To create an interesting case for macroprudential policy, we follow Jeanne and Korinek (2010) in assuming that the representative agent owns a tree that can be used as collateral. The tree delivers a dividend \( y^i_t \) in each period as long as it is held by domestic agents – this captures that domestic agents have an advantage in tending to domestic projects and creates the potential for fire-sale prices. Furthermore, domestic agents are subject to a commitment problem that limits their borrowing in period 1. In particular, an agent could invest in a scam in period 1 that would enable him to default on his lenders in period 2 and hide all his assets. Lenders can stop the scam by taking the borrower to court in period 1 and recover up to an amount \( \phi \) of his tree, but they need to re-sell the tree to other domestic agents at market price \( p^i \) lest it loses its value. This gives rise to a financial constraint on the representative agent that limits how much he can commit to repay to

\[-m^i_2 \leq \phi p^i\]

Individual agents in economy \( i \) take the asset price as given when they decide on their optimal consumption allocation. However, at the aggregate level, the asset pricing condition \( p^i = u'(c^i_2) / u'(c^i_1) \cdot y^i_2 \) defines an aggregate asset price function \( p^i(M^i_1) = (y^i_1 + M^i_1) y^i_2 \). This allows us to re-write the constraint as

\[m^i_2 + \phi p^i(M^i_1) \geq 0\]

A domestic policymaker internalizes that greater period 1 consumption raises the asset price and relaxes the borrowing constraint in period 1, which provides a rationale for macroprudential intervention. Denoting the shadow price on the constraint by \( \mu^i \), a policymaker can improve welfare when the constraint is binding by imposing a macroprudential tax on period 0 borrowing that satisfies\(^{13}\)

\[1 - \tau^i_0 = 1/ \left( 1 + \frac{\mu^i \phi p''(M^i_1)}{u'(C^i_1)} \right)\]

Since lenders are foreigners, the macroprudential tax can be interpreted as a capital control.

**Spillovers of Macroprudential Regulation or Capital Controls** A prudential policy intervention that marginally reduces period 0 borrowing and net inflows \(-dm^i_0 < 0\) creates international spillovers by shifting the demand curves of the representative agent in the three time periods by

\[-\left. \frac{dm^i}{dm^i_0} \right|_Q = \begin{pmatrix} -1 \\ -R_1/R_1(1 - \phi p''(M^i_1)) \\ -R_1R_2(1 - \phi p''(M^i_1)) \end{pmatrix}\]

The second row captures two effects: first, each unit saved in period 0 implies \( R_1 \) units of greater liquid wealth in period 1. Furthermore, the denominator captures the financial amplification effects that arise from the feedback loop between borrowing and asset prices: each additional unit

\(^{13}\) The result is derived in detail in the appendix.
of liquid wealth increases greater consumption when the period 1 financial constraint is binding, which raises the asset prices and leads to \( \phi p''(M_i^1) \) units of additional borrowing capacity. The additional borrowing raises consumption further, and so on. The total effect is captured by the geometric sum \( 1 + \phi p''(M_i^1) + [\phi p''(M_i^1)]^2 + \cdots = \frac{1}{1-\phi p''(M_i^1)} \). The third row captures that borrowers need to repay the additional borrowing in period 2. In the global equilibrium, the described shifts in demand will generally lower the world interest rate \( R_1 \) and increase the world interest rate \( R_2 \); other countries will benefit or be hurt depending on their interest rate exposure.

### 2.6 Spillovers of Exchange Rate Stabilization

Next we analyze the spillovers from stabilizing the exchange rate in a developing economy in which exchange rate fluctuations would result in undesirable redistributions among agents who do not have access to well-functioning insurance markets. Consider an economy \( i \) with two time periods \( t = 0, 1 \), a traded and a non-traded intermediate good each period and a final good. There are two categories of agents, which we call the “financial elite” and the “people” who live hand-to-mouth. The financial elite \( E \) obtain an endowment of \( \alpha y_T \) traded goods and \( \alpha y_N \) non-traded goods every period. The people are made up of two types \( j \in \{N, T\} \) who obtain their income in one of the two intermediate sectors: those in the traded sector \( T \) obtain an endowment of \( (1-\alpha) y_T \) traded goods; the people in the non-traded sector \( N \) obtain \( (1-\alpha) y_N \) non-traded goods every period \( t \). We assume for simplicity that the endowments \( (y_T, y_N) \) are constant across time and equal to each other. Each type of the people as well as the elite consist of a continuum of agents of mass 1.

Each period there is a spot market in which agents exchange traded and non-traded goods at relative prices \( p_{ij} \). After having traded, each agent consumes traded \( c_{T,t} \) and non-traded goods \( c_{N,t} \), which enter their period utility as a Cobb-Douglas composite \( c = c_T^\sigma c_N^{1-\sigma} \) where we set w.l.o.g. \( \sigma = \frac{1}{2} \) to maintain symmetry. Assuming CES intertemporal preferences, the utility of each type of agent \( j \in \{N, T, E\} \) is

\[
U_j = \sum_{t=0}^{1} \left( \frac{c_{T,t}^{\sigma} c_{N,t}^{1-\sigma}}{1-\gamma} \right)^{1-\gamma}
\]

where we assume \( \gamma > 1 \) to ensure agents have sufficient desire for intertemporal smoothing. This condition is satisfied for typical parameter values in macroeconomics.

**The People** do not have access to financial markets so they cannot borrow, save, or insure; their decision problem is purely intratemporal: they collect their endowment and trade in the spot market to maximize utility. We denote the period \( t \) wealth of people of type \( j \in \{N, T\} \) by \( w_{jt} \) so \( w_{Nt} = p_{Nt}(1-\alpha) y_N \) and \( w_{Tt} = (1-\alpha) y_T \). The indirect period \( t \) utility of type \( j \) is given by

\[
v_{jt} = v(p_{Nt}, w_{jt}) = \left( \frac{\kappa w_{jt} / p_{Nt}^{1-\sigma}}{1-\gamma} \right)^{1-\gamma}
\]

where \( \kappa = \sigma^{\sigma}(1-\sigma)^{1-\sigma} \) is a constant. For each type of the people, utility is increasing in the quantity and relative price of their endowed good. An appreciation (increase) in the real exchange
rate $p_{Ni}^i$ benefits the people in the nontraded sector at the expense of those in the traded sector, and vice versa for a depreciation. Imposing equal welfare weights, the combined period welfare of the people is

$$w_p(p_{Ni}) = v(p_{Ni}, y_T) + v(p_{Ni}, p_{Ni}y_N) = \frac{[\kappa (1 - \alpha)]^{1-\gamma}}{1 - \gamma} \left[ (y_T / p_{Ni}^{1-\gamma})^{1-\gamma} + (p_{Ni}y_N)^{1-\gamma} \right]$$

and is concave in $p_{Ni}$ around the autarky exchange rate $p_{Ni}^{aut} = 1$, which constitutes the maximum of the function (see appendix).

The Financial Elite engages in intertemporal trade $m^i$ in international financial markets and solves

$$\max_{\{c_t, c_{Ni}, m_t\}_{t=0}^\infty} U_E \ \text{s.t.} \ c_{Tt} - \alpha y_{Tt} + p_{Ni}^i (c_{Ni} - \alpha y_{Ni}) - m_t^i \leq 0 \ \forall t$$

subject to the standard intertemporal budget constraint. If the world interest rate is at the autarky level $R = 1$, then the elite does not engage in intertemporal trade and the exchange rate will be at its autarky level $p_{Ni}^{aut}$, maximizing the welfare of the people. If $R \leq 1$, the elite will borrow or lend and thus import and export traded goods. Domestic market clearing implies that the period $t$ exchange rate as a function of net inflows $M_t^i$ is

$$p_{Ni}^i (M_t^i) = \frac{1 - \sigma}{\sigma} \cdot \frac{y_T + M_t^i}{y_N}$$

with $p_{Ni}^i > 0$ so inflows appreciate the real exchange rate and vice versa. These real exchange rate fluctuations unambiguously reduce the welfare of the people.

Social Welfare If we assign a social welfare weight $\varphi \in [0, 1]$ to the people and $1 - \varphi$ to the elite, the reduced-form social welfare function of the economy is

$$V^i(M^i, m^i) = \sum_{t=0,1} [(1 - \varphi) \nu(p_N(M_t^i), c_{Ni}, y_T + p_N(M_t^i)y_N) + \varphi \nu_P(p_N(M_t^i))]$$

The uninternalized period $t$ welfare effects from external transactions are given by

$$V_{Mt}^i = [(1 - \varphi) \nu_P(p_{Ni}, w_E) + \varphi y_N \nu_w(p_{Ni}, w_E) + \varphi w_P(p_{Ni})] p_{Ni}^{\mu} (M_t^i)$$

$$= [(1 - \varphi) \kappa^{1-\gamma} w^{-\gamma} p_{Ni}^{-(1-\gamma)(1-\gamma)} (\alpha y_N - c_{Ni}) + \varphi w_P(p_{Ni})] p_{Ni}^{\mu} (M_t^i)$$

The first term in the brackets with weight $(1 - \varphi)$ corresponds to the welfare effects of exchange rate fluctuations on the elite, which depends on whether they are net buyers or net sellers of non-traded goods in the domestic economy. When the elite receives inflows of traded goods, it converts some of them into non-traded goods so $c_{Ni} > \alpha y_N$ and is hurt by real exchange rate appreciations. Conversely, when the elite sells traded goods abroad, it is hurt by domestic real exchange rate depreciations. The second term with weight $\varphi$ corresponds to the welfare effects on the people. As we already noted, they are also hurt by both the exchange rate appreciations when $M_t^i > 0$ and the depreciations when $M_t^i < 0$. We conclude that $V_{Mt}^i < 0$ when $M_t^i > 0$, and $V_{Mt}^i > 0$ when $M_t^i < 0$ for any $\varphi \in [0, 1]$. The planner will lean against both inflows and outflows by imposing the
respective taxes $\tau_i^t = -V_{Mt}/V_{mt}^i$. The greater the weight $\varphi$ on the people, the closer the allocation will be to the autarky allocation with perfect exchange rate stabilization, which maximizes the welfare of the people. In this example, capital flow management is a second-best insurance device since the people do not have access to insurance (either in a market setting or from social insurance) and labor markets are rigid, as is frequently the case in developing and emerging economies.

**Spillovers of Exchange Rate Stabilization**  
The spillovers of leaning against inflows and outflows of traded goods are given by an analogous expression to (6). They consist of reduced trading opportunities for the rest of the world, which generally increase the volatility of world interest rates.

### 3 General Framework

This section develops a general framework that nests the described examples – as well as a wide range of other open economy macroeconomic models – in order to derive broadly applicable lessons for international policy cooperation. We set up a model of a multi-country world economy in which each country encompasses optimizing private agents as well as a policymaker who maximizes domestic welfare. Both actors are subject to a set of constraints on domestic allocations and a standard budget constraint on external transactions.

**Countries**  
Consider a set of countries $\mathcal{I}$ of total measure normalized to $\omega (\mathcal{I}) = 1$. We denote by $\omega^i = \omega (\{i\})$ the measure of a given country $i \in \mathcal{I}$ in the world economy. If $\omega^i = 0$, then country $i$ is a small open economy.

**Private Agents**  
In each country $i \in \mathcal{I}$, there is a continuum of private agents of mass 1. A representative private agent obtains utility according to a function

$$U^i (x^i)$$

where $U^i (x^i)$ is increasing in each element of $x^i$ and quasiconcave, and $x^i$ is a column vector that includes two types of domestic variables. First, it includes all variables that directly provide utility to domestic agents, for example the consumption of goods and leisure. Utility is strictly increasing in such variables. Secondly, for compactness of notation, the vector $x^i$ may also include other domestic variables that we want to keep track of but that do not directly yield utility, for example the capital stock $k^i$ in models of capital accumulation. For the latter type of variables, utility is unaffected. We will provide concrete examples below.\footnote{In appendix A.2, we provide an extension of the described framework to multiple types of agents in each economy, and we show that our results continue to hold. For simplicity of notation, the main text focuses on the case where there is a representative agent in each economy.}

**External Budget Constraint**  
We denote the international transactions of the representative agent in country $i$ by a column vector of net imports $m^i$ that are traded at international prices given by the row vector $Q$. The agent may be subject to a vector of taxes/subsidies $\tau^i$ on international

\footnote{In appendix A.2, we provide an extension of the described framework to multiple types of agents in each economy, and we show that our results continue to hold. For simplicity of notation, the main text focuses on the case where there is a representative agent in each economy.}
transactions that are imposed by the country’s policymaker. The external budget constraint of the agent is
\[
\frac{Q}{1-\tau} \cdot m^i \leq T^i
\]  
where we denote by \( \frac{Q}{1-\tau} \) the element-by-element (Hadamard) division of the price vector \( Q \) by the tax vector \( 1 - \tau \) and by \( \frac{Q}{1-\tau} \cdot m^i \) the inner product of the price and quantity vectors. \( T^i \) denotes a lump-sum transfer, which we will generally use to rebate the tax revenue from \( \tau^i \) to the agent so \( T^i = \frac{\tau^i Q}{1-\tau} \cdot m^i \).

Below, we will consider both situations in which the policymaker has complete freedom in setting the tax instruments \( \tau^i \) – we will call this the case of perfect external instruments – and instances in which the policymaker faces cost, restrictions, or other imperfections in the set of external policy instruments. Likewise, we will consider situations in which the international market is an Arrow-Debreu market that is free of any imperfections and the case of international market imperfections.

**Domestic Constraints** The representative agent in country \( i \) is also subject to a collection of domestic constraints, which include domestic budget constraints and, potentially, incentive, selection, financial, or price-setting constraints as well as policy-imposed restrictions. These constraints relate the international transactions and domestic variables \((m^i, x^i)\) of the representative agent with each other. Furthermore, to capture domestic general equilibrium effects, policy interventions, and externalities, the domestic constraints also depend on the aggregate level of these variables, which we denote by the upper-case variables \((M^i, X^i)\). The representative agent takes these aggregate variables as given since he is small in the domestic economy. However, in equilibrium, individual and aggregate allocations coincide.

The following vector constraint captures all domestic constraints in considerable generality,
\[
f^i \left( m^i, x^i, M^i, X^i \right) \leq 0
\]  
\[
\text{max}_{m^i, x^i} U^i \left( x^i \right) \quad \text{s.t.} \quad (15), (16)
\]

**Policymaker** The policymaker in country \( i \) internalizes the consistency requirement that the allocations of the representative agent must coincide with the aggregate allocations so \( m^i = M^i \) and \( x^i = X^i \). She chooses the external policy instruments \( \tau^i \) and the aggregate domestic and external allocations \((M^i, X^i)\) in order to maximize the utility \((14)\) of domestic private agents subject to the domestic and external constraints \( f^i(\cdot) \leq 0 \) and \( M^i \cdot Q \leq 0 \) as well as the implementability constraints arising from problem \((17)\), which reflect that the allocations \((m^i, x^i)\) have to solve the optimization problem of private agents.

To make our setup a bit more tangible, the following examples illustrate how two common benchmark open economy models map into our framework:
Example 1 (Canonical Open Economy Model). In an infinite-horizon endowment economy i with a single consumption good, the only domestic variable is consumption so \( x^i = \{ (c^i_t)_{t=0}^\infty \} \). The vector of external transactions \( m^i = (m^i_t)_{t=0}^\infty \) and the external policy instruments \( \tau^i = (\tau^i_t)_{t=0}^\infty \) capture the net imports of the consumption good in each period, which is equivalent to the trade balance, and import tariffs or subsidies. Alternatively, we can interpret \( m^i_t \) as net capital inflows in period \( t \) and \( \tau^i_t \) as a capital control.

The utility function is given by \( U^i (x^i) = \sum_\ell \beta^\ell u (c^i_\ell) \), and the domestic constraints encompass one budget constraint for each time period, \( f^i_i (\cdot) = \{ f^i_i (\cdot) \}_{t=0}^\infty \) where \( f^i_i (\cdot) = c^i_1 - y^i_1 - m^i_1 \leq 0 \). If we normalize \( Q_0 = 1 \) then each element of the vector \( Q_t \) captures the price of a discount bond that pays one unit of consumption good in period \( t \), and the external budget constraint of the economy is given by \( 15 \). This fully describes the mapping of a canonical open economy model into our general framework.16

It is straightforward to extend the example to an economy with multiple consumption goods and/or with uncertainty by indexing all variables by good \( k \in K \) and/or state of nature \( s \in S \), for example \( (m^i_{k,s,t}, x^i_{k,s,t}) \).

