A Contagious Malady? Open Economy Dimensions of Secular Stagnation*

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Abstract

We extend the the idea of secular stagnation (Hansen (1939), Summers (2013)) to a two country open economy world with integrated financial markets. Our framework also allows us to incorporate the global savings glut hypothesis (Bernanke (2015)) into a secular stagnation framework. We consider varying degrees of capital market integration and show that, when the world natural rate of interest is negative, one or both countries may be in a secular stagnation (binding zero lower bound, deflation, and persistent output gap). Capital controls can be beneficial for the country that has a positive natural rate under autarky. In our setting, reserve accumulation may either cause or exacerbate a secular stagnation by further lowering the world natural rate of interest. Policy responses include a global increase in the inflation target or fiscal policy to raise the world natural rate of interest. The gains from monetary and/or fiscal policy coordination are substantial in our framework.

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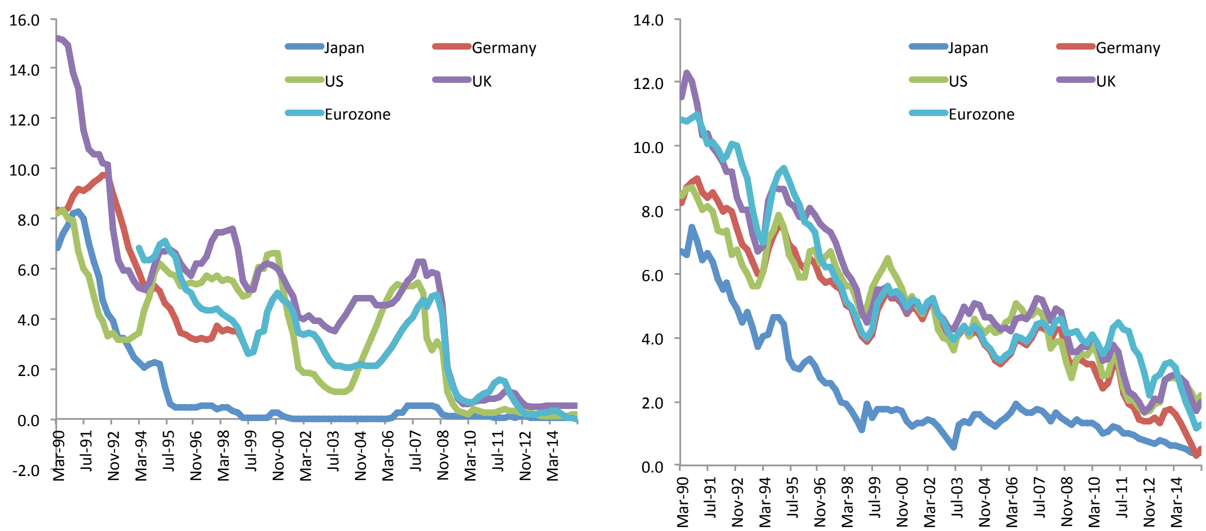
1 Introduction

In the last quarter of a century, the world has experienced a remarkable decline in the short and long-term nominal interest rates. This development is shown in Figure 1 for several leading industrial economies. At the beginning of this period, it was not uncommon to observe double digit interest rates. Today, these countries are experiencing historically low long-term rates with short-term nominal interest rates up against the zero bound. Some of the fall in nominal interest rates is explained by a worldwide fall in inflation. However, real interest rates have fallen substantially as well, suggesting more fundamental forces at work than just a reduction in inflation.

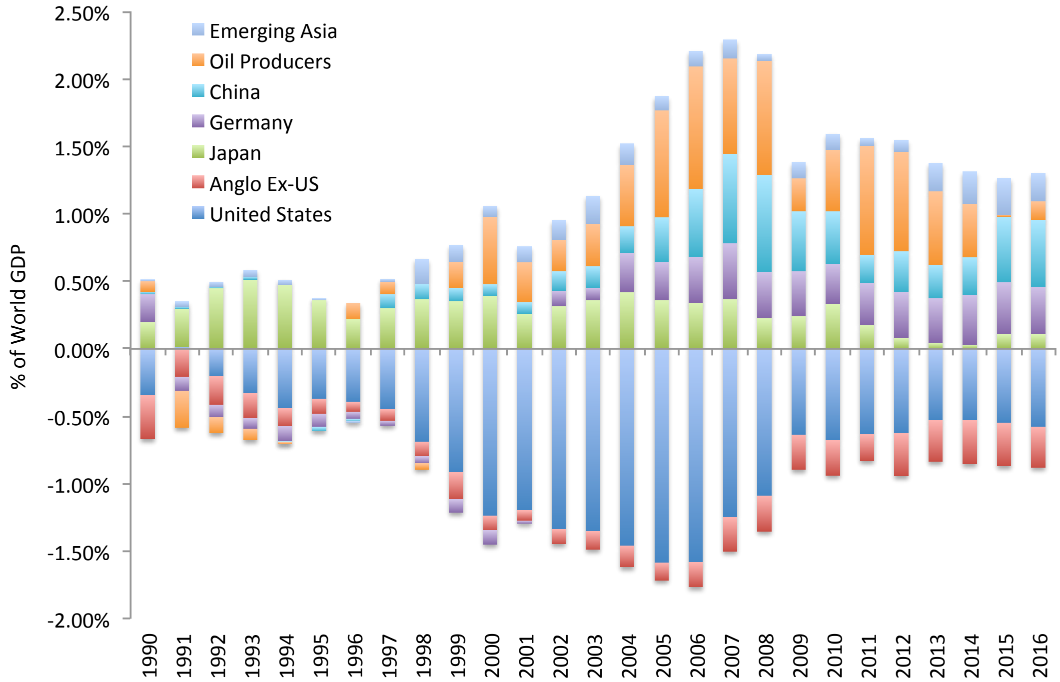
Figure 1: Short and Long-Term Interest Rates



What explains this worldwide decline in real interest rates? A leading hypothesis is that declining real rates reflect an oversupply of savings. Two prominent ideas have been put forward as a source of this. In late 2013, Summers (2013) resurrected the secular stagnation hypothesis of Alvin Hansen (1939) - that the economy could face a permanently lower and possibly negative natural rate of interest, inviting the possibility of a repeated and chronic zero bound episodes. This hypothesis was later formalized in Eggertsson and Mehrotra (2014), explicitly linking the fall in the natural rate of interest to an increase in inequality, aging of the population, the fall in relative price of investment, and tightening collateral constraints. Importantly, they show that in the presence of the zero bound and nominal frictions, a permanent reduction in the natural rate of interest translates into a permanent demand recession at the zero lower bound.