Example 2 (Production). In a neoclassical production economy, the domestic variables also includes leisure, investment, and capital so \( x^i = \{ (c^i_1, \ell^i_1, i^i_1, k^i_{1+1})_{t=0}^\infty \} \), and the utility function includes leisure, \( U^i (x^i) = \sum_\ell \beta^\ell u (c^i_\ell, \ell^i_\ell) \). Denoting labor supply by \( 1 - \ell^i_1 \), the collection of domestic constraints consists of a budget constraint \( f^i_{t,\ell} (\cdot) \) and a capital accumulation constraint \( f^i_{t,k} (\cdot) \) each period that are given by

\[
\begin{align*}
 f^i_{t,\ell} (\cdot) &= c^i_1 + \ell^i_1 - A^i_1 \left( k^i_1 \right)^a \left( 1 - \ell^i_1 \right)^{1-a} - m^i_1 \leq 0 \\
 f^i_{t,k} (\cdot) &= k^i_{1+1} - (1 - \delta) k^i_1 - i^i_1 \leq 0
\end{align*}
\]

Global Allocations

In the following, we define a feasible country i allocation for given world prices \( Q \) as a pair \( (X^i, M^i) \) that satisfies the country i domestic and external constraints \( f^i (M^i, X^i, M^i, X^i) \leq 0 \) and \( M^i \cdot Q \leq 0 \). Furthermore, we define a feasible global allocation as a collection \( (X^i, M^i)_{i \in I} \) that satisfies the domestic constraints \( f^i (\cdot) \leq 0 \forall i \in I \) and global market clearing \( \int_{i \in I} M^i d\omega (i) \leq 0 \). Finally, we call a feasible global allocation \( (X^i, M^i)_{i \in I} \) Pareto efficient if there does not exist another feasible global allocation \( \tilde{\rho} (\tilde{M}^i, \tilde{X}^i)_{i \in I} \) that makes at every country weakly better off, \( U^i (\tilde{X}^i) \geq U^i (X^i) \forall i, \) and at least one country strictly so, \( \exists j \in I \) s.t. \( U^j (\tilde{X}^j) > U^j (X^j) \) inequality.17

Definition 1 (Global Competitive Equilibrium). An equilibrium in the described world economy consists of a feasible global allocation \( (X^i, M^i)_{i \in I} \) and a set of external policy measures \( (\tau^i)_{i \in I} \) together with world market prices \( Q \) such that in each country \( i \in I, \)

\[\frac{(1 - \tau^i_{t+1}) w^i_{t+1}}{1 + r^i_{t+1}} = w^i_t - m^i_t + T^i_t \quad \forall t\]

where the interest rate \( r^i_{t+1} \) corresponds to the relative price of discount bonds in two consecutive periods, \( 1 + r^i_{t+1} = Q^i_t / Q^i_{t+1} \), and \( \tau^i_{t+1} \) corresponds to the relative tax wedge \( 1 - \tau^i_{t+1} = \left( 1 - \tau^i_t \right) / \left( 1 - \tau^i_{t+1} \right) \) on external transactions in two consecutive periods. This period-by-period formulation (together with a transversality condition \( \lim_{t \to \infty} Q^i_t w^i_t = 0 \) and the Arrow-Debreu-style formulation in our general framework are equivalent, and we will use both in our applications below.

16It is common in the open economy macroeconomics literature to keep track of the external wealth position \( w^i_t \) of a country over time and denote the external budget constraint \( 15 \) by a period-by-period law of motion

\[\frac{(1 - \tau^i_{t+1}) w^i_{t+1}}{1 + r^i_{t+1}} = w^i_t - m^i_t + T^i_t \quad \forall t\]

where the interest rate \( r^i_{t+1} \) corresponds to the relative price of discount bonds in two consecutive periods, \( 1 + r^i_{t+1} = Q^i_t / Q^i_{t+1} \), and \( \tau^i_{t+1} \) corresponds to the relative tax wedge \( 1 - \tau^i_{t+1} = \left( 1 - \tau^i_t \right) / \left( 1 - \tau^i_{t+1} \right) \) on external transactions in two consecutive periods. This period-by-period formulation (together with a transversality condition \( \lim_{t \to \infty} Q^i_t w^i_t = 0 \) and the Arrow-Debreu-style formulation in our general framework are equivalent, and we will use both in our applications below.

17Observe that our definition of Pareto efficiency is subject to the set of domestic constraints \( f^i (\cdot) \), which may capture domestic market imperfections, and which are taken as given by both domestic planners and the global planner.
• the representative agent optimizes, i.e. the individual allocations $x^i = X^i$ and $m^i = M^i$ to solve
the optimization problem of the representative agent for given prices $Q$, aggregate allocations ($X^i, M^i$) and external policy measures ($\tau^i$)
• the policymaker optimizes, i.e. the aggregate allocations ($X^i, M^i$) and external policy measures ($\tau^i$) to solve the optimization problem of the policymaker for given prices $Q$.

4 Efficient Benchmark

4.1 Conditions for Efficiency

This section establishes an efficient benchmark by showing that the global competitive equilibrium in our general framework is Pareto efficient under three general conditions. In Sections 5 to 7, we will relax these conditions one by one to study how each of them generates scope for cooperation. The appendix provides general solutions for a case in which all three conditions are simultaneously violated.

Condition 1 (Competitive Behavior). Each country $i \in I$ acts as a price-taker.

The first natural interpretation for Condition 1 is that country $i$ is a small open economy with $\omega_i = 0$ and does not produce a unique product variety. This implies that the country does not have market power over the vector of world market prices $Q$ since $dQ/dM^i = 0$.

A second interpretation of the condition is that the policymaker in country $i \in I$ acts as a price-taker. This could be because of an explicit or implicit multilateral agreement to abstain from strategic behavior, or because of an explicitly domestic policy objective that is prescribed by law and induces the policymaker to act with benign neglect towards international markets.\(^\text{18}\)

Condition 2 (Perfect External Instruments). The policymaker in each country $i \in I$ possesses a perfect set of external policy instruments $\tau^i$.

Formally, this condition implies that the policymaker in each country can choose the vector of external policy interventions $\tau^i$ without any restrictions or costs. This ensures that she has the effective means of intervening in the external allocations $M^i$ of country $i$. Depending on the structure of the model and the interpretation of the international transactions $M^i$, the external instruments can be interpreted as tariffs or capital controls. Note that the condition is silent about what powers the policymaker has in the domestic economy. The condition can be satisfied independently of whether or not the policymaker has domestic instruments to address domestic policy objectives.

We will show below in Section 6 that the condition can in fact be replaced by a weaker condition on effectively complete instruments, i.e. the policymaker needs to possess only those instruments that she actually wants to use for domestic objectives.

Condition 3 (Complete International Market). There is a complete and unrestricted international market for trading goods $M^i$ at world market price $Q$.

\(^{18}\)For example, the US Federal Reserve claims to follow a policy of acting with benign neglect towards external considerations such as exchange rates, as articulated by Bernanke (2013). Similarly, the G-7 Ministers and Governors regularly proclaim that “we reaffirm that our fiscal and monetary policies have been and will remain oriented towards meeting our respective domestic objectives using domestic instruments, and that we will not target exchange rates” (G-7, 2013).
The third condition requires that the international market is complete and free of constraints and other imperfections such as price stickiness. Note that the condition is silent about the domestic market structure in each economy $i$, which may exhibit numerous imperfections even if Condition 3 is satisfied.

Discussion The three conditions that we stated are strong and, of course, they are never strictly satisfied in the real world. This is typical whenever economists appeal to the first welfare theorem – the underlying conditions are never literally met, but they provide a useful benchmark for organizing the debate, for exploring which deviations from the efficient benchmark matter for efficiency and, in our setting, for identifying when cooperation has a chance to bear fruit.

In the following, we provide a step-by-step analysis of the global competitive equilibrium that develops a number of results that are of independent interest. We start with a lemma that greatly simplifies the analysis:

Lemma 1 (Separability). Under Condition 2, the optimal allocation of country $i \in I$ can be obtained by following a two-step procedure:

1. Solve for the optimal domestic allocation $(x^i, X^i)$ given the external allocation $(m^i, M^i)$; this defines a reduced-form utility function $V^i(m^i, M^i)$.

2. Solve for the optimal external allocation $(m^i, M^i)$ by maximizing the reduced-form utility function $V^i(\cdot)$.

Proof. See appendix A.1.

Intuitively, if the planner has perfect external instruments, the economy’s domestic allocation can be determined without considering the interactions with the external allocation – there is no need to distort the domestic allocation in order to achieve external goals. Formally, the separability result follows since the external implementability constraint on a planner with perfect instruments is slack and, given $M^i$, can be ignored when solving for the optimal domestic allocation. By contrast, if external instruments are imperfect, the planner has an incentive to manipulate her domestic allocation to better target the external allocation, as we will demonstrate in detail in section 6.

Our separability result reflects a pecking order of instruments to target external allocations: if possible, use only external instruments to achieve external objectives. If external policy instruments are imperfect, then also use domestic policy measures. This has an interesting practical implication:

Corollary 1 (Separating Domestic and External Economic Policy). Under Condition 2, the task of domestic policy-making can be assigned to a separate agency that maximizes domestic welfare taking the external allocation as given. In particular, the domestic agency does not need to internalize how its policy actions will affect external allocations.

This corollary provides conditions under which it is sufficient for agencies responsible for a domestic policymaker to have an explicitly domestic policy focus. Even though, in general equilibrium, domestic policy will affect the external allocations of private agents, this is irrelevant for
the domestic policymaker when Condition 2 is satisfied. The reason is that the external policymaker has perfect instruments to steer the external allocation of the economy.

By contrast, the agency responsible for setting external policy cannot just take the domestic allocation as given – it generally needs to internalize how the external allocation \( M^i \) affects the incentives of private agents in choosing their optimal domestic allocation. In step 2. of the lemma above, this decision problem is part of the reduced-form utility function \( V^i (\cdot) \).

The following two subsections follow the solution strategy proposed by the lemma.

### 4.2 Domestic Optimization Problem

**Private Agents** For a given external allocation \((m^i, M^i)\) and domestic aggregate allocation \(X^i\), the optimization problem of a representative agent in country \(i\) is given by the value function

\[
v^i \left( X^i, m^i, M^i \right) = \max_{x^i} U^i \left( x^i \right) \quad \text{s.t.} \quad f^i \left( m^i, x^i, M^i, X^i \right) \leq 0
\]  

(18)

Denoting the shadow prices on the vector of domestic constraints \( f^i \) by the row vector \( \lambda^i_{d^i} \), the collection of domestic optimality conditions is

\[
U^i_x = f^i_x \lambda^i_{d^i}^T
\]

(19)

where \( U_x \) denotes a column vector of partial derivatives of the utility function with respect to \( x^i \), and \( f^i_x \) is the Jacobian of derivatives of \( f^i \) with respect to \( x^i \) and is a matrix of the size of \( f^i (\cdot) \) times the size of \( x^i \). We denote the solution to problem (18) by the policy function \( x^i (X^i, m^i, M^i) \) capturing the optimal domestic choices of private agents.

**Domestic Policymaker** For a given aggregate external allocation \( M^i \), the domestic policymaker chooses the optimal domestic allocation \( X^i \) subject to the consistency conditions \( x^i = X^i \) and \( m^i = M^i \) as well as the implementability constraint (19). The planner’s problem is

\[
\max_{X^i, \lambda^i_j} U^i \left( X^i \right) \quad \text{s.t.} \quad f^i \left( M^i, X^i, M^i, X^i \right) \leq 0, \quad (19)
\]

(20)

We assign the row vector of shadow prices \( \Lambda^i_{d^i} \) to the vector of domestic constraints \( f^i \) and \( \mu^i_{d^i} \) to the collection of domestic implementability constraints. The solution to this problem defines a function \( X^i (M^i) \) that describes the optimal domestic allocation \( X^i \) for a given external allocation.

**Definition 2** (Reduced-Form Utility). Using the value function (18), we define the reduced-form utility function of a representative agent in economy \(i\) for a given pair \((m^i, M^i)\) by

\[
V^i \left( m^i, M^i \right) := v^i \left( X^i (M^i), m^i, M^i \right) = U^i \left( x^i \left( X^i (M^i), m^i, M^i \right) \right)
\]

(21)

The reduced-form utility function \( V^i (m^i, M^i) \) contains all the information we need to solve for the external allocations of country \(i\) and will play a central role in our welfare analysis. The last equality in the definition follows directly from the definition of \( x^i (\cdot) \). Note that \( V^i (m^i, M^i) \) is also
defined for off-equilibrium allocations in which \( m^i \) and \( M^i \) differ, since individual agents are in principle free to choose any allocation of \( m^i \). In equilibrium, however, \( m^i = M^i \) will hold.

For the remainder of our analysis, we will focus on the case where the partial derivatives of this reduced-form utility function satisfy \( V^i_{m^i} > 0 \) and \( V^i_{m^i} + V^i_{M^i} > 0 \) \( \forall i \): ceteris paribus, a marginal increase in individual imports \( m^i \) or a simultaneous marginal increase in both individual and aggregate imports \( m^i = M^i \) increases the welfare of a representative consumer. These are fairly mild regularity conditions that hold for the vast majority of open economy macro models. For instance, the reduced-form utility function in Example 1 is

\[
V^i_{m^i}(m^i, M^i) = \sum_t \beta^t u(y^i_t + m^i_t)
\]

satisfying the above marginal utility conditions since

\[
V^i_{m^i,t} = \beta^t u'(c^i_t) > 0
\]

∀\( t \).

4.3 External Allocations

Representative Agent  Given the reduced-form utility \( V^i (m^i, M^i) \), an international price vector \( Q \), a vector of tax instruments \( \tau^i \) on external transactions, transfer \( T^i \) and aggregate external allocation \( M^i \), the second-step optimization problem of a representative agent in country \( i \) is

\[
\max_{m^i} V^i (m^i, M^i) \quad \text{s.t. } (15) \quad (22)
\]

Assigning the scalar shadow price \( \lambda^i_e \) to the external budget constraint (15), the associated optimality condition is

\[
(1 - \tau^i)^T V^i_m = \lambda^i_e Q^T
\]

where the tax vector \((1 - \tau^i)\) pre-multiplies the column vector \( V^i_m \) in an element-by-element fashion.

The solution to problem (22) defines a reduced-form import demand function \( m^i (Q, \tau^i, T^i, M^i) \) of the representative agent. Furthermore, substituting the consistency requirement \( m^i = M^i \) and the government budget constraint \( T^i = \frac{\tau^i Q}{1 - \tau^i} \cdot M^i \), the import demand function of the representative agent defines an aggregate import demand function \( M^i (Q, \tau^i) \), which is given by the fixed point \( M^i = m^i \left( Q, \tau^i, \frac{\tau^i Q}{1 - \tau^i} \cdot M^i, M^i \right) \).

External Policymaker  Since the planner has a complete set of external policy instruments under Condition 2, the implementability constraint (23) is slack, and we can directly solve for the planner’s optimal allocation. For a given reduced-form utility function \( V^i (m^i, M^i) \), the planner solves

\[
\max_{M^i} V^i (M^i, M^i) \quad \text{s.t. } Q \cdot M^i \leq 0
\]

Assigning shadow price \( \Lambda^i_e \) to the planner’s external budget constraint, the optimality condition is

\[
V^i_{m^i} + V^i_M = \Lambda^i_e Q^T
\]

Lemma 2 (Implementation). (i) The planner can implement her optimal external allocation by setting

\[
\tau^i = -\left( \frac{V^i_M}{V^i_m} \right)^T
\]

22
where the division \( V_M^i / V_m^i \) is performed element-by-element at the optimal allocation.

(ii) There is also a continuum of alternative implementations, in which the policy instruments \( (26) \) are rescaled by a positive constant \( k^i > 0 \) s.t. \((1 - \tilde{\tau}^i) = k^i \left(1 - \tau^i\right)\).

Proof. For part (i), substituting the optimal \( \tau^i \) from \( (26) \) into the optimality condition of private agents \( (23) \) yields the planner’s optimality condition \( (25) \).

For part (ii), the rescaling of \( \tau^i \) leaves external budget constraint unaffected since tax revenue is rebated lump-sum. It proportionately rescales the shadow price \( \Lambda^i_\varepsilon \) in the optimality condition \( (25) \) by \( 1/k^i \) without affecting the real allocation of the economy.

The first part of the lemma defines a function \( \tau^i (Q) \) that implements the optimal external allocation for given world prices \( Q \). According to this implementation, the planner does not intervene in time periods/states of nature/goods for which \( V_M^i = 0 \), i.e. for which private agents fully internalize the social marginal benefit of imports. By contrast, if there is an uninternalized social benefit or cost \( V_M^i \gtrless 0 \), then \( \tau^i \leq 0 \), so the planner subsidizes or taxes inflows of \( m^i_t \). Furthermore, the planner’s optimal policy \( \tau^i (Q) \) defines a reduced-form aggregate import demand function \( M^i (Q) = M^i (Q, \tau^i (Q)) \).

Part (ii) of the lemma observes that the incentive of private agents to shift consumption across time/states of nature/goods only depends on the relative price of goods. Multiplying all after-tax prices by a constant and changing initial net worth by the corresponding amount is equivalent to changing the numeraire.

Conversely, part (ii) of the lemma also implies that no policy intervention is necessary if the private and social marginal benefit of import goods are proportional for all goods, i.e. if \( V_M^i = h^i V_m^i \) for some scalar \( h^i \in (-1, \infty) \). In that case, setting \( \tilde{\tau}^i = 0 \) will implement the same allocation as \( \tau^i = -\left(V_M^i / V_m^i\right)^T \), as can be verified by setting \( k^i = \frac{1}{1+h^i} \) in the lemma. To rule out degenerate cases, we will assume that \( h^i \) that satisfies \( V_M^i = h^i V_m^i \) when we speak of a country that exhibits externalities in the following.

### 4.4 Welfare Properties of Equilibrium

We now turn to the welfare properties of the described global equilibrium.

**Theorem 1** (Efficiency of Global Equilibrium). (i) Under Conditions 1 to 3, the global competitive equilibrium allocation is Pareto efficient.

(Efficient Spillovers). (ii) By implication, any spillovers that arise from optimal domestic and external policy interventions constitute efficient pecuniary externalities.

Proof. A formal proof is provided in appendix A.1.

Intuitively, the theorem is a version of the first welfare theorem, with two modifications. First, it is applied to an environment in which there are two layers of actors – private agents and a policymaker in each country. Secondly, it only applies to the external allocations of each country – there can be any number of domestic market imperfections or targeting problems, and the external allocations are still Pareto efficient under the conditions of the theorem.