A complementary narrative was offered by Bernanke (2015) prior to the crisis as an explanation for large US current account deficits. This is the global savings glut hypothesis. According to this hypothesis, the current account deficits in the US prior to the Great Recession were not

Figure 2: Global Imbalances



necessarily a consequence of US domestic conditions. Rather, they were a consequence of global developments. In particular Bernanke identified large reserve accumulation, both by East Asian countries and oil producing ones, as a contributing factor. Bernanke also suggested than demographic changes (aging) in several leading economies – Germany, Japan and China ? were raising global saving and thus contributing to global imbalances.

Figure 2 displays the global imbalances leading up to the financial crisis. Japan, a major contributor to global imbalances, was the country that first hit the ZLB in the late 1990s. The excess savings in Japan is then further amplified in the early 2000s by players such as Germany, oil producers, China and other emerging Asian economies. These countries all started exporting savings with the US serving as the primary recipient of funds. In the US, the current account deficit rose to 4% of US GDP prior to the crisis (we also observe an increasing share directed to other Anglo Saxon countries: UK, Canada, Australia and New Zealand, post 2000).

Post crisis, Bernanke recently reviewed the literature on secular stagnation. He argued that the literature on secular stagnation has paid insufficient attention to global factors. The data cited above suggest that global factors surely must have played a role. The goal of this paper is to fill this gap. This paper integrates the global savings glut hypothesis into a model of secular stagnation and explore the transmission of secular stagnation across countries. We do this by considering a two-country variation of the Eggertsson and Mehrotra formalization of Summers?s secular stagnation hypothesis. We first show how the two perspectives fit naturally together.

Indeed, we view the global saving glut and the secular stagnation as two sides of the same coin - the forces articulated in each theory can generate a persistent decline in the world natural rate of interest and thus create the possibility of a long lasting slump and protracted periods of a binding zero bound. What the global saving glut adds, relative to the closed economy model of secular stagnation, is that it focuses attention more closely to the role of international financial markets in generating this outcome and transmitting secular stagnation from one country to another.

Broadly speaking, what emerges from our analysis is that international financial markets naturally work as a transmission mechanism for low real interest rates - or secular stagnation. Large amounts of excess savings in Japan, Germany, or in oil producing nations triggers downward pressures on interest rate in these countries. But with open financial accounts, this leads to current account flows into the rest of the world, triggering a global fall in interest rates. The source of the excess saving can either be from the private sector (e.g. due to population aging) or due to government policy (e.g. reserve accumulation). We show, for example, that the accumulation of foreign reserves, in the form of US Treasuries, leads to downward pressure on the world natural rate of interest, as long as these purchases are not offset by a corresponding increase in domestic debt. If the forces pushing down the real interest rate are strong enough, the world natural rate of interest can be negative, the world economy can find itself at zero interest rates with subpar growth. The picture that emerges, then, is of a world plagued with excess savings and a binding zero bound - a world in which the excess saving in one country has strong negative externalities on its trading partner via current account surpluses that push the natural rate of interest down. Since the nominal interest rate is stuck at zero, this downward pressure will manifest itself as slack demand.

Under perfect capital integration, we characterize the possible equilibria when the world natural rate of interest is negative. If the inflation target is insufficient for equilibrium rates to attain the natural rate, then output must fall below its full-employment level. However, in contrast to the closed economy case, multiple equilibria can emerge. In a symmetric stagnation steady state, both countries experience output gaps and nominal interest rates at the zero lower bound. This equilibria captures the salient features of the global economy - particular the US and Eurozone - since 2008.

By contrast, there also exist asymmetric stagnation steady states with one country at full employment while the other is at the zero lower bound. The equilibrium world interest rate must be higher than in a symmetric stead state to generate sufficient deflation for one country to fully absorb the world output gap. This equilibria captures elements of the pre-2008 global environment and, in particular, the interaction of Japan and the US where capital flows towards the US pull down interest rates and raise household indebtedness while deflation and the zero lower bound characterized the Japanese lost decade. Relative to a symmetric stagnation, global imbalances are more pronounced in an asymmetric stagnation.

But if international financial markets transmit secular stagnation, would closing markets be beneficial for the country with positive interest rates in financial autarky? The answer to this question may be yes and we show this formally in the model. Financial autarky raises welfare by maintaining full employment and higher consumption levels but may reduce welfare by impairing the ability of household to smooth consumption. In any case, superior policies exist and we do not endorse capital controls in countries such as the US and UK as a desirable way to escape secular stagnation.

Instead of closing international capital markets, there are several other policy options that are more appealing. In particular we show that a natural answer to a secular stagnation scenario is fiscal policy, either in the form of direct spending financed by taxation on the working population or via increase in public debt. Since the problem in a secular stagnation is excessive savings, the issuance of government debt is the most natural solution to raising the natural rate of interest as it directly increases the supply of instument that can store value.

One lesson that emerges in when considering secular stagnation in an open economy is that fiscal policy has strong positive externalities on a country's trading partners giving rise to large gains from policy cooperation. One problem that emerges is that since the benefits of fiscal policy are borne by all countries alike in the global economy, each country may try to free ride on the demand stimulus of others. We show the gains from coordinated fiscal policy versus the Nash equilibrium from countries that cannot coordinate fiscal expansions. With imperfect capital integration, a form of neo-mercantilism emerges where countries in secular stagnation wish to accumulate foreign assets to boost domestic demand at the expense of foreign demand.

Another key lesson is that while fiscal policy has positive externalities, a monetary expansion in one country relative to the other, that is, an increase in the inflation target in Japan vs. the US, will come completely at the expense of the trading partner. Our analysis, thus suggest a strong beggar-thy neighbor component of monetary policy.