The role of the three conditions is as follows. Condition 2 implies that the policymaker has sufficient instruments to freely choose the external allocation of the economy – otherwise, the private
agents in the economy may choose inefficient allocations that leave room for Pareto improvements because they neglect domestic externalities, as shown in Section 7. Conditions 1 and 3 capture the typical requirement for the first welfare theorem that agents act as price-takers and trade in a complete market.\footnote{Given that the planner in each country internalizes all domestic externalities, the excess demand \( M^i \) of each country correctly reflects the country’s social marginal valuation of international transactions. The social marginal rates of substitution of all traded goods are equated across countries, and the resulting equilibrium is Pareto efficient.} Part (ii) of the theorem follows naturally: any time policymakers engage in domestic or external policy intervention, global prices \( Q \) and quantities \( (M^i)_{i \in I} \) will adjust. These general equilibrium effects – or spillovers, as they are called in the policy debate – are the natural mechanism by which the world economy re-equilibrates. However, an immediate implication of point (i) is that such spillovers are Pareto efficient. They constitute pecuniary externalities that are mediated by a complete market for \( M^i \). As such, they do generate redistributions between countries, but do not impinge on Pareto efficiency.

This insight may explain why the political debate about international policy spillovers and global cooperation is at times so vexing – spillovers generate winners and losers but, after they have taken place, there is no scope for Pareto improvements. Attempts at global cooperation are then a zero-sum game.

**Tatonnement and Arms Race** The equilibrium adjustment process (tatonnement) may sometimes involve dynamics that look like an arms race, even though Theorem 1 applies and the spillovers and resulting allocation are Pareto efficient:

**Example 3** (Tatonnement and Arms Race). Consider a world economy in which countries engage in intertemporal trade, similar to the example in sections 2.1 and 2.2. Assume a set of emerging economies \( \tilde{I} \subset I \) with measure \( \mu(\tilde{I}) > 0 \) experiences negative externalities from capital inflows in a given time period \( t \) that increase in a convex fashion. Formally, this is captured by reduced-form welfare functions that satisfy \( V_{M^i} < 0 \) and \( V_{MM^i} < 0 \) for \( i \in \tilde{I} \). Assume an exogenous shock that increases the supply of capital flows in period \( t \) from the rest of the world \( I \setminus \tilde{I} \). This leads to greater capital flows to the emerging economies and greater externalities from capital flows. Each of the affected countries will optimally increase capital controls, but this deflects some of the capital flows to the rest of the world economy – including the other emerging economies. In response to these deflected capital flows, each of the emerging economies finds it optimal to raise capital controls even more, leading to further deflection and so forth, until a new equilibrium with greater intervention in the emerging economies and a lower world interest rate is reached.\footnote{Giordani et al. (2014) provide careful evidence for such capital flow deflection dynamics and the resulting potential for an arms race of capital account intervention.} Such dynamics may give the appearance of an arms race, but they represent the natural mechanism through which the world economy re-equilibrates. They are Pareto efficient under the conditions of Theorem 1.
4.5 Policy Cooperation and Pareto Improvements

Achieving Pareto improvements rather than merely Pareto-efficient allocations generally requires international policy cooperation. Theorem 1 emphasized that domestic and external policy interventions and the resulting competitive allocations and spillovers are Pareto efficient, even though they may involve considerable redistributions between countries. This subsection describes two mechanisms by which cooperation among policymakers can undo the redistributions inherent in spillovers so as to generate Pareto improvements.

The first mechanism involves lump-sum transfers, which can trivially undo the wealth redistributions that arise from spillovers and achieve a Pareto-superior allocation. To describe this mechanism, let us start from a global competitive allocation that satisfies Proposition 1 in which policymakers employ the instruments \((\tau_i)_{i \in I}\) to internalize the externalities \(V^i_M\) of external transactions, as described in Lemma 2. Now assume that there is a perturbation in a set of countries \(\tilde{I}\) with positive mass \(\omega(\tilde{I}) > 0\) that leads to a change in the externalities from external transactions \(\tilde{V}^i_M \neq V^i_M\) and thus calls for a change in policy instruments to \((\tilde{\tau}_i)_{i \in \tilde{I}}\) according to the lemma.

**Proposition 1** (Pareto-Improving Intervention with Transfers). *Starting from an initial global competitive allocation \((X^i, M^i)_{i \in I}\) with external policy instruments \((\tau_i)_{i \in I}\) and prices \(Q\), a global planner who identifies domestic externalities \(\tilde{V}^i_M / V^i_m \neq \tau^i\) can achieve a Pareto improvement by setting the interventions \(\tilde{\tau}^i = -\tilde{V}^i_M / V^i_m \forall i\) and providing compensatory international transfers \(\tilde{T}^i\) across countries that satisfy \(\int_{i \in \tilde{I}} \tilde{T}^i d\omega (i) = 0\).*

**Proof.** Denote the net imports and world prices in the global planner’s equilibrium resulting from \((\tilde{\tau}^i)\) and transfers \((\tilde{T}^i)\) by \((\tilde{M}^i)\) and \(\tilde{Q}\). Assume the planner provides cross-border transfers

\[
\tilde{T}^i = \tilde{Q} \cdot (M^i - \tilde{M}^i)
\]

These transfers satisfy \(\int_{i \in \tilde{I}} \tilde{T}^i d\omega (i) = 0\) since both allocations clear markets. Furthermore, given the transfers, private agents in each country \(i\) can still afford the initial allocation. Since \(\tau^i \neq \tilde{\tau}^i \forall i \in \tilde{I}\), the global planner’s allocation differs from the initial allocation. Given that the initial allocation is still feasible for each country but is not chosen, revealed preference implies that every country is better off under the new allocation.

Explicit compensatory transfers across sovereign nations may be difficult to implement in practice. However, since countries interact with each other along a multitude of dimensions in today’s globalized world, political horse-trading that amounts to implicit transfers is widespread.

As a second mechanism to generate Pareto improvements, a global planner who can coordinate external policy instruments may correct the externalities of individual economies while holding world prices constant so that no wealth effects arise. As a result, the planner’s intervention generates a global Pareto improvement at a first-order approximation.

A practical example of this type of cooperation is when exporting countries impose voluntary export restraints, which meets the policy objectives of importing countries that want to reduce the quantity of their imports, but at the same time avoids the adverse price effect that exporters would
experience if importers imposed tariffs.21

The following lemma demonstrates how a global planner can manipulate world prices by simultaneously adjusting the instruments in all countries worldwide; building on the lemma, we will then show how this mechanism can be used to hold world prices fixed so as to avoid redistributions when engaging in optimal policy interventions.

**Lemma 3.** Consider a global competitive equilibrium with an external allocation \((M^i_j)_j \in \mathcal{I}\), external policy instruments \((\tau^i_j)_j \in \mathcal{I}\) and world prices \(Q\). A global planner can change world prices by \(dQ\) while keeping the external allocations of all countries constant by adjusting the policy instruments in each country \(j \in \mathcal{I}\) by

\[
\left( d\tau^i_j \right)^T = -\left( M^i_\tau \right)^{-1} M^i_Q (dQ)^T
\]

**Proof.** We set the total differential of the net import demand function \(M^i_j (Q, \tau^i_j)\) of each country \(j\) with respect to world prices and policy instruments to zero,

\[
dM^i_j = M^i_Q (dQ)^T + M^i_\tau (d\tau^i_j)^T = 0
\]

and rearrange to obtain equation (27). \(\square\)

In the following proposition, we assume an exogenous change \(dV^i_m\) in the externalities of a subset of identical countries \(i \in \tilde{\mathcal{I}} \subset \mathcal{I}\) with mass \(\tilde{\omega} = \omega (\tilde{\mathcal{I}})\). If these countries did not respond to the change, their welfare would be affected by \(dV^i_m \cdot M^i\). If they respond by unilaterally adjusting their external policy instruments \(d\tau^i_j = -dV^i_m / V^i_m > 0\) per Lemma 2, world market prices \(Q\) would change. Some countries would gain whereas others would lose from the resulting redistribution. The change in world prices and the redistribution can be avoided using the following policy:

**Proposition 2 (Pareto-Improving Intervention, No Transfers).** Assume an exogenous marginal increase in the externalities of a subset of countries \(\tilde{\mathcal{I}}\) that calls for an adjustment \(d\tau^i\) in their optimal unilateral taxes. A global planner can correct for the increase in externalities while keeping world prices constant \(dQ = 0\) to avoid wealth effects by adjusting \(\forall j \in \mathcal{I}\)

\[
\left( d\tau^i \right)^T = \begin{cases} 
I - \tilde{\omega} \left( M^i_\tau \right)^{-1} M^i_Q (M_Q)^{-1} M^i_\tau (d\tau^i)^T & \text{if } j \in \tilde{\mathcal{I}} \\
-\tilde{\omega} \left( M^i_\tau \right)^{-1} M^i_Q (M_Q)^{-1} M^i_\tau (d\tau^i)^T & \text{if } j \notin \tilde{\mathcal{I}}
\end{cases}
\]

where we define \(M_Q \equiv \int_{i \in \mathcal{I}} M^i_Q d\mu (i)\) as the slope of the world excess demand function. In the resulting equilibrium, net imports \((M^i_j)_j \in \mathcal{I}\) are marginally altered but world prices are unchanged. By the envelope theorem, welfare is unchanged at a first-order approximation.

**Proof.** If the domestic planner implemented the unilaterally optimal changes \(d\tau^i\) in the set of countries \(\tilde{\mathcal{I}}\), then world prices would move by \((dQ)^T = -\tilde{\omega} (M_Q)^{-1} M^i_\tau (d\tau^i)^T\). According to Lemma 3, the move in world prices can be undone if the taxes of all countries \(j \in \mathcal{I}\) are simultaneously

21For example, voluntary export restraints were employed heavily in textile trade, leading to the multilateral Multi-Fiber Agreement in the early 1970s and the Agreement on Textiles and Clothing in 1994 (see e.g. Suranovic, 2016). They were also imposed on automobile exports by Japan in the early 1980s (see e.g. Feenstra, 1984).
adjusted by \(- \left( M^j_i \right)^{-1} M^j_i (dQ)^T \), which delivers the equation for \( j \notin \tilde{I} \). For the countries affected by the shock \( j \in \tilde{I} \), we need to add the optimal unilateral change in intervention \( d\tau^i \) to this adjustment. In the resulting equilibrium, the change in the externality \( d\tau^i \) is accounted for but world market prices are unchanged at a first-order approximation. Furthermore, by the envelope theorem, the change in welfare that results from the equilibrium adjustment in quantities \( dM^j_i \) is zero,

\[
dV^j_i \bigg|_{dQ=0} = \left( V^j_m + V^j_M \right)^T \cdot dM^j_i = 0 \tag{22}
\]

Intuitively, the global planner performs a coordinated shift in all demand and supply curves in source and destination countries that changes the equilibrium quantities of the externality-generating flows while keeping world prices constant. The following example describes this in a simplified setting:

**Example 4 (Pareto-Improving Intervention, Symmetric Countries).** Consider a world economy that consists of a unit mass \( I \) of identical economies as described in Example 1. Assume a set of countries \( \tilde{I} \) with mass \( \tilde{\omega} = \omega (\tilde{I}) \) experiences a marginal increase in an externality that calls for a change \( d\tau^i \) in their optimal unilateral external policy instruments. Since the economies are identical, \( M^j_i (M^j_i)^{-1} = (M^j_i)^{-1} M^j_i = I \forall i, j \) and the formulas in Proposition 2 simplify drastically. A Pareto improvement at a first-order approximation can be achieved by setting \( \forall i \in I \)

\[
d\tilde{\tau}^i = \begin{cases} (1 - \tilde{\omega}) d\tau^i & \text{if } i \in \tilde{I} \\ \tilde{\omega} d\tau^i & \text{if } i / \in \tilde{I} \end{cases}
\]

The planner would share the burden of policy intervention between countries in \( \tilde{I} \) and the rest-of-the-world according to the relative size of the two blocks. The larger the set of countries \( \tilde{I} \), the greater the impact of their policy interventions on world prices, and therefore the more of the intervention the planner would shift to other countries so as to keep world prices \( Q \) constant and avoid redistributions. Conversely, a small open economy \( i \) with measure zero \( \omega^i = 0 \) has no impact on world prices and does not create spillovers. Therefore the above formula implies that there is no need to share the burden of policy intervention with other countries.

5 Monopolistic Behavior

5.1 Optimization Problem

We drop Condition 1 and analyze countries that exert market power to demonstrate how the resulting Pareto inefficiency creates scope for coordination. In the current section, we continue to assume complete external instruments. In subsection 5.3 we consider a situation when a country has both monopoly power and incomplete external instruments, i.e. both Conditions 1 and 2 are violated.

\[\text{For non-infinitesimal changes in } \tau^i, \text{ changes in net imports } \Delta M^j_i \text{ have second-order effects on welfare (i.e. effects that are negligible for infinitesimal changes but growing in the square of } \Delta M^j_i \text{) even if world prices are held constant. Under certain conditions, e.g. if there are only two types of countries in the world economy, a global planner can undo these second-order effects via further adjustments in the world prices } Q.\]
Consider a country $i$ with positive measure $\omega^i > 0$ in the world economy. Assume that the country’s policymaker internalizes her pricing power over the world market price $Q$. Specifically, the country’s policymaker internalizes how the optimal policies of the policymakers $\tau^j(Q)$ and the optimal allocations of private agents $M^j(Q, \tau^j(Q))$ in the remaining set of countries $I^{-i} = I \setminus \{i\}$ depend on the world market price $Q$.

This gives rise to a rest-of-the-world excess demand function

$$M^{-i}(Q) = \int_{j \in I^{-i}} M^j(Q, \tau^j(Q)) \, d\omega(j)$$

which can be inverted to obtain an inverse rest-of-the-world excess demand function $Q^{-i}(M^{-i})$. Since global market clearing requires $\omega^i M^i + M^{-i}(Q) = 0$, the country $i$ planner internalizes that her external allocations $M^i$ imply world prices $Q = Q^{-i}(-\omega^i M^i)$, and she solves the optimization problem

$$\max_{M^i} V^i(M^i, M^i) \quad \text{s.t.} \quad Q^{-i}(-\omega^i M^i) \cdot M^i \leq 0 \quad (28)$$

The optimality conditions are

$$V^i_m + V^i_M = \Lambda^i Q^T \left(1 - \mathcal{E}^i_{Q,M}\right) \quad \text{with} \quad \mathcal{E}^i_{Q,M} = \omega^i Q^{-i} M^i / Q^T \quad (29)$$

where the column vector $\mathcal{E}^i_{Q,M}$ represents the inverse demand elasticity of imports of the rest of the world and consists of four elements: the country weight $\omega^i$ reflects the country’s market power in the world market; the Jacobian square matrix $Q^{-i}_M = \partial Q^{-i}/\partial M^{-i}$ captures how much world market prices respond to absorb an additional unit of exports from country $i$. The column vector $M^i$ post-multiplies this matrix to sum up the marginal revenue accruing to country $i$ from the different goods as a result of monopolistically distorting each good. Finally, the result is normalized element-by-element by the price vector $Q$ to obtain elasticities.

Intuitively, a monopolistic planner equates the social marginal benefit of imports $V^i_m + V^i_M$ to the marginal expenditure $Q^T \left(1 - \mathcal{E}^i_{Q,M}\right)$ rather than to the world price $Q^T$ times a factor of proportionality $\Lambda^i$. The planner introduces a distortion to shift world prices in her favor and extract monopoly rents from the rest of the global economy. The intervention constitutes a classic inefficient beggar-thy-neighbor policy.

**Proposition 3 (Optimal Monopolistic Intervention).** (i) If Condition $T$ is violated for a given country $i$, the country’s policymaker optimally exerts market power by her external policy instruments to

$$1 - \tau^i = \frac{1 + V^i_M / V^i_m}{1 - \mathcal{E}^i_{Q,M}} \quad (30)$$

where all divisions are performed element-by-element.

---

23It is immaterial for our analysis whether the optimal policy $\tau^j(Q)$ in a given country $j$ in the rest of the world arises from price-taking behavior as described in Section 4 or monopolistic behavior as described in the current section. The only important assumption is that the excess demand functions $M^j(Q, \tau^j(Q))$ are well-defined and continuously differentiable. This rules out that policymakers respond directly to each other’s external instruments $\tau^j$, for example using discontinuous trigger strategies. Our assumption is common in the analysis of international policy cooperation (see e.g. Bagwell and Staiger 2002).
(ii) The policymaker does not distort its domestic policies to exert monopoly power.

(iii) An equilibrium in which a domestic planner in a country with $\omega^i > 0$ exerts market power is Pareto-inefficient and creates scope for policy cooperation.

Proof. For part (i), the tax vector $\hat{\tau}_i^t$ ensures that the private optimality condition of consumers (23) replicates the monopolistic planner’s Euler equation (29). For part (ii) observe that Condition 2 ensures that external considerations such as market power do not enter the optimality conditions of the policymaker for domestic policies $X_i$. Part (iii) follows immediately from Proposition 1.

To provide some intuition for part (i), consider an economy with positive measure $\omega^i > 0$ in the world economy that has, for simplicity, no domestic externalities so $V^i_M = 0$. Assume that the off-diagonal elements of the matrix $Q^{-i}_M$ are small compared to the diagonal elements, i.e. that income effects are small in comparison to substitution effects, and consider a good indexed by $t$ that is ordinary so $\partial M^{-i}_t / \partial Q_t < 0$, i.e. the rest of the world imports less if the price of good $t$ goes up. If country $i$ is a net importer $M^i_t > 0$ of the good, then the elasticity $\mathcal{E}^i_{Q,M,t}$ is negative and the optimal monopolistic tax on imports $\hat{\tau}^i_t > 0$ is positive and reduces imports. Similarly, if country $i$ is a net exporter $M^i_t < 0$, then the optimal monopolistic tax $\hat{\tau}^i_t < 0$ reduces the country’s exports. This captures the standard trade-reducing effects of monopolistic interventions.

The intuition for part (ii) is closely related to the optimal targeting principle established by Bhagwati and Ramaswami (1963): if the goal of a policymaker is to distort international prices, then she uses the instruments that affect international prices in the most direct way possible, and those are the external instruments $\tau_i$. It is undesirable to introduce distortions in domestic optimality conditions, given that the planner can affect external allocations directly. Conversely, when external instruments are not available, for example because they have been restricted by international agreements on trade or financial flows or “single markets,” then a second-best way of exerting market power is to distort domestic instruments. This explains why international trade agreements increasingly include provisions on domestic policies, as we discuss in more detail in subsection 5.3. Some of the literature on fiscal or monetary policy cooperation assumes implicitly that countries do not have the instruments to target external transactions directly and proceeds to study how countries distort fiscal or monetary instruments to internalize terms-of-trade effects as a second-best device. Although this may be an appropriate assumption in many cases, we believe that is useful to be explicit about it.

Part (iii) is a natural corollary and is a well-known implication of monopolistic behavior. When one or several players in an economic system exert market power, the resulting Pareto inefficiency creates scope for cooperation, as described in further detail in Propositions 1 and 2.

5.2 Identifying Monopolistic Behavior

The effects on Pareto efficiency and thus the scope for global cooperation depend crucially on whether policymakers use their external instruments to correct for domestic distortions or to exert market power. Unfortunately there is no general recipe for distinguishing between the two motives for intervention. In fact, the following result is a straightforward corollary of proposition 3:

Corollary 2 (Observational Equivalence of Corrective and Monopolistic Intervention). The external spillover effects of a given policy intervention $\tau_i$ in a country $i$ are equivalent no matter if the policy is
imposed to correct domestic market distortions or to exert market power.

Proof. Consider two economies $i$, $j$ of identical size $\omega^i = \omega^j > 0$ and with the identical excess demand functions $M^i(Q, \tau^i)$. Assume that economy $i$ exhibits $V^i_m \neq 0$ and sets its external policy instruments $\tau^i$ purely to correct domestic market imperfections according to lemma 2 without exerting market power. Assume that economy $j$ has no externalities $V^j_m = 0$ and sets its external policy instruments $\tau^j$ purely to exert market power as described in proposition 3. If $1 + V^j_m/V^i_m = 1/\left(1 - E^j_i\right)$, then the two economies will employ identical policy interventions which will create identical spillovers.

The corollary captures that the spillover effects of an intervention in external allocations are the same, no matter what the motive for intervention. It is easy for policymakers to invoke market imperfections, domestic objectives or different political preferences to justify an arbitrary set of policy interventions in the name of domestic efficiency, and it is difficult for the international community to disprove them.24

Monopolistic Behavior and Direction of Intervention However, the direction of the optimal monopolistic policy intervention is often instructive to determine whether it is plausible that a given intervention is for monopolistic reasons. If an observed intervention is inconsistent with this optimal direction, it is unlikely that it was conducted for monopolistic reasons. For this purpose, let us discuss the components of the elasticity term $E^i_{QM} = -\omega^i Q^i M^i / Q^T$:

Country Size $\omega^i$ The optimal monopolistic intervention is directly proportional to the country’s measure $\omega^i$ in the world economy, since this reflects the country’s impact on the world market. If a small open economy with $\omega^i \approx 0$ and undifferentiated exports engages in external intervention, it is unlikely to be driven by monopolistic motives.

Responsiveness of World Price $Q^i$ Monopolistic intervention requires that world market prices are responsive to changes in quantities. If there are, for example, close substitutes to the goods traded by a country so $Q^i \approx 0$, it is unlikely that the country’s intervention is monopolistic.

Direction and Magnitude of Flows $M^i$ The optimal monopolistic intervention on a good indexed by date $t$, good $k$, state of nature $s$ is directly proportional to the magnitude of the country’s net imports. The larger $M^i_{t, k, s}$ in absolute value, the greater the revenue benefits from distorting the price $Q_{t, k, s}$. By contrast, if $M^i_{t, k, s} \approx 0$, the optimal monopolistic intervention is zero. The economic interpretation depends on the specific setting: In intertemporal trade, $Q_t$ reflects the interest rate and $M^i_t$ captures date $t$ net capital inflows or, equivalently, the date $t$ trade balance. When $M^i_t \approx 0$, it is impossible for country $i$ to distort the intertemporal price. By contrast, a large net capital inflow, or trade deficit, $M^i_t > 0$ invites monopolistic inflow taxes $\tau^i_t > 0$ to keep world interest rates

24More specifically, for any reduced-form utility function $V^i(m^i, M^i)$ and intervention $\tau^i$, a planner can claim an alternative reduced-form utility function $\hat{V}^i(m^i, M^i)$ such that $\tau^i$ implements the optimal competitive planner allocation under that utility function,

$$\hat{V}^i(m^i, M^i) = V^i(m^i, M^i) - \hat{\pi}^i \cdot \left( V^i_n M^i \right)$$

The reduced-form utility function $\hat{V}^i(\cdot)$ can in turn be interpreted as deriving from a fundamental utility function $\hat{U}^i(\cdot)$ and a set of constraints $\hat{\pi}^i(\cdot)$ that justify it.
lower, or vice versa for a country with a large net outflow/trade surplus $M^i_t < 0$. In a stochastic setting, $Q_{t,s}$ reflects the state price of payoffs in state $s$ at date $t$. Each country has – by definition – monopoly power over its own idiosyncratic risk. Optimal risk-sharing implies inflows (imports) in bad states and exports in good states of nature. Monopolistic intervention restricts risk-sharing so as to obtain a higher price for the country’s idiosyncratic risk and to reduce the price of insurance from abroad. If a country encourages insurance (e.g. by encouraging FDI and forbidding foreign currency debt; see [Korinek, 2010]), then the motive is unlikely to be monopolistic. In trade policy, it is well-known that monopolistic intervention implies tariffs $\tau^i_{t,k} > 0$ on imported goods $k$ with $M^i_{t,k} > 0$ and taxes on exports $\tau^i_{t,k} < 0$ for $M^i_{t,k} < 0$ (see e.g. [Bagwell and Staiger, 2002]).

5.3 Monopolistic Use of Domestic Policy Instruments

Proposition (ii) demonstrated that a policymaker with complete instruments will only use external instruments $\tau^i$ not domestic policies to exert monopoly power. This subsection assumes a policymaker who has both market power (violating condition [1]) and incomplete instruments (violating condition [2]) and shows that this makes it optimal to distort domestic policies in pursuit of monopolistic objectives. Restrictions on external policy instruments are of increasing practical relevance since many international agreements have swept away countries’ ability to intervene in private trade or capital flows.

Intuitively, a planner who aims to exert market power but does not have complete external instruments proceeds in two steps, which reflect that the separation results of lemma [1] no longer hold. First, she analyzes in which direction she would like to distort the external transactions of the economy to optimally exert market power. Secondly, she internalizes how each domestic instrument of hers will affect the external transactions of the economy, and she distorts the domestic instruments accordingly. We formally consider an example in which we assume a planner has no external instruments whatsoever in appendix A.3.

**Example 5 (Monopoly Power and Fiscal Policy, With and Without External Instruments).** We continue the Example 2.4 on fiscal policy but consider a policymaker who takes into account monopolistic considerations. For simplicity, we set $\alpha = 1$.

First, we consider a policymaker who has complete external instruments $\tau^i$ and maximizes utility (9) subject to the intertemporal budget constraint (28). In that case, the optimality condition that determines the mix of private and public spending (10) is unchanged from that of a competitive policymaker, $u^i(G^i_t) = u^i(G^i_0)$ or $C^i_t = G^i_t$ for $t = 0, 1$, consistent with proposition (ii). However, the monopolistic planner will impose taxes (30) on the external transactions of private agents

$1 - \tau^i = \frac{1}{1 - \mathcal{E}^i_{Q,M}}$ \(\text{where } \mathcal{E}^i_{Q,M} = \left( \frac{\partial Q_1}{\partial Q_1} / \partial Q_1 / \partial M^i_0 \right) \cdot M^i_1 / Q_1 \)

where $Q_1 = 1 / R$. Consider w.l.o.g. a borrowing country that imports $t = 0$ goods and exports $t = 1$ goods so $M^i_0 > 0 > M^i_1$. In that case, we find $(\mathcal{E}^i_{Q,M})_0 > 0 > (\mathcal{E}^i_{Q,M})_1$ and therefore $\tau^i_0 < 0 < \tau^i_1$, i.e. the country

\^[25\] If private capital accounts are closed, optimal monopolistic intervention consists of reduced/increased foreign reserve accumulation. For example, when policymakers reduce reserve accumulation because they are concerned that they are pushing down the world interest rate too much, this is classic non-competitive behavior and is equivalent to monopolistic capital controls. This corresponds to statements by some Chinese policymakers that were concerned about pushing down US Treasury yields because of their reserve accumulation in 2014.
both taxes $t = 0$ imports and taxes $t = 1$ exports in order to push up the relative price $Q_1$ of its exports. In the intertemporal interpretation of the model, this amounts to pushing down the interest rate at which it borrows.

In the absence of external instruments (i.e. if we restrict $\tau^i \equiv 0$), a monopolistic policymaker distorts government spending to pursue her monopolistic objective in a less efficient manner. Specifically, the policymaker maximizes utility subject to the intertemporal budget constraint and the implementability constraint given by the private Euler equation $Q_1 u'(C^i_0) = u'(C^i_1)$ or $C^i_0 = Q_1 C^i_1$ under log-utility, to which we assign shadow price $\mu$. The planner’s optimality conditions can be combined to yield

$$u'(C^i_0) = u'(G^i_0) - \mu$$
$$u'(C^i_1) = u'(G^i_1) + \mu Q_1$$

The shadow price satisfies $\mu > 0$ and implies that the planner now distorts the optimal mix between private and public spending to further her monopolistic objectives. In particular, $t = 0$ government spending is reduced compared to private spending, $G^i_0 < C^i_0$, and vice versa, $G^i_1 > C^i_1$, for $t = 1$ spending. Both interventions serve to push up the relative price $Q_1$ or, in the intertemporal interpretation, to push down the world interst rate $R = 1/Q_1$ at which the country borrows.

Since the planner’s optimization problems with and without external instruments are identical except for the additional implementability constraint in the latter case, which is binding, welfare in the latter case is strictly inferior.

6 Imperfect External Policy Instruments

This section considers how violations of Condition 2 (Complete External Policy Instruments) lead to Pareto inefficiency and create scope for policy coordination. There is only one way in which a country’s set of external policy instruments can be classified as perfect – captured in Condition 2 – but a myriad of ways in which external policy instruments can be imperfect. These include situations when some of the external instruments $\tau^i$ are costly to implement, restricted, missing, or cannot be fine-tuned, as well as instances when setting external instruments involves fiscal considerations or time consistency problems.

This section develops a representative model of incomplete policy instruments that captures implementation costs, missing instruments and coarse policy instruments. We discuss circumstances under which the set of external policy instruments is effectively complete, even if there are limitations on some instruments. Then we demonstrate how a global planner can generically achieve Pareto improvements when some countries face imperfect external policy instruments.

6.1 A Model of Imperfect External Instruments

We capture imperfections in the set of instruments of a policymaker in country $i$ by assuming that there is a function $\Gamma^i(\tau^i)$ that describes the cost of imposing the vector of policy instruments $\tau^i$.

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26Let us remind the reader that it is only incomplete external policy instruments (i.e. those targeting the external transactions $M^i$) that are relevant for Condition 2 and that justify global coordination; incomplete or imperfect domestic policy instruments (i.e. those targeting domestic variables $X^i$) do not violate Condition 2 and do not create inefficiencies that can be addressed via global coordination, as stated in our main Theorem 1.
The function $\Gamma^i$ is non-negative, convex, and satisfies $\Gamma^i(0) = 0$. For simplicity, we assume that the cost represents a utility cost so it enters the planner’s objective as $\max_{X^i, M^i, \tau^i} U^i(X^i) - \Gamma^i(\tau^i)$ subject to the usual constraints.\footnote{Our results are largely unchanged if we assume that the cost is a resource cost that is subtracted from the external budget constraint $Q \cdot M^i + \Gamma^i(\tau^i) \leq 0$, except that the function $\Gamma^i(\cdot)$ is pre-multiplied by the shadow price on the external budget constraint $A^i$. Similar results can be derived if the cost of policy intervention depends on quantities transacted, e.g. $\Gamma^i(\tau^i, M^i)$, which may capture for example the costs associated with attempts at circumvention.}

**Perfect External Instruments** Formally, a policymaker in country $i$ has a perfect set of external instruments if $\Gamma^i \equiv 0$, i.e. if the policymaker can impose any arbitrary policy instruments $\tau^i$ and thus implement any feasible external allocation without incurring implementation costs. In the baseline setup of section \footnote{Our setup could also capture further types of imperfections in instruments such as asymmetries between taxes and subsidies on external transactions, or instruments that are fixed at non-zero levels.} the set of policy instruments is perfect by construction.

**Imperfect External Instruments** Conversely, a policymaker in country $i$ has an imperfect set of external instruments if some instruments are costly to impose, i.e. $\exists \tau^i$ s.t. $\Gamma^i(\tau^i) > 0$. This includes the case in which the cost of a particular configuration of instruments is infinite. By implication, there may be some feasible external allocations that the planner cannot implement costlessly as a decentralized allocation. Our setup captures several different types of imperfections in instruments:

- **Costly Instruments** The most straightforward interpretation of the specification is that it is costly to impose policy instruments. If we index the elements of vector $\tau^i$ by the letter $t$, a cost function of the simple quadratic form $\Gamma^i(\tau^i) = \sum_t \gamma^i_t (\tau^i_t)^2/2$ may capture implementation costs that arise from policy intervention and that grow in the square of the intervention. The cost may vary across different elements of the vector $\tau^i$ by adjusting $\gamma^i_t$, for example by setting $\gamma^i_t = \beta^i_t \phi^i$. More generally, implementation costs could also be asymmetric for taxes and subsidies.

- **Missing or Restricted Instruments** In the limit case $\gamma^i_t \to \infty$ for some $t$, the cost function captures that instrument $\tau^i_t$ is not available. For $\gamma^i_t \to \infty \forall t$, country $i$ has no external policy instruments. Similarly, if $\Gamma^i(\tau^i) = \sum_t \gamma^i_t (\tau^i_t - \tau^i_{\bar{t}})^2/2$ then $\gamma^i_t \to \infty$ captures that instrument $t$ is restricted to $\tau^i_{\bar{t}}$.

- **Coarse Instruments** If we index the vector $\tau^i$ along several dimensions, for example to capture different goods $k = 1..K$ or different states of nature $s \in \Omega$ in each time period $t$, a cost function of the form $\Gamma^i(\tau^i) = \sum_t \sum_{k=2}^K \gamma^i_t \left(\tau^i_{tk} - \tau^i_{t1}\right)^2/2$ or $\Gamma^i(\tau^i) = \sum_{s \in \Omega} \gamma^i_t (\tau^i_{ts} - \tau^i_{t0})^2/2$ captures that is is costly for the planner to differentiate her policy instruments across different goods or across different types of state-contingent financial flows, e.g. flows with different risk profile. In the limit case $\gamma^i \to \infty$, the function captures that the planner is completely unable to differentiate the instruments and has to set $\tau^i_{tk} = \tau^i_{t1} \forall k$ or similarly $\forall s \in \Omega$\footnote{Our results are largely unchanged if we assume that the cost is a resource cost that is subtracted from the external budget constraint $Q \cdot M^i + \Gamma^i(\tau^i) \leq 0$, except that the function $\Gamma^i(\cdot)$ is pre-multiplied by the shadow price on the external budget constraint $A^i$. Similar results can be derived if the cost of policy intervention depends on quantities transacted, e.g. $\Gamma^i(\tau^i, M^i)$, which may capture for example the costs associated with attempts at circumvention.}.

**Effectively Perfect External Instruments and Efficiency** Sometimes a policymaker with an incomplete set of instruments has nonetheless all the instruments she wants to use. We say that the policymaker’s external instruments are effectively perfect if the cost of implementing the planner’s optimal policy is zero, $\Gamma^i(\tau^i) = 0$. If a policymaker’s has a perfect set of external instruments, it is...
also effectively perfect, but not vice versa. In many instances this allows us to weaken Condition 2 (Perfect External Instruments) to

**Condition 2’ (Effectively Perfect External Instruments).** The policymaker in country \( i \in I \) possesses an effectively perfect set of external policy instruments.

It is easy to see that the efficiency result of Theorem 1 still holds under the weaker Condition 2’.

The case \( V^i_M = 0 \) of no externalities is a trivial example in which the set of instruments of the policymaker in country \( i \) is effectively perfect, even if she has no instruments whatsoever. In that case, imperfections in actual policy instruments are irrelevant since the efficient allocation requires zero intervention.

**Effectively Perfect at the Global Level**  
Even if some countries have instruments that are *not* effectively perfect, it is possible that coordination can restore a situation in which the imperfections do not matter. We call the worldwide set of external policy instruments of all countries \((\tau^i)_{i \in I}\) **effectively perfect at the global level** if a global planner can implement a Pareto efficient equilibrium in which \( \Gamma^i (\tau^i) = 0 \forall i \in I \). The intuition is that a planner may be able to fix the externalities of one country with imperfect instruments by using the instruments of another country with perfect instruments – what matters for efficiency is that the social rates of substitution between different goods are equated, not how this is accomplished.