At the heart of this result is that when the world natural rate of interest is negative, then either one or both countries can be in a secular stagnation - we have inherent multiplicity of equilibria. If both are in a secular stagnation, and one of the countries manages to escape it, than it will do so necessarily at the expense of the other. If both countries successfully commit to inflation, however, neither country experiences secular stagnation and both gain. The monetary policy side thus - just as in the case of the fiscal one - highlights the importance of policy coordination.

1.1 Related Literature

Our paper builds on several strands of literature. We have already alluded to Summers (2013) and Hansen (1939) which was then formalized in Eggertsson and Mehrotra (2014) We also build on an emerging literature of models of economic stagnation including Kocherlakota (2013), Schmitt-

Grohé and Uribe (2013), Benigno and Fornaro (2015), Bianchi and Kung (2014), Guerron-Quintana and Jinnai (2014). Our model is closest to recent work by Caballero, Farhi and Gourinchas (2015) that examines the interaction of safe asset shortages with nominal frictions and the zero lower bound.

Our model is similar in structure to the model of Coeurdacier, Guibaud and Jin (2015) which examines how financial integration accounts to declining real interest rates and capital flows from emerging markets to advanced economies. Our model also shares features of models that examine the global demand for safe assets and the persistent US current account deficit: e.g. Caballero, Farhi and Gourinchas (2008), Caballero and Farhi (2014), Gourinchas and Jeanne (2013). Interestingly, when the natural rate turns negative, our model can generate a current account deficit for debtor countries in steady state.

Finally, our results on the gains from monetary and fiscal coordination build on earlier work by Clarida, Galı and Gertler (2002), Dixit and Lambertini (2003) and Benigno and Benigno (2006).

The paper is organized as follows. Section 2 examines determinants of the world natural rate of interest in an endowment economy while Section 3 extends this framework to model the global savings glut. Section 4 extends our model to include monetary policy and nominal wage rigidities. Section 5 and 6 analyzes and characterizes both symmetric and asymmetric stagnation equilibria. Section 7 examine monetary policy responses, while Section 8 examines fiscal policy responses.

2 Capital Integration and the Natural Rate of Interest

We start by showing how the real interest rate is determined in an endowment economy, allowing for varying degrees of financial integration. To consider cases in between autarky and full financial integration we introduce a constraint on international capital flows. Our focus is to show how the real interest rate is affected by the degree of financial market integration.

There are two countries, domestic and foreign. Each country trades a one period risk free bond with returns r_t and r_t^* respectively. Without loss of generality, we focus here on the case in which $r_t \geq r_t^*$, - a situation in which the returns on the asset in the domestic economy dominates that in the foreign country so long as capital markets are imperfectly integrated.

Consider a simple overlapping generation economy. Households live for three periods: they are born in period 1 (young), earn income in period 2 (middle aged), and retire in period 3 (old). For now, we assume there are no aggregate savings, but that the generations can borrow and lend to one another. We suppose that only the middle age receive income, Y_t and Y_t^* respectively. This will imply that the middle aged generation in each country lend to the young in order to save for retirement. A key constraint we impose is on the borrowing of the young. The young are constrained by a borrowing limit $(1 + r_t)B_t^y \leq D_t$ and $(1 + r_t^*)B_t^{*y} \leq D_t$ as in Eggertsson and Krugman (2012). Implicitly, we think of this limit as emerging from some type of incentive constraint, however, for our purposes, we take this limit to be exogenous.

If the real interest rate is higher in one country than the other then savings will flow to the country with the highest yield. If there are no constraints on capital flows, then the real interest rate is equalized across the two countries. We impose a simple quantity constraint on international capital flows which we denote K_t. In particular, we assume that the the domestic debt held by the foreign country has to be lower than some K_t. Again, implicitly, we are assuming this constraint reflects some sort of incentive constraint, perhaps due to incomplete enforcement of contracts across national borders, home bias for investors, or other limits to arbitrage, but for the purpose of our analysis, we will simply treat it as exogenous. One could similarly interpret this as representing some form of "capital controls", since it places a direct quantity limit on how much capital can move across countries. If the constraint is not binding, then the real interest rates must be equalized across the two countries.[1]

Formally, consider the following overlapping generation model. A domestic household born at time t has the following utility function:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} \mathbb{E}_t \left\{ \log\left(C_t^y\right) + \beta \log\left(C_{t+1}^m\right) + \beta^2 \log\left(C_{t+2}^o\right) \right\}$$

subject to the following (real) budget constraints:

$$C_t^y = B_t^y \tag{1}$$

$$C_{t+1}^m = Y_{t+1} - (1+r_t)B_t^y - A_{t+1}^D - A_{t+1}^I \tag{2}$$

$$C_{t+2}^o = (1+r_{t+1})A_{t+1}^D + (1+r_{t+1}^*)A_{t+1}^I \tag{3}$$

$$(1+r_t)B_t^i \leq D_t \tag{4}$$

$$0 \leq A_{t+1}^I \leq K_{t+1} \tag{5}$$

Here C_t^i denotes consumption for each generation i, B_t^y borrowing in a one period risk-free bond that pays an interest rate r_t. A_t^D is the asset holding of the middle aged household of the domestic bond that carries interest rate r_t while A_{t+1}^I is the middle generation holding of the foreign asset. The foreign economy has the same set of preferences and faces the same set of constraints. We assume that there is no short-selling of the foreign asset. While the middle generation can accumulate a positive position in A_{t+1}^I, which earns interest r_t^*, it cannot issue its own debt in the foreign currency at rate r_t^*.

We consider an equilibrium in which the borrowing constraint for the young is binding:

$$C_t^y = B_t^y = \frac{D_t}{1+r_t} \tag{6}$$

[1] We derive similar results when there is a credit spread function that depends on the level of the capital flow between the two countries. We adopt the quantity restriction here given that the resulting equilibrium conditions are a generalization of the case considered in Eggertsson and Mehrotra (2014).