**Example 6 (Effectively Perfect Instruments at the Global Level).** Consider a world economy with two countries \( I = \{1, 2\} \) that are described by the reduced-form utility functions \( V^i (m^i, M^i) \). Assume that country 1 suffers from externalities to inflows of some goods so \( V^1_M < 0 \) but does not have any policy instruments to correct for them. Furthermore, assume that country 2 does not suffer from externalities \( V^2_M = 0 \) but has a complete set of external instruments.

If country 1 had the instruments to do so, it would impose optimal taxes \( \tau^1 = - (V^1_M / V^1_m)^T \), and the resulting equilibrium would satisfy \( (1 - \tau^1) V^1_m / \lambda^1_e = Q = V^2_m / \lambda^2_e \). However, since the country has no policy instruments, the global competitive equilibrium coincides with the laissez-faire equilibrium – country 2 has no incentive to intervene in its external transactions.

Under cooperation, country 2 would set its policy instrument \( 1 - \tau^2 = 1 - V^2_m / \lambda^2_e \), i.e. it would tax outflows to correct for the negative externalities of inflows in country 1, and the allocation would satisfy \( (1 - \tau^1) V^1_m / \lambda^1_e = Q = V^1_m / \lambda^1_e \) or \( (1 - \tau^1) V^1_m / \lambda^1_e = (1 - \tau^1) V^2_m / \lambda^2_e \), also equating the social marginal rates of substitution between the two countries. Aside from a change in world market prices, the equilibrium is unchanged from the one under optimal taxes in country 1.

**6.2 Uncoordinated Policy with Imperfect External Instruments**

In the following, we describe a version of our baseline model with imperfect instruments but in which, for simplicity, we assume away domestic policies so that policymakers only need to focus on external allocations. We maintain conditions (1) (No Market Power) and (3) (Complete International Markets). A planner who has no domestic choice variables maximizes the reduced-form utility function \( V^i (m^i, M^i) \) of her country net of implementation costs and subject to the
external budget and implementability constraints

$$\max_{M^i,\tau^i,\lambda^i} V^i(M^i, M^i) - \Gamma^i(\tau^i) \quad \text{s.t.} \quad Q \cdot M^i \leq 0, V^i_m = \lambda^i \left( \frac{Q}{1 - \tau^i} \right)^T$$

A detailed description of the optimization problem is given in appendix (A.4).

We denote the vector of shadow prices on the implementability constraint by $\mu^i$. These shadow prices capture the extent of mis-targeting for each element of $M^i$. We rewrite the planner’s optimality condition $FOC(M^i)$ as

$$\left( \mu^i \right)^T = \left( V_{mm}^i + V_{mM}^i \right)^{-1} \left[ V^i_m + V^i_M - \Lambda^i Q^T \right]$$

The expression in square brackets reflects the social benefit $V^i_m + V^i_M$ minus the social cost of net imports $\Lambda^i Q$. If the planner had perfect instruments, she would equate the two and the expression would be zero for each element of the vector of external transactions. Under imperfect instruments, the marginal net benefits of imports in the square brackets is positive for all goods for which greater net imports are desirable and vice versa. The expression is pre-multiplied by the negative semi-definite matrix $\left( V_{mm}^i + V_{mM}^i \right)^{-1}$, which captures the curvature of the $V^i$, i.e. how much the marginal social benefit changes in the quantity of net imports. As a result, $\mu^i$ reflects how much a marginal unit of net imports improves the mis-targeting problem. $\mu^i$ is negative for those elements of the import vector $M^i$ that are less than optimal and positive for those that are more than optimal.

The planner’s optimality condition $FOC(\tau^i)$ can be written as

$$\left( 1 - \tau^i \right) \Gamma^i(\tau^i) \cdot \left( 1 - \tau^i \right)^T = 0$$

where all multiplications are performed element-by-element.\(^{29}\) When there are excessive flows of a good $k$ so $\mu^i_{e,k} > 0$, the planner imposes a positive tax $\tau^i_k > 0$ that leads to a positive marginal cost $\Gamma^i_k > 0$; conversely, if flows of a good are insufficient $\mu^i_{e,k} < 0$, the planner imposes a subsidy.

The following proposition characterizes how the policymaker in an individual country sets costly instruments:

**Proposition 4** (National Planner with Costly Instruments). At the uncoordinated optimum, (i) the weighted average marginal cost of a national planner’s instruments is zero,

$$\Gamma^i(\tau^i) \cdot \left( 1 - \tau^i \right)^T = 0$$

(ii) and the national planner sets

$$1 - \tau^i = \frac{\Lambda^i}{\lambda^i} + \frac{\Gamma^i}{\left( V^i_M \right)^T} \left[ V_{mm}^i + V_{mM}^i \right]$$

\(^{29}\)In the limit case of fully missing, restricted or coarse instruments, which we captured by $\gamma_i \to \infty$ for quadratic cost functions in section 6.1, the derivatives $\Gamma^i_k(\cdot)$ need to be replaced by the more general concept of subgradients, which are set-valued, $\Gamma^i_k(\cdot) = (-\infty, \infty)$. Intuitively, this captures that a function with kinks does not have a unique derivative but admits tangents of many different slopes. Given this concept, the relevant value for $\Gamma^i_k(\cdot)$ is determined by the restriction on the instruments, and the described optimality condition still holds.
Proof. See appendix (A.4).

Point (i) of the proposition reflects that it is optimal for the planner to impose a combination of taxes and subsidies such that the average marginal cost is zero. Given the constraints on her instruments, the planner chooses her allocations such that some flows are too low and others too high compared to the complete instruments case. This minimizes the total implementation cost. Suppose, for example, that there is one good \( k \) that creates a negative externality. Given a convex cost of intervention, it would be inefficient to tax this good and not intervene in the markets for other goods since private agents only care about *relative* after-tax prices of goods to guide their allocations. Under costly instruments, an optimizing planner would instead reduce the tax on good \( k \) and impose a small subsidy on all other goods to minimize total intervention costs.

To interpret the tax formula in point (ii), observe first that the expression reduces to equation (26) if the denominator is one. This would reflect optimal policy under complete instruments. Under costly instruments, the marginal cost term \( \Gamma^i(\tau) \) in the denominator, weighted by the curvature of the reduced-form utility function, increases or reduces the level of policy intervention to account for the costs. Furthermore, the ratio of the social to the private shadow value of wealth \( \Lambda^i e / \lambda^i e \) in the denominator adjusts the average level of controls to ensure that point (i) is satisfied. For example, if externalities \( V^i M \) are on average positive, then the planner’s marginal valuation \( \Lambda^i e \) of will be above \( \lambda^i e \) and vice versa.

We illustrate the weighted average marginal cost criterion in the following example:

**Example 7 (Costly Instruments).** Consider a country \( i \) in which inflows in period 2 due to a linear learning externality so \( y^i_1(M^i_0) = y^i_0 - \eta^i M^i_0 \). Assume there is no discounting, that world prices satisfy \( Q = (1, 1) \) and that external policy instruments impose a quadratic utility cost of implementation \( \Gamma^i(\tau^i) = \gamma^i \tau^i \cdot \tau^i T / 2 \). The utility of country \( i \) is

\[
V^i \left( m^i, M^i \right) - \Gamma^i \left( \tau^i \right) = u \left( y^i_0 + m^i_0 \right) + u \left( y^i_0 - \eta^i M^i_0 + m^i_1 \right) - \Gamma^i \left( \tau^i \right)
\]  

(34)

Let us start from an equilibrium in which \( \eta^i = 0 \) and assume a small increase \( d\eta^i > 0 \) in the externality. Given the costly instruments, the planner will set \( \tau^i_0 = d\eta^i / 2 = -d\tau^i_1 \) such that condition (32) is satisfied. The planner taxes the externality-generating inflows (or, equivalently, subsidizes outflows) in period 0 but subsidizes inflows (and taxes outflows) in period 1. The planner internalizes that higher \( m^i_1 \) implies lower \( m^i_0 \) by the external budget constraint, and that she can correct the externality while saving on implementation costs by spreading her intervention across both periods.

6.3 Global Coordination under Imperfect External Instruments

A global planner who faces the same restrictions on instruments will maximize the weighted sum of welfare

\[
\max_{Q(\mathbf{M}, \mathbf{\tau}, \mathbf{\Lambda}^i) \in \mathcal{I}} \int_{i \in I} \left\{ \theta^i \left[ V^i \left( M^i, M^i \right) - \Gamma^i \left( \tau^i \right) \right] - \mu^i \cdot \left[ V^i_m - \lambda^i e Q \left( \tau^i \right) \right]^T - \nu \cdot M^i \right\} d\omega (i)
\]

(30)

To see how point (ii) of proposition maps into our example, observe that \( \Gamma^i = 0 \) and that \( \Lambda^i e / \lambda^i e = 1 - d\eta^i / 2 \) since the negative learning effects reduce the value of external wealth.
for a given set of welfare weights \((\theta^i)_{i \in I}\). The extent of mis-targeting is now captured by

\[
\left(\mu_c^i\right)^T = \left(V_{mm}^i + V_{mM}^i\right)^{-1} \left[ \theta^i (V_m^i + V_M^i) - v \right] - 1
\]

As in our analysis of national planners, a positive element in \(\mu_c^i\) means that the global planner would like more inflows of the respective good to country \(i\) and vice versa.

The main difference between the allocation of an individual planner in country \(i\) and the global planner is the following:

**Proposition 5 (Global Optimum with Costly Instruments).** A global planner sets to zero not only the weighted average marginal cost of instruments within each country, described in (32) \(\forall i \in I\), but also the weighted average marginal cost of instruments for each good across all countries,

\[
\int_{i \in I} \left[ \left(1 - \tau^i\right) \theta^i \Gamma^i (\tau^i) \right] d\omega (i) = 0 \quad (35)
\]

**Proof.** See appendix (A.4).

The argument for the first part of the proposition is the same as in the individual country case. However, unlike in the competitive allocation, a global planner also sets the average marginal distortion across all countries for each good equal to zero. If some countries impose taxes on inflows for certain elements of \(M^i\), others must impose taxes on outflows. In short, the planner spreads her intervention across inflow and outflow countries in proportion to their cost of intervention.

We illustrate our findings in the following two examples:

**Example 8 (Sharing the Regulatory Burden).** Consider a world economy consisting of two sets of atomistic economies \(I_1\) and \(I_2\) of equal measure that have the structure described in example 7 with utility functions given by (34) and cost functions \(\Gamma^i (\tau^i) = \frac{\gamma^i \tau^i}{2} (\tau^i)^T\) for \(i \in \{1, 2\}\). Assume an equilibrium with \(\eta^i = 0 \forall i \in \{1, 2\}\) and consider the effects of a small increase \(d\eta^1 > 0\) in the externalities in set \(I_1\). In the uncoordinated equilibrium, policymakers in set \(I_1\) behave as described in example 7, but the resulting allocation violates condition (35) since \(\tau_0^1 > 0 > \tau_1^1\) but \(\tau^2 \equiv 0\), indicating inefficiency.

A global planner would share the regulatory burden among both sets of countries to minimize the total cost of intervention. Specifically, the planner would set the policy instruments in accordance with the relative cost of intervention,

\[
\begin{align*}
\frac{\gamma^1 d\eta}{2 (\gamma^1 + \gamma^2)} &= \frac{\gamma^2 d\eta}{2 (\gamma^1 + \gamma^2)} = -d\tau_1^1 \quad \text{and} \quad -d\tau_2^1 = \frac{\gamma^1 d\eta}{2 (\gamma^1 + \gamma^2)} = -d\tau_0^1
\end{align*}
\]

This guarantees that the sum of interventions equals the increase in the externality \(d\eta\) and that both optimality conditions (32) and (35) are satisfied. If the cost of intervention is equal among the two sets of countries, then the respective fractions are 1/4, i.e. the planner corrects one quarter of the externality in each time period in each country to implement a constrained efficient equilibrium.

For given \(\gamma^1 > 0\), we can analyze two interesting limit cases: first, if it becomes cost-less to impose external policy instruments in set \(I_2\) (\(\gamma^2 \rightarrow 0\)), then the planner would only intervene in those countries and

\[\text{Ghosh et al. (2014) provide empirical evidence for the practical feasibility of burden-sharing between inflow and outflow countries.}\]
fully correct the externality there, leaving $\tau^1 = 0$, as in Example 6. Conversely, if it becomes prohibitively costly to employ policy instruments in set $I_2$ ($\gamma^2 \to \infty$), then the planner would only intervene in the country in $I_1$ and leave $\tau^2 = 0$.

Example 9 (Wasteful Competitive Intervention). Assume a world economy that consists of a set of identical atomistic countries $I$ with reduced-form utility functions $V^i(m^i, M^i)$ that all suffer from externalities $V^i_M \neq 0$ and from implementation costs $\Gamma^i(\tau^i) = \gamma \tau^i \cdot \tau^i T / 2$. Following proposition 4, the national planners in all countries impose the same non-zero policy instruments $\tau^i \neq 0$ and incur the same costs $\Gamma^i(\tau^i) > 0$. This clearly violates the optimality condition (35) since the policy instruments have the same sign.

A global planner who puts equal weight on the countries recognizes that the competitive interventions in all countries are wasteful – since the countries are identical, there is not trade $M^i = 0 \forall i$ and all countries could save the cost $\Gamma^i(\tau^i)$ without changing global allocations by coordinating to reduce their policy instruments to zero. Technically, the global competitive equilibrium violates condition (35) since the policy instruments of all countries have the same sign. The only way the planner can satisfy this optimality condition is to move all controls to zero. Observe that the set of instruments is effectively complete at the global level even though all $N$ countries have incomplete/imperfect instruments.

Our examples illustrate that two important rationales for global cooperation are to save on implementation costs by (i) sharing the regulatory burden in the most cost-effective way possible and (ii) avoiding wasteful competition.

Scope for Pareto Improvement One caveat to the cooperative agreements described in this section is that Pareto improvements generally requires lump-sum transfers. Sharing the regulatory burden generally involves a shift in world market prices that may create winners and losers. However, there are two special cases under which cooperation along the lines described above does generate Pareto-improvements: (i) if there is no trade, as in our example 9 on wasteful competition, so that changes in terms-of-trade have no redistributive effects, and (ii) if the savings from lowering the cost of intervention $\Gamma^i(\tau^i)$ are large enough to offset terms-of-trade losses in all countries that experience such losses.

6.4 Domestic Policy under Imperfect External Instruments

We now return to the full setup of our baseline model with domestic policy $X^i$ in order to study the effects of incomplete external policy instruments on domestic allocations.

Lemma 4 (Non-Separability). (i) If a domestic planner faces a set of external policy instruments that is not effectively complete, she will generically distort her domestic policy choices $X^i$ as a second-best device to target external transactions. (ii) If the set of policy instruments in the world economy is not effectively perfect at the global level, a global planner will coordinate the use of both external and domestic policies to achieve a superior global allocation.

Proof. For (i), we add the utility cost $-\Gamma^i(\tau^i)$ to the optimization problem of national planners in appendix A.1 and observe that $\mu^e_i \neq 0$ if the planner’s set of external policy instruments is
not effectively complete; therefore the optimality condition $FOC(X^i)$ of a domestic planner is generically affected (except in knife-edge cases when $\partial f^i_m/\partial X^i = 0$ happens to hold, i.e. when domestic policy has no effect whatsoever on external transactions).

For (ii) we proceed in the same manner, and the global planner faces the same optimality conditions $FOC(X^i)$. The shadow prices $\mu^i_e$ on the global planner’s external implementability constraints are generically non-zero if external policy instruments are not perfect at the global level. Therefore the global planner internalizes how domestic choices affect external transactions.

Intuitively, the planners in both cases consider not only domestic objectives in setting the domestic policy instruments $X^i$ but also how their choices will improve external allocations, which they can only target indirectly given the imperfect external instruments. Suppose inflows of good $k$ are excessive ($\mu^i_{e,k} > 0$). If a domestic choice variable $X^i_h$ is complementary to $m^i_k$, then the planner will reduce $X^i_h$ to bring down $m^i_k$, and vice versa for substitutes.

7 International Markets Imperfections

This section discusses how global cooperation can improve outcomes if there are imperfections in international markets, which violate Condition 3 (Perfect International Markets). We already noted in the introduction, borrowing from Leo Tolstoy’s quote on unhappy families, that each imperfect market is imperfect in its own way, i.e. that there is a myriad of different forms of market imperfections. Like in the section on imperfect instruments, we must therefore limit our focus on specific instances of international market imperfections.

We consider a setting that nests the two main imperfections that are considered by the literature in this area: market incompleteness and price stickiness in the international arena. We first show that if each country has a full set of external policy instruments $\tau^i$, cooperation to deal with these two imperfections will be limited to adjusting the external policy instruments of countries, and there is no need to coordinate domestic policy. Then we provide several examples of imperfections in international markets and how cooperation can improve outcomes.

7.1 A Simple Model of International Market Imperfections

To capture international market incompleteness and price stickiness in a general way, we borrow from the closed-economy setting of Farhi and Werning (2016) and impose a set of constraints on international transactions and prices of the form

$$\Phi\left(\left(m^i\right)_{i\in I}, Q\right) \leq 0 \quad (36)$$

To provide a few of examples for this constraint, suppose the market for a good $k$ is missing, this corresponds to a constraint $m^i_k = 0 \forall i$. If risk markets in period $t$ are absent, this is captured by a constraint $m^i_{t,s} = m^i_{t,s'}$ for any two states $s,s'$ in period $t$. Price stickiness can be captured by constraints of the form $Q^i = \tilde{Q}$, and so on.

We could further generalize constraint (36) by assuming that different countries face different prices in external markets as is the case e.g. under local currency pricing. An important assumption inherent in (36) is that the constraint does not directly depend on domestic variables. Nonetheless, the constraint covers a wide range of models of international market imperfections.
We add this constraint to our baseline model and observe:

**Lemma 5** (Separability under Imperfect International Markets). A global planner will only coordinate the use of external policy instruments $\tau^i$ not domestic instruments $X^i$, as long as Condition 2 (Perfect External Instruments) is satisfied.