In equilibrium, the middle generation lend to the young to save for their retirement. Their savings decision satisfies a consumption Euler equation:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t (1 + r_t) \frac{1}{C_{t+1}^o} \tag{7}$$

while the old consume all their income - interest of domestic and foreign savings.

$$C_t^o = (1 + r_{t-1}) A_{t-1}^D + (1 + r_{t-1}^*) A_{t-1}^I \tag{8}$$

The residents of the foreign economy satisfy the same conditions where we denote each variable with a star. The model is closed by bond market clearing in each country. For the domestic market it is given by:

$$N_t B_t^y = N_{t-1} A_t^D + N_{t-1}^* A_t^{I*} \tag{9}$$

while the foreign bond market clearing condition is given below:

$$N_t^* B_t^{y*} = N_{t-1}^* A_t^{D*} + N_{t-1} A_t^I \tag{10}$$

2.1 Financial Autarky

Let us first consider the case in which $K_t = 0$. In this case we have full financial autarky. It is now straightforward to characterize the equilibrium defined by the relationships above. Define population growth as $1 + g_t = \frac{N_t}{N_{t-1}}$. We can then write:

$$(1 + g_t) B_t^y = A_t^D \tag{11}$$

The left-hand side is the demand for loans, L_t^d, and the right-hand side the supply of loans, L_t^s. The demand of loans in the domestic economy is given by equation (6) so that:

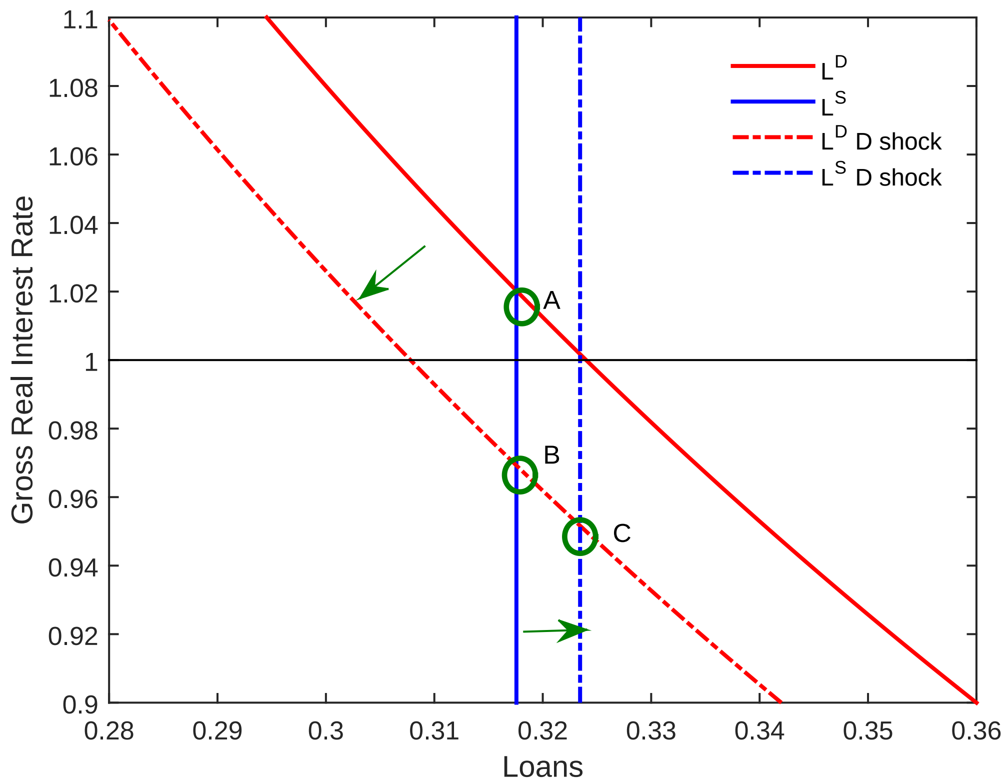
$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t$$

We obtain the supply of loans, assuming perfect foresight, by substituting the consumption of the old (8) to solve for the consumption of the middle aged using (7). We then substitute the resulting expression for C_t^m into the middle aged budget constraint (2), use (6) and solve for A_t^D to obtain:

$$L_t^s = \frac{\beta}{1 + \beta} (Y_t^m - D_{t-1})$$

Figure 3 depicts the demand and supply for loans in the domestic economy. The demand for loans increases as the real interest rate fall. This is because lower interest rate increases the borrowing capacity of the young, thus allowing them to take on more debt. As we can see, as emphasized by Eggertsson and Mehrotra (2014), both the debt deleveraging shock D_t as well as slowdown in population growth can reduce the real interest rate. Either mechanism will shift

Figure 3: Equilibrium in the asset market



down the demand for loans, as shown in the figure (point B), leading to a drop in the real interest rate.

We see that the supply for loans is the same for any given interest rate.[2] This assumption implies that the middle aged are simply saving a fixed fraction of their disposable income which then determines the supply of savings. Hence the supply of savings is a vertical line in Figure 3. One interesting mechanism that shifts the supply for loans is a permanent debt deleveraging shock. It leads to a reduction in D_{t-1} triggering a further reduction in the real interest rate by shifting out the supply for loans as shown by point C in Figure 3. The fact that the young can take on less debt with a permanent decrease in D means that they have less debt to pay off when middle age and thus higher supply of savings.

Equating demand and supply of loans, we obtain an expression for the real interest rate in the domestic economy under autarky:

$$1 + r_t^{aut} = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{Y_t - D_{t-1}} \tag{12}$$

[2]This is not a general feature of the model, but is due to the assumption of log preferences and the fact that all income is accrued in middle aged. Eggertsson and Mehrotra (2014) treat the more general cases, that we omit here for simplicity.

Analogously, we can obtain the real interest rate in the foreign economy:

$$1 + r_t^{aut*} = \frac{1+\beta}{\beta} \frac{(1+g_t) D_t^*}{Y_t^* - D_{t-1}^*}$$

Observe the most important implication of our supply and demand framework for loanable funds: There is no inherent reason to expect the equilibrium real interest rate, as given by r_t^{aut} or r_t^{aut*} to be either positive or negative. This depends on the relative size of demand and supply for loanable funds. While we show above how population dynamics and debt deleveraging may affect these relationship, the earlier literature has also emphasized other forces which could easily be incorporated such as an increase in income inequality (which increases the supply of savings) or a fall in the relative price of investment or even an increase in uncertainty. In this setting, we can investigate a new one: how is the natural rate affected by the opening of international financial markets?

2.2 Perfect Financial Integration

Let us now integrate financial markets in this environment. Before, equilibrium in each country was determined by the demand and supply relationship in Figure 3. With full integration, interest rates are equalized across countries:

$$1 + r_t = 1 + r_t^* \tag{13}$$

In this case we can rewrite each of the household problem in terms of net asset position $A_{t+1}^m = A_{t+1}^D + A_{t+1}^I$ and obtain an international financial market clearing condition:

$$N_t B_t^y + N_t^* B_t^{y*} = N_{t-1} A_t^m + N_{t-1}^* A_t^{m*} \tag{14}$$

which closes the model.[3]

Let us now characterize the equilibrium defined by these new equations. We denote the relative size of domestic and foreign economy by $\omega_{t-1} = \frac{N_{t-1}}{N_{t-1}^* + N_{t-1}}$. The bond market clearing condition becomes:

$$\omega_{t-1} (1+g_t) B_t^y + (1-\omega_{t-1}) (1+g_t^*) B_t^{y*} = \omega_{t-1} A_t^m + (1-\omega_{t-1}) A_t^{m*} \tag{15}$$

where the left-hand side is world demand for loans, $L_t^{W(d)}$ and the right-hand side is world supply, $L_t^{W(s)}$. World loan demand is now given by:

$$L_t^{W(d)} = \omega_{t-1} (1+g_t) L_t^d + (1-\omega_{t-1}) (1+g_t^*) L_t^{d*} = \frac{1}{1+r_t} (\omega_{t-1} (1+g_t) D_t + (1-\omega_{t-1}) (1+g_t^*) D_t^*)$$

[3]For a given set of exogenous processes $\{D_t, N_t, Y_t\}$ and $\{D_t^*, N_t^*, Y_t^*\}$ an equilibrium in the global economy is now characterized by a collection of stochastic processes $\{C_t^y, C_t^o, C_t^m, r_t, B_t^y, A_t^m\}$ and $\{C_t^{y*}, C_t^{o*}, C_t^{m*}, r_t^*, B_t^{y*}, A_t^{m*}\}$ that solve (1), (2), (6), (7), (8) for the domestic and the foreign households respectively. In place of the two equations for each economy given by (12) previously, we now have a no-arbitrage condition (13) and an international bond-market clearing condition given by (14).

where (13) implies the last equality.

The supply of loans is derived in exactly the same manner as in the case of financial autarky:

$$
\begin{aligned}
L_t^{W(s)} &= \omega_{t-1} L_t^s + (1-\omega_{t-1}) L_t^{s*} \\
&= \frac{\beta}{1+\beta}\omega_{t-1}(Y_t - D_{t-1}) + \frac{\beta}{1+\beta}(1-\omega_{t-1})\left(Y_t^* - D_{t-1}^*\right)
\end{aligned}
$$

The world natural real interest rate is now given by:

$$
1 + r_t^W = \frac{1+\beta}{\beta} \frac{\omega_{t-1}(1+g_t) D_t + (1-\omega_{t-1})(1+g_t^*) D_t^*}{\omega_{t-1}(Y_t - D_{t-1}) + (1-\omega_{t-1})\left(Y_t^* - D_{t-1}^*\right)} \tag{16}
$$

leading to the first proposition.

Proposition 1. *If $r_t^{aut} > r_t^{aut^*}$, then $r_t^{aut} > r_t^W > r_t^{aut^*}$.*

Proof. Follows directly from the expression for the world interest rate under integration and the domestic/foreign interest rates under autarky. □

The proposition above is the key result of this section and straightforward to interpret. Consider two countries that do not have integrated financial markets and thus two different interest rates. If these countries open up their financial markets, the new world interest rate will lie in between the two autarky interest rate. Importantly, just as in the last section, there is no reason to expect this interest rate to be either positive or negative. The proposition also clarifies another straightforward intuition. If the home country is small (i.e. ω_{t-1} close to zero), then domestic conditions will have a minimal effect on the new world interest rate. In the limit, as ω_{t-1} approaches 0, we have the open economy assumption in which the world interest rate is exogenous from the perspective of the home country. Furthermore, we see that it is the overall population dynamics in both countries that have an effect on the world interest rate. Thus, if the foreign country is going through a demographic slowdown with a fall in g_t^* this will put downward pressures on the real interest rate (although ultimately the weight on that country will also go down over time).

Observe that in this equilibrium, as long as $r_t > r_t^*$ in autarky then capital will flow into the domestic economy. The domestic economy's net foreign asset position under full integration is given by:

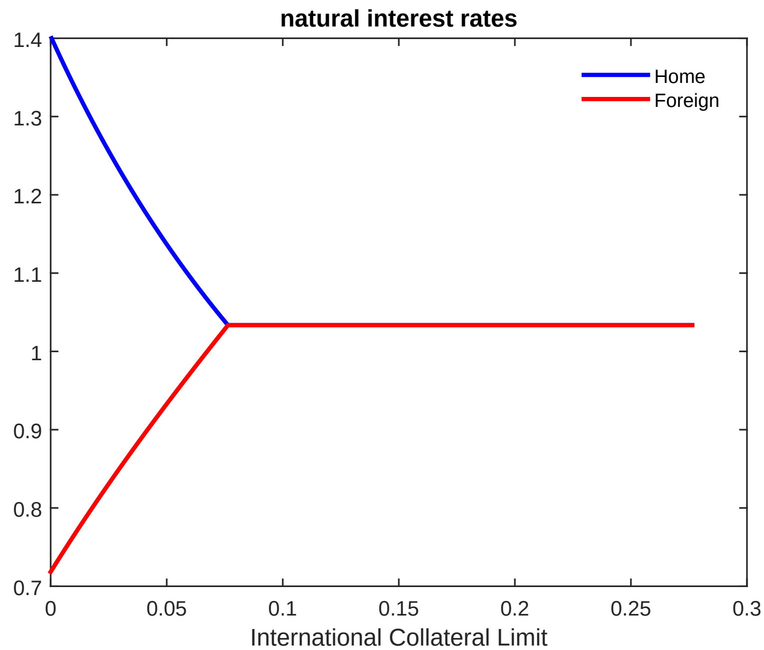
$$
NFA_t^{full} = \frac{\beta}{1+\beta}(Y_t - D_{t-1}) - (1+g_t)\frac{D_t}{1+r_t} < 0
$$