*Proof.* We add the constraint (36) to the planner’s optimization problem in Appendix A.1 and observe that the constraint only affects the optimality conditions for $M^i$ and $Q$, not those for domestic policies. Owing to Condition 2, the implementability constraint on external transactions is slack so $\mu^i_e = 0$, and the optimality conditions for the domestic allocation $X^i$ are unaffected by international market incompleteness. \hfill \square

Intuitively, the result captures again the separability between domestic and external choices under perfect external instruments. Conversely, if external instruments were imperfect, implementability constraints are generically binding so $\mu^i_e \neq 0$ and the optimal domestic allocation is distorted to further external objectives.

Given perfect external instruments and the separability result, the Lagrangian of the global planner is

$$L = \max_{Q, (M^i)_{i \in I}} \int \left\{ \theta^i \left[ V \left( M^i, M^i \right) \right] - \nu M^i \right\} d\omega (i) - \phi \Phi \left( \left\{ M^i \right\}_{i \in I}, Q \right)$$

with optimality conditions

$$\text{FOC} \left( Q \right) : \quad \phi \Phi Q = 0 \quad (37)$$

$$\text{FOC} \left( M^i \right) : \quad \theta^i \left( V_m^i + V_M^i \right) = \nu + \phi \Phi M^i \quad (38)$$

Observe that the price vector $Q$ does not play an allocative role in this setting. This immediately leads to the following result:

**Proposition 6** (Freedom in Setting International Prices and Resolving Imperfections). If $\text{rank} \Phi Q \geq \text{dim} \Phi$ then a global planner sets prices to fully resolve the market imperfections captured by constraint (36), including problems associated with international price stickiness and pecuniary externalities from prices in constraints.

*Proof.* The condition in the proposition ensures that first-order condition (37) has a unique solution for the vector of shadow prices $\phi$, which is the degenerate solution $\phi = 0$. This indicates that the vector constraint (36) is not binding at the optimum. \hfill \square

Intuitively, the rank condition ensures that all the constraints in the vector constraint (36) are responsive to price changes, and that the planner has sufficiently many price instruments to meet all the constraints, rendering them irrelevant. Since prices play no role in the actual allocation of resources [cf. optimality condition (A.23)], the planner then simply sets the instruments $\{ \tau^i \}_{i \in I}$ so as to implement the optimal real allocation that would prevail in the absence of the international market imperfections.

When the rank condition in the proposition is violated, then the constraints (36) will generically distort the planner’s optimal allocation in international markets. Examples include instances of
non-existing or incomplete markets, e.g. constraints of the form $m_{ik} = 0$ for some $k$, or $m_{it,s} = m_{it,s'}$ for some $t, s \neq s'$. In these examples, the market imperfections are independent of prices so rank $\Phi_Q = 0$.

To illustrate the scope for global coordination according to Proposition 6, we consider two more specific settings.

7.2 Price Stickiness in International Markets

To consider price stickiness in international markets, we introduce two deviations from our baseline setup: First, we include restrictions on $Q$ in constraint (36) to directly reflect the price stickiness. For example, when some prices are perfectly rigid, this is captured by $Q_k = \bar{Q}_k$ for some $k$. More generally, we could assume constraints capturing Phillips curve-type behavior of prices that link changes in prices to quantities demanded. Secondly, when prices cannot fulfill their usual role in clearing markets, we must assume an alternative clearing mechanism. Following the tradition of the New Keynesian literature, we assume that for any good that exhibits sticky prices, producers satisfy demand by adjusting their output in equal proportion to ensure that $\int_{i \in \tilde{I}} M_i d\omega(i) = 0$.

Example 10 (Rigid Prices). We consider a world economy that is made up of countries as described in section 2.1 with reduced-form utility function $V^i(m', M') = u(y^i_0 + m^i_0) + u(y^i_1 + m^i_1)$. We assume the vector of world market prices is rigid at $Q = \bar{Q} = (1,1)$ and that market non-clearing is reflected in demand-determined period 0 output being different from potential output, $\bar{y}^i_0 \neq \bar{y}^i_0 = \arg \max_y y - d^i(y)$ where $d^i(\cdot)$ reflects a convex disutility of production in a GHH-style utility function.

In an uncoordinated equilibrium, the national planner of each country set $V^i_m = \Lambda^i_j \bar{Q}$ or, equivalently, $u'(c^i_0) = u'(c^i_1)$, i.e. total consumption is constant across periods for all countries. Output in the set of countries $\tilde{I}$ adjusts so that $\int_{i \in \tilde{I}} \bar{y}^i_0 d\omega(i) = \int_{i \in \tilde{I}} y^i_1 d\omega(i)$. Under global coordination, a planner simply sets the tax instruments of all countries so as to emulate what would be the equilibrium price in an unrestricted market.

7.3 Pecuniary Externalities from Prices in Constraints

Another type of international market imperfection that relates to Proposition 6 occurs when international transactions are subject to constraints that depend on market prices, giving rise to pecuniary externalities.

7.4 Classic Global Externalities

Our first example is a classic global externalities problem, which arises for example from emissions of greenhouse gases and ozone-depleting substances, from over-use of antimicrobials that fosters antimicrobial resistance, from the destruction of biodiversity, or from nuclear proliferation.

We follow Arrow (1969) in representing externalities as missing markets for goods that capture the external effects of different countries on each other. Assuming there are $K$ different externalities, we expand our definition of the set of international goods by the vector $\tilde{M} = (\tilde{M}_{jk})_{j \in \tilde{I}, k=1...K}$,
which has size \( \dim \mathcal{I} \times K \), i.e. we add one good for each country and externality.\(^{33}\) We then denote by \( \hat{M}_{ik}^i \) with \( i \neq j \) the externality imposed by country \( i \) on country \( j \) in category \( k \), and by \( \hat{M}_{ik}^i \) the total “net imports” of the externality experienced by country \( i \). Market clearing implies that \( \omega_i^i \hat{M}_{ik}^i + \int_{j \in \mathcal{I} \setminus \{i\}} \hat{M}_{jk}^i d\omega(i) = 0 \forall i, k \), i.e. that the total externality experienced by country \( i \) consists of the sum of what is created by the other countries \( \mathcal{I} \setminus \{i\} \). We can then define the complete vector of international transactions of each country by \( M^i = (\tilde{M}^i, \hat{M}^i) \) where \( \tilde{M}^i \) represents the usual external transactions in privately traded goods, as in the preceding sections, and similar for \( Q = (\hat{Q}, \hat{Q}) \).

**Uncoordinated Equilibrium** In the uncoordinated equilibrium, individual countries rationally do not internalize the externalities that they generate. Mapping this into our general framework, this corresponds to the following constraints on international trade, which nest into constraint (36) as follows:

\[
\hat{Q} = 0 \quad \text{and} \quad \omega_i^i \hat{m}_{ik}^i = - \int_{j \in \mathcal{I} \setminus \{i\}} \hat{M}_{jk}^i d\omega(i) \forall i, k
\] (39)

In other words, externalities are unpriced in the uncoordinated equilibrium, and inflows into country \( i \) correspond to the externalities generated by all other countries \( j \in \mathcal{I} \setminus \{i\} \).

Assuming that Conditions 1 (Competitive Behavior) and 2 (Perfect Instruments) hold for the privately traded goods \( \tilde{M}^i \), the optimality conditions of private agents and national planners pin down external transactions in private goods so

\[
\frac{V^i_{\tilde{M}} + V^i_{\tilde{M}}}{\Lambda^i} = \hat{Q} = \frac{V^j_{\hat{M}} + V^j_{\hat{M}}}{\Lambda^j} \forall i, j \in \mathcal{I}
\]

In words, for privately traded goods, marginal rates of substitution are equated for all countries. However, the constraints (39) imply that the externalities remain unpriced and are therefore generically at inefficient levels. When we imposed the constraints (39), observe that we implicitly assumed that Condition 2 (Perfect External Instruments) was violated for trade in externalities – if a legal framework existed such that each country could levy a tax on the externalities that it imports, then efficiency would be restored, as emphasized by Coase (1960). We will further explore the role of imperfections in instruments when market prices are restricted in section tk below.

**Global Optimum** The goal of policy cooperation in the described setting is to internalize the externalities, which can be accomplished in the following way:

**Proposition 7** (Internalizing Global Externalities). *In the described setting, a global planner does not need to intervene in markets for privately traded goods, but imposes taxes on global externalities to ensure that*

\[
\theta^i V^i_M = \theta^j V^j_M \forall i, j \in \mathcal{I}
\]

\(^{33}\)For simplicity, we limit our attention to the case in which the externalities experienced by a country only depend on the total global amount of an externality-generating activity. As described in Arrow (1969), it is easy to relax this and make externalities specific to both the generating and receiving country by introducing \( \dim \mathcal{I}^2 \times K \) goods to capture externalities.
In particular, the planner’s optimality condition also includes the goods that we introduced to captures externalities so \( \theta^i V^i_M = \theta^j V^j_M \).

We illustrate this principle in the following example:

**Example 11 (Environmental Externalities).** Suppose a set of countries \( I \) that each consume a good \( y^i \) which they value according to a CES utility function \( u(c^i) = (c^i)^{1-\sigma}/(1-\sigma) \). The production of one unit of the good creates one unit of private disutility and increases global pollution by one unit, which imposes disutility \( \eta \) on each consumer worldwide. The utility function of the representative agent in country i is thus \( U^i(y^i) = u(y^i) - y^i - \eta \int_{j \in I} Y^j d\omega(j) \). We assume that the national policymaker in each country has a costless tax instrument to control domestic production. In this simple example, countries only affect each other via pollution externalities and there is no international trade. To map the example into our general framework, denote by \( \hat{M}^i_j = Y^i \forall j \neq i \) the pollution externalities that country i imposes on country j. This constraint is part of the constraint set \( f^i(\cdot) \leq 0 \). Furthermore, denote by \( \hat{M}^i_i \) the pollution imported by country i, which reduces utility by \( \eta \hat{M}^i_i \). The incompleteness in markets corresponding to the externality problem is captured by (39).

In the uncoordinated equilibrium, the country i policymaker will only internalize the externalities that country i agents impose on the country itself, resulting in the optimality condition \( u'(Y^i) = 1 + \eta \omega^i \) – the larger the country, the more of the pollution externalities it will internalize. There is overproduction of the pollution good since exports of pollution are unpriced, \( V^i_{M,j} = \hat{Q}_j = 0 \), even though each country suffers disutility \( V^i_{M,j} = -\eta \) from importing a marginal unit of pollution.

Global coordination internalizes these externalities and ensures that private agents equate the marginal benefit of consumption to the global social cost of production,

\[
u'(y^i) = 1 + \eta \]

This can be achieved either in Coasian fashion, by defining clear international property rights on the environment so that one unit of pollution trades at the efficient market price \( \eta \), or by taxing domestic production at rate \( \eta \).

## 8 Conclusions

This paper investigates under what conditions the coordination of national economic policies is desirable from a global welfare perspective. We develop a benchmark under which national policymaking delivers efficient outcomes and spillovers constitute pecuniary externalities that cancel out at the global level. This benchmark depends on three conditions: (i) national policymakers act competitively in the international market, (ii) they possess an effectively complete set of instruments to control external transactions and (iii) international markets are free of imperfections.

We develop an analogue of the first welfare theorem that holds at the level of national economic policymakers under these conditions.

We establish this benchmark result in a very general framework that nests a wide range of open economy macro models. We also provide a number of examples in which our efficiency results apply, counter to the intuition of many commentators and policymakers. Then we investigate each of the three conditions required for efficiency in more detail. We show how violations of the
conditions generally lead to inefficiency and provide some guidelines for how global cooperation among national policymakers can improve outcomes. Furthermore, we also discuss guidelines for how to ascertain whether the conditions for our benchmark result are satisfied, for example how to detect monopolistic behavior and how to identify symptoms of targeting problems.

An important research area is how to implement Pareto-improving cooperation in practice. Willett (1999) discusses a range of political economy problems that make international policy cooperation difficult. Bagwell and Staiger (2002), among others, provide an analysis of how to achieve agreements to abstain from monopolistic beggar-thy-neighbor policies if countries are sufficiently symmetric.
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A Mathematical Derivations of Main Results

A.1 Combined Optimization Problem [tx]

Proof of Lemma 1 (Separability) We derive the optimality conditions of the full optimization problem of private agents and a national planner under conditions (1) to (3) and show that they lead to identical optimality conditions as the two-step procedure described in sections 4.2 to 4.3. Since they are also subject to the same constraints, the two solution procedures are equivalent, proving the lemma.

The Lagrangian of the combined optimization problem of a private agent is

\[ w^i \left( m^i, x^i; M^i, X^i \right) = \max_{m^i, x^i} U^i \left( x^i \right) - \lambda_d^i \cdot f^i \left( m^i, x^i, M^i, X^i \right) - \lambda_e^i \left[ \frac{Q}{1 - \tau^i} \cdot m^i - T^i \right] \]

The optimality conditions are given by the vector equations

\[ \text{FOC} \left( x^i \right) : U^i_x = f^i_x T^i \lambda_d^i \]  \hspace{1cm} (A.1)

\[ \text{FOC} \left( m^i \right) : \lambda_d^i f^i_m + \lambda_e^i \frac{Q}{1 - \tau^i} = 0 \]  \hspace{1cm} (A.2)

These two conditions represent domestic and external implementability constraints on the national planner’s problem. By substituting the transfer \( T^i = \frac{r Q}{1 - \tau^i} \), the external budget constraint on private agents reduces to \( Q \cdot M^i \leq 0 \), and the optimization problem of the planner in country \( i \) is given by the Lagrangian

\[ \mathcal{L}^i = \max_{M^i, X^i, \tau^i, \lambda_d^i, \lambda_e^i} U^i \left( X^i \right) - \Lambda_d^i \cdot f^i \left( M^i, X^i, M^i, X^i \right) - \Lambda_e^i \left[ Q \cdot M^i \right] - \mu_d^i \cdot \left[ U^i_x - f^i_x T^i \lambda_d^i \right] - \mu_e^i \cdot \left[ \lambda_d^i f^i_m + \lambda_e^i \frac{Q}{1 - \tau^i} \right]^T \]

This yields the optimality conditions (with all multiplications and divisions in the FOC (\( \tau^i \)) calculated element-by-element)

\[ \text{FOC} \left( X^i \right) : U^i_x = \left( f^i_{xx} + f^i_{xM} \right) T^i \Lambda_d^i T + U^i_x T^i \mu_d^i T - \mu_d^i \left( f^i_{xx} + f^i_{xM} \right) \Lambda_d^i T + \mu_e^i \left( f^i_{mx} + f^i_{mX} \right) \Lambda_d^i T \]  \hspace{1cm} (A.7)

\[ \text{FOC} \left( M^i \right) : 0 = \left( f^i_m + f^i_M \right) T^i \Lambda_d^i T + \Lambda_e^i Q^T - \mu_d^i \left( f^i_{xm} + f^i_{xM} \right) \Lambda_d^i T + \mu_e^i \left( f^i_{mm} + f^i_{mM} \right) \Lambda_d^i T \]  \hspace{1cm} (A.8)

\[ \text{FOC} \left( \tau^i \right) : 0 = \mu_e^i \left[ \frac{\lambda_d^i Q}{(1 - \tau^i)^2} \right] \]  \hspace{1cm} (A.9)

\[ \text{FOC} \left( \lambda_d^i \right) : 0 = -\mu_d^i f^i_{xM} + \mu_e^i f^i_m T \]  \hspace{1cm} (A.10)

\[ \text{FOC} \left( \lambda_e^i \right) : 0 = \mu_e^i \left( \frac{Q}{1 - \tau^i} \right)^T \]  \hspace{1cm} (A.11)
where we denote by \( \mu^i_{j} f_{i}^{j} \lambda_i^j T \) the tensor product \(^{34}\)

\[
\mu^i_{j} f_{i}^{j} \lambda_i^j T := \frac{\partial \mu^i_{j} f_{i}^{j} \lambda_i^j T}{\partial x^i} = \sum_h \lambda_{d,h} f_{h,x} T \mu^i_{d} T = \left( \begin{array}{c}
\sum_{i=1}^{H} \sum_{h=1}^{a} \lambda_{d,h} f_{h,x} T \mu^i_{d} T \\
\vdots \\
\sum_{i=1}^{H} \sum_{h=1}^{a} \lambda_{d,h} f_{h,x} T \mu^i_{d} T
\end{array} \right)
\]

which has the same dimension as vector \( x^i \), and similar for the tensor products \( \mu^i_{j} f_{i}^{j} \lambda_i^j T \), \( \mu^i_{m} f_{i}^{m} \lambda_i^m T \), etc.

Given the complete set of external instruments \( \tau^i \), optimality condition FOC \((\tau^i)\) implies that the vector of shadow prices on the external implementability constraint satisfies \( \mu^i_{e} = 0 \) – the planner sets the vector \( \tau^i \) to whichever levels she wants without facing trade-offs. By implication, the last term in the other four optimality conditions drops out, allowing us to separate the problem into two blocks.

The optimality conditions \((A.7)\) and \((A.10)\) with \( \mu^i_{e} = 0 \) replicate the optimality conditions of the optimal domestic planning problem \((20)\) in section 4.2. Together with the domestic constraint \((A.3)\) and the domestic implementability condition \((A.5)\), these four conditions are identical to the four conditions that pin down the optimal domestic allocation for given \( M' \) in section 4.2 and yield identical solutions for the four domestic variables \( (x^i, \lambda_{d}, \lambda_{e}, \mu^i_{d}) \).