The trade balance is simply the change in the net foreign asset position adjusted for interest payments and population growth. In the case of the domestic economy, the trade balance is given by the following expression:

$$
TB_t = NFA_t - \frac{1+r_{t-1}}{1+g_{t-1}} NFA_{t-1}
$$

In steady state, if the real interest rate exceeds the growth rate of population, the trade balance takes the opposite sign of the net foreign asset position. Debtor countries with negative NFA

Figure 4: Effect of an increase in international lending on natural rate



positions must run a trade surplus in steady state. However, if $r < g$, the trade balance and NFA share the same sign. Debtor countries can run perpetual trade deficits.

2.3 Imperfect Financial Integration

We have already illustrated the outcome in the case in which each country is in financial autarky or under full integration. Now, let's consider the intermediate case in which the constraint on K_t is binding. For concreteness, let us suppose that $r_t > r_t^*$ and $K_t = \bar{K}$. Observe that for this to be the case, we need that $NFA_t^{full} < -\bar{K}$, i.e.that the financial flows under full integration cannot be achieved. In this case our formula for real interest rates in each country generalizes to:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{Y_t - D_{t-1} + \frac{1-\omega_{t-1}}{\omega_{t-1}} K_t} \tag{17}$$

Analogously, we can obtain the real interest rate in the foreign economy:

$$1 + r_t^* = \frac{1+\beta}{\beta} \frac{(1+g_t^*)D_t^* + \frac{1+r_t}{1+\beta} K_t}{Y_t^* - D_{t-1}^* - K_t} \tag{18}$$

Figure 4 shows this graphically. The x axis reflects ranges of K_t, and the y axis shows the gross real interest rate. In autarky, the foreign interest rate is negative while the domestic rate is positive. We see that as K_t increases, the two interest rate converge. Finally when $K_t = NFA_t^{full}$ then the two interest rates equalize and constraint (5) is not binding. The point of convergence may happen at either positive or negative world interest rate, depending on the parameters. We will see in the next section that the assumption of incomplete integration will be helpful to make sense of the fact that Japan has been experiencing conditions consistent with a secular stagnation for a far longer period than the rest of the world.

3 Government Debt and the Global Savings Glut

The global saving glut hypothesis is that the reduction in the real interest rate observed throughout the world in recent years has been triggered by reserve accumulation by East Asian and oil producing countries. So far, our formula for the real interest rate only depended on demographic factors and the households borrowing limit in the two countries, along with possible constraints on international capital flows. We now incorporate the global saving glut hypothesis into our model by introducing government debt for the two countries. We will see that a global savings glut can be triggered by the accumulation of domestic government bonds (Treasuries) by the foreign government.

We denote a the lump sum tax levied on each generation by T_t^y, T_t^m, T_t^o. The domestic government issues public debt and levies taxes on each generation to make interest payments on past government debt and finance some government expenditure G_t:

$$B_t^g + \frac{1}{1+g_{t-1}} T_t^o + T_t^m + (1+g_t) T_t^y = G_t + \frac{1+r_{t-1}}{1+g_{t-1}} B_{t-1}^g \tag{19}$$

Our aim here will not be to analyze fiscal policy in general (we defer that discussion until we have incorporated endogenous production), but instead clarify how foreign reserve accumulation can contribute to the drop in the natural rate of interest. For now, assume that $T_t^y = 0$ and $G_t = 0$. Also, as in Eggertsson and Mehrotra (2014), we assume for simplicity that each government adopts a particular fiscal rule that rules out any effects of taxes on loan supply:

$$T_{t+1}^o = \beta \left(1 + r_t\right) T_t^m \tag{20}$$

The overall level of taxes will adjust to ensure the government budget constraint is satisfied. The foreign government also issues public debt and levies taxes on each generation to make interest payments on past government debt. However, the foreign government also chooses to accumulate some of the debt issued by the other country, IR_t:

$$B_t^{g*} + \frac{1}{1+g_{t-1}^*} \left(T_t^{o*} + (1+r_t)\, IR_{t-1}\right) + T_t^{m*} + (1+g_t^*) T_t^{y*} = G_t^* + \left(1 + r_{t-1}^*\right) B_{t-1}^{g*} + IR_t \tag{21}$$

Here the left-hand side of the equation tallies the revenues of the government while the right-hand side gives government expenditures. We express the variables in term in terms of spending/reserves per middle age person. In particular, a positive level of IR_t denotes foreign reserve assets accumulated by the foreign government which are in the form of the bond issued by the domestic government.

Fiscal policy impacts interest rates through its effects on the asset market clearing conditions:

$$N_t B_t^y + N_{t-1} B_t^g - N_{t-1}^* IR_t = N_{t-1} A_t^D + N_{t-1}^* K_t \tag{22}$$

$$N_t^* B_t^{y*} + N_{t-1}^* B_t^g = N_{t-1}^* A_t^{D*} \tag{23}$$

To avoid needless notation, we assume symmetric country size so that $\omega_t = 1/2$.[4] The equilibrium real interest rate in debtor and creditor countries is given by:

$$1 + r_t = \frac{(1+g_t) D_t}{\frac{1+\beta}{\beta}(Y_t - D_{t-1}) + K_t - B_t^g + IR_t} \tag{24}$$

$$1 + r_t^* = \frac{1+\beta}{\beta} \frac{(1+g_t) D_t^* + \frac{1+r_t}{1+\beta} K_t}{Y_t^* - D_{t-1}^* - K_t - \frac{1+\beta}{\beta} B_t^{g*}} \tag{25}$$

Observe that an increase in public debt for the home or foreign country raises their respective natural rates. However, holding constant the level of home public debt constant, accumulation of Treasuries by the foreign government - an increase in IR_t, puts downward pressure on the natural rate of interest as this policy is increasing the global supply of savings. The effect of reserve accumulation on global rates is consistent with the argument advanced in Bernanke (2015). Given the fiscal rule we have considered, this increase in foreign holdings of Treasuries has no effect on the saving behavior of private agents.[5] Hence we have seen that the global saving glut is a natural complement to other forces that that may trigger secular stagnation, like slowdown in the population or deleveraging shocks. A final point to emphasize is that IR_t, i.e. it reflects the savings choices of the government. An important implication of this is while we would not expect private capital from from one country in our mode to another unless there is an interest rate differential that gives people the inventive to invest abroad, no such interest rate differential is needed for this policy choice. This matters, since a large driver of current account deficits we documented in the introduction stems from countries such as China or oil producing countries. It is not obvious rates of returns in the US dominate the returns in these countries. The fact that those countries still choose to invest in US Treasuries then experts a negative force on the natural rate of interest in the US, which as we will see, can have negative consequences when we take production and prices into account. Foreign reserve accumulation, in this way, can thus exert a negative externality on the US.

[4] We also assume equal population growth rates to keep world population shares constant.

[5] In principle, foreign reserve accumulation could potentially crowd-in or crowd-out private saving. Full crowd-out would imply Ricardian equivalence meaning that the world natural rate is independent of the foreign government's acquisition of foreign reserves. Our OLG structure breaks the Ricardian equivalence.

4 Prices, Production and Exchange Rates

That the real interest rate is negative needs not to be problem in itself. It only becomes a problem once we introduce the zero lower bound and nominal frictions. We now introduce nominal price determination, the zero lower bound, production and nominal frictions. Critically, we will be assuming that each country may run its own monetary policy. Accordingly each country has a currency which determines the price level in terms of that nominal unit. On the production side, will assume frictions in the adjustment of nominal wages defined in the price level of each country. For concreteness, and save on space, we define the equilibrium under perfect capital mobility. Towards the end of the section we show on how the full model is adjusted to consider the intermediate cases of incomplete financial integration.

4.1 Prices

We follow the literature by introducing nominal price determination via Woodford "cashless" economy. Each country now trades, in addition to the real bond, a nominal bond denominated in each countries price level. We assume that households in either country can hold these nominal bonds implying arbitrage equations between the real and the nominal bonds within a country, but also arbitrage equations across nominal assets denominated in different currencies.[6] Let us denote the domestic price level by P_t and the foreign price level with P_t^*. The nominal exchange rate is $S_t = \frac{P_t}{P_t^*}$

The presence of the two nominal bonds implies two new Euler equations for the middle generation in each country:

$$\frac{1}{C_t^m} = (1+i_t)\beta E_t \tfrac{1}{C_{t+1}^o} \tfrac{P_t}{P_{t+1}} \tag{26}$$

$$\frac{1}{C_t^{m*}} = (1+i_t^*)\beta E_t \tfrac{1}{C_{t+1}^{o*}} \tfrac{P_t^*}{P_{t+1}^*} \tag{27}$$

Each middle generation household also must be indifferent between holding debt denominated in the domestic or foreign currency:

$$(1+i_t^*)E_t \frac{1}{C_{t+1}^o} \frac{P_t^*}{P_{t+1}^*} = (1+i_t)E_t \frac{1}{C_{t+1}^o} \frac{P_t}{P_{t+1}} \tag{28}$$

and between holding real and nominal debt:

$$(1+r_t^W)E_t \frac{1}{C_{t+1}^o} = (1+i_t)E_t \frac{1}{C_{t+1}^o} \frac{P_t}{P_{t+1}} \tag{29}$$

where r_t^W is the real interest rate which is equal across the two countries under perfect integration.

[6] In equilibrium, we assume that the nominal bonds are in zero net supply. Hence these equations are only important for pricing, i.e. the resulting pricing equations for these nominal bonds is what pins down the nominal price level in each country, see equation (26)-(29). This is convenient because it implies that, in equilibrium, the budget constraint will be identical to in the endowment economy so that the previous derivations continue to hold.

4.2 Monetary Policy

We assume that each country follows a strict inflation targeting regime, so that:

$$\Pi_t = \bar{\Pi} \text{ if } i_t \geq 0 \text{ otherwise } i_t = 0 \text{ and } \Pi_t < 1 \tag{30}$$

$$\Pi_t^* = \bar{\Pi}^* \text{ if } i_t^* \geq 0 \text{ otherwise } i_t^* = 0 \text{ and } \Pi_t^* < 1 \tag{31}$$

Each country will set its nominal interest rate so as to achieve its inflation target. If the inflation target cannot be achieved then the central bank sets its nominal interest rate equal to zero. The zero interest rate then closes the model instead of the inflation target. This assumption conveniently abstracts altogether from how these particular equilibria are implemented while focusing on the possible problems a country may face if it cannot achieve its inflation target due to the zero bound.

This policy specification is quite useful to organize our thinking about global secular stagnation and limit equilibrium conditions to the essentials. In particular, it reduces the scenarios we need to consider to *only 4 possible cases* at *any given time t*.

If both countries follow a strict inflation targeting, then the possible scenarios at time t are:

Scenario 1: Full-Employment: Both countries set $\Pi_t = \bar{\Pi}$ and $\Pi_t^* = \bar{\Pi}^*$ while $i_t \geq 0$ and $i_t^* \geq 0$

Scenario 2: Global Secular Stagnation: Both countries miss their inflation targets $\Pi_t < \bar{\Pi}$ and $\Pi_t^* < \bar{\Pi}^*$ and set $i_t = i_t^* = 0$

Scenario 3: Foreign Secular Stagnation: Home sets $\Pi_t = \bar{\Pi}$ while $i_t \geq 0$. Foreign misses its inflation target and sets $i_t^* = 0$.