Given the envelope theorem, the optimality condition \((A.8)\) can equivalently be written as \( \partial L^i / \partial M^i = dV^i / dM^i - \Lambda^i_1 Q^T = 0 \), where the latter condition coincides with the optimality condition \((25)\) defining the optimal external allocation in section 4.3. The optimality conditions \((A.9)\) and \((A.11)\) for \( \mu^i_{e} = 0 \) capture that the planner can set the product \( \lambda_{e} (1 - \tau^i) \) such as to precisely meet the constraints \((A.6)\) where the planner has one scalar degree of freedom, as we also emphasized in section 4.3. The three optimality conditions together with the two constraints yield an identical set of solutions for the five external variables \( (M', \tau^i, \lambda_{d}, \lambda_{e}, \mu^i_{e}) \) as we described in section 4.3. This shows that the two-step procedure yields identical solutions as the combined optimization problem, proving the lemma.

**Proof of Theorem 1 (Efficiency of Global Equilibrium)**

An allocation is Pareto efficient if it maximizes the weighted sum of welfare of all countries for some set of country welfare weights \( \{\theta^i\}_{i \in I} \) subject to the global resource constraint as well as the domestic constraints \( f^i(\cdot) \) and the domestic implementability constraints \((A.1)\) for each country \( i \in I \). Given the complete set of external instruments and following the same arguments as in the proof of lemma \(4\) above, the planner can directly choose the external allocations \( \{M^i\}_{i \in I} \) and solve

\[
\max_{\{M^i, X^i\}_{i \in I}} \int_{i \in I} \theta^i U^i(X^i) d\omega(i) \quad \text{s.t.} \quad (A.1), f^i(M^i, M', X^i, X^i) \leq 0 \forall i \in I \quad \text{and} \quad \int_{i \in I} M^i d\omega(i) = 0
\]

By the definition of \( V^i(M', M^i) \), we can restate this problem in terms of reduced-form utilities of the optimal external allocations \( m^i = M^i \) as

\[
\max_{\{m^i\}_{i \in I}} \int_{i \in I} \theta^i V^i(M^i, M^i) d\omega(i) \quad \text{s.t.} \quad \int_{i \in I} M^i d\omega(i) = 0
\]

Assigning the shadow price \( v \) to the vector of resource constraints, the optimality conditions of the global planner are

\[
\theta^i (V^i_m + V^i_M) = v^T \forall i \in I
\]

Any global competitive equilibrium allocation also satisfies these optimality conditions if we use the shadow price on the global resource constraint \( v = Q \) and assign the welfare weights \( \theta^i = 1 / \Lambda^i_1 \), where \( \Lambda^i_1 \) are the national planners’ shadow prices on the external budget constraints. Therefore any competitive equilibrium allocation is Pareto efficient.

---

\(^{34}\)Since the function \( f^i(\cdot) : \mathbb{R}^{2K + 2L} \rightarrow \mathbb{R}^H \) is vector-valued, its second derivative with respect to, say, \( x^i \) is a three-dimensional tensor of dimensions \( \dim f^j x \dim x^i \times \dim x^i \).
A.2 Optimization Problem with Heterogeneous Agents

This appendix extends the benchmark model of section 3 to multiple types of heterogeneous agents and repeats the analysis of appendix A.1. We assume that each economy $i \in I$ contains a set $J^i$ of types of atomistic agents with total measure normalized to $\bar{w}^i (J^i) = 1$. We denote the functions and individual, and aggregate variables of each type $j \in J^i$ by the two superscripts $ij$, for example $U^i_{ij} (x^{ij})$, or $m^i_{ij}$ and $X^i_{ij}$.

The Lagrangian of the combined optimization problem of an agent $j \in J^i$ and the associated optimality conditions are

$$
\begin{align*}
L^i = \max_{\{M_i^i, X_i^i, \tau^i, \lambda^i_d, \lambda^i_e\}_{j \in J^i}} \int_{j \in J^i} \left\{ U^i_{ij} \left( X^i_{ij} \right) - \Lambda^i_d \cdot f^i \left( M^i_{ij}, X^i_{ij}; \{ M^i_{ih}, X^i_{ih} \}_{h \in J^i} \right) \right. \\
- \Lambda^i_e \left[ \frac{Q}{1 - \tau^i} \cdot m^i_{ij} - T^i \right] \\
- \mu^i_d \cdot \left[ U^i_{x} - f^i_{x} \lambda^i_d \right] \\
- \mu^i_d \cdot \left[ \lambda^i_{j} f^i_{m} + \lambda^i_e \frac{Q}{1 - \tau^i} \right] \}
\end{align*}
$$

A set of optimality conditions similar to equations (A.7) to (A.11) can be derived. Given a complete set of external instruments $\tau^i_{ij} \forall j \in J^i$, the shadow prices on the external implementability constraints will again satisfy $\mu^i_{ij} = 0 \forall j \in J^i$, delivering a separability result analogous to lemma 1. This makes it straightforward to extend theorem 50 to heterogeneous types of agents.

A.3 Optimal Monopolistic Use of Domestic Instruments (Section 5.3)

We formally consider the optimal monopolistic use of domestic instruments for the case in which a country’s policymaker has no external instruments at all ($\tau^i \equiv 0$) but has a full set of domestic instruments so there are no domestic targeting problems. This allows us to ignore the domestic optimization problem of private agents and let the planner directly choose the vector $X^i$ as part of her optimization problem

$$
\begin{align*}
\max_{M^i, X^i, \lambda^i_d} U^i (X^i) \quad \text{s.t.} \quad f^i (M^i, M^i, X^i, X^i) \leq 0, \\
Q^{-i} \left( -\omega^i M^i \right) \cdot M^i \leq 0, \\
\lambda^i_d f^i_m = -\lambda^i_e Q^{-i} \left( -\omega^i M^i \right)
\end{align*}
$$

The domestic constraint and external budget constraint in the first two lines are the usual ones, but the planner internalizes the effects of her external allocations on world market prices $Q^{-i} \left( -\omega^i M^i \right)$. The implementability constraints in the third line reflect that the planner lacks external policy instruments. We assign
the usual shadow prices $\Lambda_d^i$, $\Lambda_e^i$ and $\mu_e^i$ to these constraints and obtain the optimality conditions

$$\text{FOC} \left( M^i \right) : \quad 0 = \left( f_m^i + f_M^i \right)^T \Lambda_d^iT + \Lambda_e^iT \left( 1 - \omega^i \varepsilon^i_{Q,M} \right)$$

$$+ \mu_e^i \left( f_m^i + f_M^i \right)^T \lambda_d^iT - \omega^i \mu_e^i Q_{-i}^T \lambda_e^iT$$

$$\text{FOC} \left( \chi^i \right) : \quad U_X^i = \Lambda_d^i f_X^i + \mu_e^i \left( f_m^i + f_M^i \right) \lambda_d^iT$$

$$\text{FOC} \left( \lambda_e^i \right) : \quad 0 = \mu_e^i \cdot Q^T$$

The optimality condition on $\lambda_e^i$ implies that the price-weighted sum of shadow prices on the external implementability constraint (IC) is zero $\sum_k \mu_{e,k}^i Q_k = 0$, i.e. the planner chooses the allocation such that the cost of under- and overshooting on some of her individual targets cancels out. Some of the shadow prices on the external IC will therefore in general be positive and some will be negative. In particular, we can re-write the optimality condition on $M^i$ as

$$\mu_e^i = - A^{-1} \left[ \left( f_m^i + f_M^i \right)^T \Lambda_d^iT + \Lambda_e^i \left( 1 - \omega^i \varepsilon^i_{Q,M} \right) \lambda_d^iT - \omega^i Q_{-i}^T \lambda_e^iT \right]$$

where $A = \left( f_m^i + f_M^i \right)^T \lambda_d^iT - \omega^i Q_{-i}^T \lambda_e^iT$. To interpret this expression, consider an $A$ that is close to diagonal and assume that there are no domestic externalities so $f_M^i = 0$. In the absence of monopolistic behavior, the square brackets in this expression would be zero by the private optimality condition of consumers $\lambda_d^iT f_m^i + \lambda_e^i Q = 0$. By contrast, for a monopolistic planner, the vector of shadow prices $\mu_e^i$ captures the monopolistic cost (benefit) of increasing imports of each good and is therefore negative for net imports and positive for net exports.

The first-order condition $\text{FOC} \left( \chi^i \right)$ for the optimal domestic allocation and policy measures reflects that the planner equates the marginal benefit of consumption to the marginal cost $\Lambda_d^i f_X^i$ augmented by the monopolistic benefits or costs $\mu_e^i \left( f_m^i + f_M^i \right) \lambda_d^iT$. If a domestic consumption action increases net imports of a good that the planner wants to increase for monopolistic reasons $\left( \mu_{e,k}^i > 0 \right)$, the planner will consumer more of it, and vice versa.

### A.4 Optimal Use of Costly Instruments (Section 6)

#### Simplified National Planning Problem under Costly Instruments

The Lagrangian of the simplified optimization problem of a national planner under costly instruments described in section 6 is

$$\mathcal{L} = \max_{M^i, \tau^i, \lambda_e^i} V^i \left( M^i, M^i \right) - \Gamma^i \left( \tau^i \right) - \Lambda_e^i \left[ Q \cdot M^i \right] - \mu_e^i \left[ V_m^i - \lambda_e^i \frac{Q}{1 - \tau^i} \right]^T$$

and delivers the associated optimality conditions (with all multiplications and divisions in the $\text{FOC} \left( \tau^i \right)$ calculated element-by-element)

$$\text{FOC} \left( M^i \right) : \quad V_m^i + V_M^i = \Lambda_e^i Q^T + \left( V_m^i + V_M^i \right) \left( \mu_e^i \right)^T$$

(A.12)

$$\text{FOC} \left( \tau^i \right) : \quad \Gamma^i = \mu_e^i \left[ \frac{\lambda_e^i Q}{(1 - \tau^i)^2} \right]$$

(A.13)

$$\text{FOC} \left( \lambda_e^i \right) : \quad 0 = \mu_e^i \cdot \left[ \frac{Q}{1 - \tau^i} \right]^T$$

(A.14)

---

35The diagonal elements of the matrix are likely to be an order of magnitude larger than the off-diagonal elements as long as goods aren’t strong complements or substitutes.
From FOC \((M^i)\) we express the shadow prices on the implementability constraint as
\[
\left(\mu^i_e\right)^T = \left(V_{mm}^i + V_{mM}^i\right)^{-1} \left[\left(V_m^i + V_M^i\right) - \Lambda_e^i Q^T\right] \tag{A.15}
\]
We combine the domestic IC \(V_m^i = \lambda_e^i Q / (1 - \tau^i)\) with FOC \((\tau^i)\) to obtain
\[
\left(1 - \tau^i\right) \Gamma^i_T = \mu^i_e \left(V_m^i\right)^T \tag{A.16}
\]
with element-wise multiplication.

**Proof of Proposition 4** For point (i), we use FOC \((\tau^i)\) to obtain \(\mu^i_e = \left[\frac{(1 - \tau^i)^2}{\lambda_e^i Q}\right] \Gamma^i_T\), where all multiplications and divisions are element-wise, and substitute this into FOC \((\lambda^i_e)\). Dropping the constant \(\lambda^i_e\), we obtain the result \((1 - \tau^i) \cdot \Gamma^i_T = 0\).

For point (ii), we use the implementability constraint \((1 - \tau^i) V_m^i / \lambda_e^i = Q^T\) to substitute out \(Q^T\) from the planner’s FOC \((M^i)\). Furthermore, we use the FOC \((\tau^i)\) and the implementability constraint to substitute \(\mu^i_e = \left[\frac{(1 - \tau^i)^2}{\lambda_e^i Q}\right] \Gamma^i_T\). We obtain
\[
V_m^i + V_M^i = \Lambda_e^i / \lambda_e^i \left(1 - \tau^i\right) V_m^i + \left(V_{mm}^i + V_{mM}^i\right) \left[\left(1 - \tau^i\right) V_m^i\right]^{T} \tag{A.16}
\]
This is a vector equation of the size of \(M^i\). We divide element-by-element by \(V_m^i\) and rearrange terms to find
\[
1 + V_m^i / V_M^i = \left(1 - \tau^i\right) \left[\frac{\Lambda_e^i}{\lambda_e^i} + \left(V_{mm}^i + V_{mM}^i\right) \left(\Gamma^i_T\right)^T \left(V_m^i\right)^T\right] \tag{A.20}
\]
The tax formula in the proposition follows immediately.

**Global Planning Problem** The Lagrangian of a global planner in our setup with costly instruments is
\[
\mathcal{L} = \max_{Q, (M^i, \tau^i, \lambda^i_e) \in \mathcal{I}} \int_{\mathcal{I}} \left\{\theta^i \left[V^i (M^i, M^i) - \Gamma^i (\tau^i)\right] - \mu^i_e \cdot \left[V_m^i - \lambda_e^i Q / (1 - \tau^i)\right]^T - v M^i\right\} d\omega (i)
\]
The global planner’s optimality conditions are
\[
FOC (Q) : 0 = \int_{\mathcal{I}} \left\{\mu^i_e \left[\frac{\lambda_e^i}{1 - \tau^i}\right]\right\} d\omega (i) \tag{A.17}
\]
\[
FOC \left(M^i\right) : \theta^i \left(V_m^i + V_M^i\right) = \left(V_{mm}^i + V_{mM}^i\right) \left(\mu^i_e\right)^T + v \tag{A.18}
\]
\[
FOC \left(\tau^i\right) : \theta^i \Gamma^i_T = \mu^i_e \left[\frac{\lambda_e^i Q}{(1 - \tau^i)^2}\right] \tag{A.19}
\]
\[
FOC \left(\lambda_e^i\right) : 0 = \mu^i_e \cdot \left[\frac{Q}{1 - \tau^i}\right]^T \tag{A.20}
\]
The analogous expression to \(A.15\) is
\[
\left(\mu^i_e\right)^T = \left(V_{mm}^i + V_{mM}^i\right)^{-1} \left[\theta^i \left(V_m^i + V_M^i\right) - v\right]
\]
Furthermore, expression \(A.16\) can be derived along the same lines as above.
**Proof of Proposition 5** The proof follows along the same lines as the proof of proposition 4(i). For the second condition, we substitute \( \mu_i = \frac{\lambda_i}{(1 - \tau^i)} \) = \((1 - \tau^i) \theta^i \Gamma_i^\ell / Q_i \), with multiplications and divisions performed element-wise, into FOC \((Q)\) and multiply by the constant \(Q\) to obtain the result that \( \int_{i \in I} (1 - \tau^i) \theta^i \Gamma_i^\ell d\omega(i) \).

### A.5 Optimal Behavior in Model of Imperfect International Markets

**National Planning Problem under Costly Instruments** We set up the Lagrangian of a national planner facing imperfect markets as described in Section 7. Under Condition 2 (Complete External Instruments), we can formulate the problem in terms of choosing quantities to maximize reduced-form utilities. A national planner thus solves the Lagrangian

\[
\mathcal{L}^i = \max_{M^i} V^i(M^i) - \Lambda^i \left[ Q \cdot M^i \right] - \phi^i \Phi \left( \{M^j\}_{j \in I}, Q \right)
\]

and obtains the straightforward optimality condition

\[
\text{FOC} \left( M^i \right) : V^i_M + V^i_{M^i} = \Lambda^i Q^T + \phi^i \Phi^i M^i \tag{A.21}
\]

For any market in which the constraint binds, \( M^i_k \) is restricted and the relevant entry in the vector of shadow prices \( \phi^i_k \) picks up the social cost of the market imperfection.

**Proof of Proposition [no proposition here]**

**Global Planning Problem** The Lagrangian of a global planner in our model of international market imperfections is

\[
\mathcal{L} = \max_{Q, \{M^i\}_{i \in I}} \int_{i \in I} \left\{ \theta^i \left[ V \left( M^i, M^i \right) \right] - v M^i \right\} d\omega(i) - \phi^i \Phi \left( \{M^i\}_{i \in I}, Q \right)
\]

The global planner’s optimality conditions are

\[
\text{FOC} \left( Q \right) : \Phi Q d\omega(i) = 0 \tag{A.22}
\]
\[
\text{FOC} \left( M^i \right) : \theta^i \left( V^i_M + V^i_M \right) = v + \phi^i \Phi^i M^i \tag{A.23}
\]

The first optimality condition captures that international market prices \( Q \) do not play an allocative role for the global planner. Since the set of instruments \( \{\tau^i\}_{i \in I} \) is perfect at the international level, the global planner can choose \( Q \) so as to optimally satisfy the constraint \( \Phi(\cdot) \), and then set all the \( \tau^i \)'s in a manner that implements the desired allocation \( \{M^i\}_{i \in I} \) subject to the constraint \( \Phi(\cdot) \). We summarize the implications in the following corollary:

**Corollary 3 (Freedom in Setting International Prices).** A global planner under perfect external instruments can set prices without regard for allocative effects. This enables the global planner to resolve problems associated with (i) international price stickiness and (ii) pecuniary externalities from prices in binding constraints.

This implies that, under complete instruments,

**Proof of Proposition 5** The proof follows along the same lines as the proof of proposition 4(i). For the second condition, we substitute \( \mu_i = \frac{\lambda_i}{(1 - \tau^i)} \) = \((1 - \tau^i) \theta^i \Gamma_i^\ell / Q_i \), with multiplications and divisions performed element-wise, into FOC \((Q)\) and multiply by the constant \(Q\) to obtain the result that \( \int_{i \in I} (1 - \tau^i) \theta^i \Gamma_i^\ell d\omega(i) \).
B Online Appendix: Analytical Details and Further Results on Examples in Section 2

B.1 Spillovers of Current Account Intervention

This appendix extends the analysis of section 2.2 from learning-by-exporting externalities to learning-by-doing externalities to verify the robustness of our findings.

We consider a country $i$ in which labor input generates output according to the linear production function $y_i' = A_i' L_i'$ at disutility $d(\ell_i')$ which satisfies $d', d'' > 0$. Period 0 productivity $A_0'$ is exogenous, but period 1 productivity, $A_1' = A_1' (L_0')$ with $A_1'' > 0$, is an increasing function of aggregate period 0 employment $L_0'$. Furthermore, we assume that the policymaker in country $i$ cannot employ a labor subsidy to internalize the externality, for example because of a lack of institutional capacity to target such subsidies without creating abuse, or because of a lack of fiscal revenue. As a result, current account intervention is a second-best device to internalize the learning externalities. The restriction on labor market instruments implies that the country $i$ planner faces the implementability constraint

$$A_0' u' (C_0') = d' \left( L_0' \right)$$

The planner in country $i$ thus solves the optimization problem

$$\max_{L_0', M_0'} \left( A_0' L_0' + M_0' \right) - d \left( L_0' \right) + u \left( A_1' (L_0') L_1' + M_1' \right) - d \left( L_1' \right) + \gamma' \left[ A_0' u' (A_0' L_0' + M_0') - d' \left( L_0' \right) \right] - \Lambda_e \left[ M_0' + M_1' / R \right]$$

The optimality conditions for $L_0'$ and $M_0'$ are

$$FOC \left( L_0' \right) : A_0' u' (C_0') - d' \left( L_0' \right) + u' \left( C_1' \right) A_1'' L_1' + \gamma' \left[ (A_0')^2 u'' (C_0') - d'' \left( L_0' \right) \right] = 0$$

$$FOC \left( M_0' \right) : u' \left( C_0' \right) + \gamma' A_0' u'' \left( C_0' \right) = \Lambda_e$$

Substituting the implementability constraint, the first condition can be use to express the shadow price on the implementability constraint as

$$\gamma' = \frac{u' \left( C_1' \right) A_1'' L_1'}{d'' \left( L_0' \right) - (A_0')^2 u'' (C_0')} > 0$$

The optimality condition on $L_1'$ coincides with the implementability constraint, and that on $M_1'$ corresponds to the standard condition $u' \left( C_1' \right) = \Lambda_e / R$. The two optimality conditions on $M_1'$ can be combined to yield the generalized Euler equation $u' \left( C_0' \right) + \gamma' A_0' u'' \left( C_0' \right) = Ru' \left( C_1' \right)$. The planner can implement this optimality condition by subsidizing period 0 net inflows, i.e. by imposing the tax rate

$$\tau_0 = \gamma' \cdot \frac{A_0' u'' \left( C_0' \right)}{u' \left( C_0' \right)} = \frac{u' \left( C_1' \right) A_1'' L_1'}{d'' \left( L_0' \right) - (A_0')^2 u'' (C_0')} \cdot \frac{A_0' u'' \left( C_0' \right)}{u' \left( C_0' \right)} < 0$$

\[36\] It is well known that if such an instrument was available, then a labor subsidy of $s_0^f = A_1'u' L_1' : u' (c_1') / u' (c_0')$ would correct the distortion and no further intervention is required.
C Online Appendix: Three Simple Illustrations of Policy Cooperation

This appendix illustrate in a simple and teachable manner how deviations from the three efficiency conditions lead to Pareto inefficient equilibria that call for global cooperation. We do so in a multi-country setting that builds on the simple example of current account intervention in a single country that we developed in section 2.2.

General Equilibrium Let us assume that there are two sets $\mathcal{I}^A$ and $\mathcal{I}^D$ of countries with measure $\omega(\mathcal{I}^A) = \omega(\mathcal{I}^D) = 1/2$ that are each made up of identical atomistic countries and that, respectively, represent advanced and developing countries. The setup of each country is as described in section 2.2. The only distinction between advanced and developing countries is that advanced countries no longer experience learning externalities so their period 1 endowment is constant. For simplicity we assume that $y_1^A(-M_0^A) = y_0^A M_0^A$ and $y_1^D(0) = y_0^D$ but $y_1^D(-M_0^D) > 0$, capturing the learning externalities. As discussed in section 2.2, this gives rise to the reduced-form welfare functions

$$V^i(m^i, M^i) = u(y_0^i + m_0^i) + u(y_1^i (-M_0^i) + m_1^i)$$

with marginal utility of private net imports is $V_m^i = (u'(c_0^i), u'(c_1^i))^T$ and uninternalized social marginal utility $V_M^i = (-y_1^i(-M_0^i) \cdot u'(c_1^i), 0)^T$, which satisfies $V_M^i \equiv (0, 0)^T \forall i \in \mathcal{I}^A$ for advanced economies but $V_M^i < 0 \forall i \in \mathcal{I}^D$ for developing countries.

Laissez-Faire Equilibrium In the global laissez-faire equilibrium, private agents in each country $i$ take the aggregate allocation $M^i$ as given and solve the optimization problem

$$\max_{m^i} V^i(m^i, M^i) \quad \text{s.t.} \quad Q \cdot m^i \leq 0$$

Assigning shadow price $\lambda^i$, it is easy to see that the optimality condition is $V_m^i = \lambda^i Q^T$ or, equivalently, $\frac{u'(c_0^i)}{u'(c_1^i)} = R$. The allocation $m^i = M^i = (0,0)^T$ together with the price vector $Q = (1,1)$ represents an equilibrium of the system since the endowment of both types of countries is constant, implying perfect consumption smoothing for private agents under zero net imports. However, the laissez-faire equilibrium is sub-optimal since private agents in developing countries neglect the potential gains from learning externalities.

National Planner Allocation The national planners in advanced countries find it optimal not to intervene and set $\tau^i = (0,0) \forall i \in \mathcal{I}^A$, since their countries do not suffer any domestic market imperfections and are atomistic. However, the national planners in developing countries $i \in \mathcal{I}^D$ internalize the learning externalities. As we emphasized in section 2.2, their optimality condition is

$$V_m^i + V_M^i = \Lambda^i Q$$

(A.24)

Furthermore, they find it optimal to subsidize period 0 net exports by imposing the tax wedge $\tau_0^i = -\frac{V_m^i}{V_m^0} = y_1^i \cdot \frac{u'(c_1^i)}{u'(c_0^i)} > 0$ and to set $\tau_1^i \equiv 0$.

Spillovers In the resulting global equilibrium, developing countries will be net exporters in period 0 and net importers in period 1, and vice versa for advanced countries, so $m_0^D = -m_0^A < 0$ and $m_1^D = -m_1^A > 0$. Furthermore, the world interest rate will decline below the laissez-faire level $R < 1$. These quantity and price adjustments represent spillovers from the interventions of developing countries.

37 In the described example, advanced countries happen to be better off from the interventions of devel-
Global Optimum. The key question of our paper is under what conditions the equilibrium among national planners is socially efficient. To answer this question in the current example, we will compare the equilibrium among national planners with the allocations that would be chosen by a global planner who maximizes the sum of worldwide welfare for a given set of Pareto weights, which we assume $\phi^A$ and $\phi^D$ for advanced and developing countries. Substituting the global market-clearing condition $M^A = M = -M^D$, the planner’s problem can be described as

$$
\max_M \phi^A V^A(M, M) + \phi^D V^D(-M, -M)
$$

(A.25)

with associated optimality condition

$$
\phi^A [V^A + V^A_M] = \phi^D [V^D + V^D_M]
$$

(A.26)

Consider a national planning allocation, which satisfies the optimality conditions (A.24) for $i = A, D$. We combine the conditions for both types of countries and observe $V^A_M = V^D_M = 0$ to obtain

$$
\frac{V^A_m}{\Lambda^A} = Q^T = \frac{V^D_m + V^D_M}{\Lambda^D}
$$

It can be easily seen that the optimality conditions of the national planners coincide with the optimality conditions of a global planner (A.26) with welfare weights $\phi_i = 1/\Lambda^i$ for $i = A, D$. The national planning allocation also satisfies global market clearing and is therefore globally Pareto efficient.

Intuitively, the national planners described in the example ensure that each country equates the social marginal benefit of transacting with the rest of the world to the common vector of world market prices. Since (i) the described national planners act competitively, (ii) they have sufficient external policy instruments and (iii) the international market is complete, the outcome is Pareto efficient. Even though the interventions of developing countries have spillover effects on advanced countries, these effects are Pareto efficient; in fact, they are necessary for the efficient functioning of the market.

In the following, we illustrate the case for global cooperation by relaxing, in turn, each of the three conditions required for efficiency.

Deviating from Condition (i): Monopoly Power. If there is a single large developing country $D$ instead of a unit mass of atomistic countries, then the policymaker in country $D$ has market power and finds it optimal to internalize how the country’s international transactions $M^D$ affect world prices. Market clearing implies $M^A + M^D = 0$ and so the Euler equation of advanced countries defines a world interest rate schedule as a function of the international transactions of the developing country, $R(M^D) = u'(y^A_0 - M^D) / u'(y^A_1 - M^D)$ or, in vector notation, $Q(M^D) = (1, 1/R(M^D))$. A planner in country $D$ who optimally exerts market power will solve the optimization problem

$$
\max_{M^D} V^D(M^D, M^D) \quad \text{s.t.} \quad Q(M^D) \cdot M^D \leq 0
$$

The associated optimality condition is

$$
V^D_m + V^D_M = \Lambda^D Q^T (1 - \xi_{Q,M})
$$

opposing countries. We could easily describe examples in which advanced countries are worse off: if $y^A_1 > y^A_2$ so that advanced countries are net lenders in the laissez faire equilibrium, then a marginal decline in the interest rate would hurt them, representing a negative spillover effect.

The laissez faire equilibrium is clearly not globally efficient – combining the optimality conditions of private agents, we obtain $V^A_m / \Lambda^A = Q^T = V^D_m / \Lambda^D$. This is inconsistent with the planner’s optimality condition (A.26) no matter what set of welfare weights $(\phi^A, \phi^D)$ the global planner employs since the first element of the vector $V^D_M \neq 0$ but the second element $V^D_M = 0$. 

\[38\]
where $\mathcal{E}_{Q,M} = -[\partial Q/\partial M^D \cdot M^D] / Q^T$ is a vector of demand elasticities of world prices which satisfies $\mathcal{E}_{Q,M_0} < 0 < \mathcal{E}_{Q,M_1}$, with the division performed element-by-element. The expression captures that the planner in the developing country internalizes that manipulating her import and export decisions enables her to improve the country’s terms-of-trade vis-à-vis advanced countries. The resulting allocation can be implemented by setting the external policy instruments to

$$\left( 1 - \frac{\tau_0^D}{1 - \tau_1^D} \right) = \frac{V_i^M / V_i^m}{1 - \mathcal{E}_{Q,M}^i} \left( \frac{1 + y_1^m \cdot \frac{u'(C_1^0)}{u'(C_0^0)}}{\left( 1 - \frac{u'(C_0^0)}{u'(C_1^0)} \right) M^D} \right) \frac{1}{\left( 1 + \frac{u'(C_1^0)}{u'(C_0^0)} M^D \right)}$$

where all divisions are performed element-by-element. This implies that $\tau_0^D > 0 > \tau_1^D$—in addition to internalizing the growth externalities, the planner recognizes that restricting exports in period 1 and restricting imports in period 1 increases the world interest rate, which allows the country to earn a higher return on its savings.

Interestingly, the monopolistic national planner subsidizes exports in period 0 at a lower rate than a national planner in an atomistic country (as captured by the denominator in the expression for the period 0 tax rate), i.e. she forgoes part of the benefit of internalizing the learning externalities in order to manipulate the world interest rate. The spillovers created by the monopolistic national planner are thus smaller than those created by a price-taking (efficient) national planner. As a result, advanced countries benefit less from valuable intertemporal trading opportunities with the developing country.

Since the planner imposes monopolistic wedges, the allocation is clearly not Pareto efficient and worldwide welfare is reduced. The deviation from price-taking behavior creates a clear scope for global coordination: global policymakers can increase worldwide welfare by forestalling monopolistic behavior.

### Deviating from Condition (ii): Incomplete Instruments

Let us return to the setup with atomistic countries without market power but assume that developing countries $D$ have imperfect external policy instruments. For simplicity, assume that they are completely unable to affect the external allocations of private agents so $\tau_1^D = (0, 0)$ at all times, but that advanced countries $A$ have a full set of external policy instruments $\tau^A$ that can be set to arbitrary levels.

In the uncoordinated national planning allocation, policymakers in developing countries do not engage in policy intervention because they are not able to; policymakers in advanced countries do not engage in policy intervention and set $\tau^A = (0, 0)$ because they do not see any domestic rationale to intervene in markets. The resulting allocation is identical to the global laissez-faire allocation. As we showed earlier, this allocation is not Pareto efficient because it neglects the learning externalities.

Again, there is a clear scope for global policy coordination: the global optimum described above requires that the social marginal products of the two types of countries are equated, $(V_m^D + V_m^A) / \Lambda^D = V_m^A / \Lambda^A$. This allocation can be replicated if the policymakers in advanced countries set their policy instruments to

$$1 - \tau^A = \frac{1}{1 - \tau^D} \quad \text{or} \quad \tau^A = \left( 1 - \frac{1}{1 - \tau^D}, \frac{u'(C_1^0)}{u'(C_0^0)} \right)$$

to internalize the externalities of developing countries and if developing countries provide a transfer to finance the policy intervention.

The transfer is of the exact same magnitude as what developing countries would have used to finance their own export subsidies if that instrument was available, so no extra government revenue is required. However, international transfers may be politically contentious. If we rule out transfers, a global planner would still want to use the policy instruments of advanced countries to internalize any externalities of developing countries with incomplete instruments, but the allocation of Proposition ?? can no longer be replicated. In other words, implementing globally efficient allocations would generally go hand in hand with redistributions.  

\[\text{57}\]
adjusts to $\hat{Q} = (1/(1 - \tau^D), 1/R)$. At this new price vector and given the transfer payment, the original optimal allocation in the global optimum described above is feasible for both types of countries and the social marginal products of the two types of countries are equated since $V^D_m / A^D = \hat{Q} = (1 - \tau^D)V^D_m / \Lambda^D$.

Intuitively, it does not matter if developing countries subsidize exports or advanced countries subsidize imports in period 0 – the resulting allocation is the same.

**Deviating from Condition (iii): International Market Imperfections** The third area that requires global coordination are international market imperfections. To illustrate a relevant example, loosely inspired by Jeanne (2014), let us assume that international financial transactions are restricted to take place in the currencies of advanced countries, which are in a liquidity trap and face a zero interest rate so the international price vector is $Q = (1, 1)$. Furthermore, as is common in the New Keynesian literature, assume that period 0 output in advanced countries is demand-determined and adjusts so as to clear the market. In other words, when developing countries increase exports, the world interest rate cannot decline, but advanced countries import more and experience a decline in demand for domestic output and thus in $y^A_1$. Furthermore, assume that period 0 output in each advanced country $A$ is produced at a continuously differentiable convex utility cost $d(y^A_0)$ that satisfies $d(0) = d'(0) = 0$ and $d'(1) = 1$.

When the zero-lower-bound in advanced countries is binding, period 0 output is determined by the Euler equation:

$$u'(C^A_0) = u'(C^A_1) \quad \text{or} \quad y^A_0(M^A) = y^A_1 - M^A + M^A_1$$

The reduced-form utility function of a representative advanced country $A$ is

$$V^A(M^A) = u(y^A_0(M^A) + m^A_0) - d(y^A_1(M^A)) + u(y^A_1 + m^A_1)$$

The national planner in an advanced country $A$ recognizes that imports lead to aggregate demand externalities and sets her external policy instruments to

$$\tau^A = -\frac{V^A_m}{V^A_M} = \left(1 - \frac{d'(y^A_0)}{u'(C^A_0)} \right) \left(1 - \frac{u'(C^A_0)}{u'(C^A_1)} \right)$$

We can interpret the term $1 - d'(y^A_0)/u'(C^A_0) > 0$ as the analogon of the labor wedge in New Keynesian models, i.e. as the cost of the demand shortage – an additional unit of output would cost $d'(y^A_0)$ but bring utility benefit $u'(C^A_0)$. The national planner would thus tax period 0 imports which take away from domestic demand and subsidize period 1 imports, which create a future boom and by implication boost today’s output [see equ. (A.27)]. The national planners in developing countries would continue to operate as in the baseline example [5] above. Given the sticky price vector $\hat{Q} = (1, 1)$, the resulting global equilibrium is described by the equilibrium condition

$$\frac{V^A_m + V^A_M}{\Lambda^A} = \hat{Q} = \frac{V^D_m + V^D_M}{\Lambda^D}$$

(A.28)

From this condition, it is apparent that the price mechanism cannot play its usual role of efficiently allocating goods across countries – prices do not reflect the relative social valuation of goods, but are given exogenously.

A global planner solves the optimization problem (A.25) with optimality condition (A.26). It can easily be seen that the equilibrium described by (A.28) can be improved upon: at the described uncoordinated allocation, developing countries internalize learning externalities by equating the marginal benefit of imports/exports in the two periods to the fixed world price vector; however, period 0 exports from developing countries create negative demand externalities for advanced countries. A marginal reduction in period 0 exports from developing countries would come at a second-order cost for developing countries (since they were at their point of optimality, given world prices $\hat{Q}$) but would provide a first-order benefit of $u'(C^D_0) - d'(y^D_0) > 0$ to advanced countries.

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40 For a detailed derivation of how a typical New-Keynesian setup with a binding zero-lower-bound determines output see appendix B