Scenario 4: Domestic Secular Stagnation: Home Secular Stagnation: Home misses its inflation target $\Pi_t < \bar{\Pi}$ and sets $i_t = 0$. Foreign sets $\Pi_t^* = \bar{\Pi}^*$ while $i_t^* \geq 0$

4.3 Production

We assume that firms are price takers on product and labor markets. Additionally, we assume that wages are downwardly rigid. As we will see, this assumption is sufficient to generate a long-run trade-off between inflation and output. That is all we need to generate our central results.[7]

Households supply labor inelastically at $\bar{L}$. Only the middle aged supply labor. There is one firm per middle aged household. Firms hire labor to produce output. Firms maximize profits, taking wages and prices as given:

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t \tag{32}$$

$$\text{s.t. } Y_t = L_t^{\alpha} \tag{33}$$

[7] In Eggertsson and Mehrotra (2014), we have experimented with alternative and more forward looking behavior, e.g. Calvo pricing, but found that it added much complexity with little additional insight. In that environment, the long run trade-off between inflation and output stems from inefficient price dispersion.

The optimality condition for firm labor demand is:

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \tag{34}$$

If prices and wages are flexible the model is closed by setting aggregate demand equal to aggregate labor supply:

$$L_t = \bar{L} \tag{35}$$

Under this assumption the economy is identical to the endowment economy we have already studied, except for the determination of nominal prices and exchange rates.[8]

What will separate our model from the endowment economy is that we replace the market clearing relationship (35) with the assumption that wages do not fully adjust. In particular, we assume that workers will never be willing to supply labor to firms if the firm offers a wage that falls below some wage norm $\tilde{W}_t$ (the classic example of this is the Keynesian idea that workers will never accept wages lower than last years nominal wages). This constraint is asymmetric, that is, workers would happily accept higher wages. Accordingly, if the wage rate implied by competitive markets is above $\tilde{W}_t$, then wages get bid up and the market clears. What this assumption implies is that if the wage norm is binding, then real wages can be higher than they would need to be for the market to clear. In this case, we assume that employment is rationed.

To be more specific, we assume that wages are downwardly rigid and given by:

$$W_t = \max\{\tilde{W}_t, W_t^{flex}\}$$

where $\tilde{W}_t$ is a wage norm determined by:

$$\tilde{W}_t = \gamma W_{t-1}\bar{\Pi} + (1-\gamma)P_t\alpha\bar{L}^{\alpha-1}$$

When $\gamma = 1$ and $\bar{\Pi} = 1$ wages are perfectly downwardly rigid and when $\gamma = 0$, wages are perfectly flexible since the term:

$$W_t^{flex} \equiv P_t\alpha\bar{L}^{\alpha-1}$$

is the nominal wage rate when wages are flexible. Observe that we allow for possibility that the wage norm is rising at the inflation target of the central bank, $\bar{\Pi}$,this will allow us to model secular stagnation in a way that is consistent with no deflation but an inflation rate below the central bank's inflation target. If $W_t > W_t^{flex}$, then $L_t < \bar{L}$ because firms will not hire all the available labor endowment - employment is rationed. Let us denote output when labor is fully employed as $Y^f = \bar{L}^\alpha$.

Together these imply an aggregate supply curve of the form:

$$Y_t = \begin{cases} Y^f & \text{if } \Pi_t \geq (\frac{Y^f}{Y_{t-1}})^{\frac{1-\alpha}{\alpha}} \\ \left[\gamma \frac{Y_{t-1}^{\frac{\alpha-1}{\alpha}}}{\Pi_t} + (1-\gamma){Y^f}^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} & \text{otherwise} \end{cases} \tag{36}$$

[8]See Eggertsson and Mehrotra (2014) for further discussion of this.

Analogously, for the foreign economy, we have:

$$Y_t^* = \begin{cases} Y^{*f} & \text{if } \Pi_t^* \geq \left(\frac{Y^{*f}}{Y_{t-1}^*}\right)^{\frac{1-\alpha}{\alpha}} \\ \left[\gamma^* \frac{Y_{t-1}^{*\frac{\alpha-1}{\alpha}}}{\Pi_t^*} + (1-\gamma^*) Y^{*f\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} & \text{otherwise} \end{cases} \tag{37}$$

We now adjust the middle generation household budget constraint to take account of labor income and profits - we replace Y_t in (2) with $\frac{W_t}{P_t}L_t + \frac{Z_t}{P_t}$. Noting that $Y_t = \frac{W_t}{P_t}L_t + \frac{Z_t}{P_t}$, we observe that the budget constraints takes on exactly the same form as before, and hence the first order conditions for the each generation maximization problem we derived in the endowment economy still apply. Hence, following the same steps as before, we can express the world real interest rate as:

$$1 + r_t^W = \frac{1+\beta}{\beta} \frac{\omega D_t + (1-\omega) D_t^*}{\omega(Y_t - D_{t-1}) + (1-\omega)(Y_t^* - D_{t-1}^*)}$$

which can be rewritten as an 'IS curve'

$$\omega Y_t + (1-\omega) Y_t^* = \frac{1+\beta}{\beta} \frac{\omega D_t + D_t^*}{1 + r_t^W} + \omega D_{t-1} + (1-\omega) D_{t-1}^* \tag{38}$$

We now have all the pieces together to explicitly define the equilibrium in the model.

Definition 1. *An equilibrium is a collection of stochastic processes* $\{Y_t, C_t^y, C_t^m, C_t^o, B_t^y, B_t^m, i_t, \Pi_t\}$ *for the domestic economy and* $\{Y_t^*, C_t^{y*}, C_t^{m*}, C_t^{o*}, B_t^{y*}, B_t^{m*}, i_t^*, \Pi_t^*\}$ *for the foreign and a world real interest rate* $\{r_t^W\}$ *that solves (1), (2), (6), (7), (8), (26), (36) (and corresponding 7 equations for the foreign economy) along with (28) and (29) and (38).*

The definition above is written for the model under full capital market integration. The equilibrium can be similarly defined under incomplete integration with the key modification being replacing the world IS curve (38) with the two IS curves (17) and (18).

5 Stagnation under Imperfect Integration: US and Japan, 2000-2008

With the model fully set up, we now wish to illustrate how this framework can be used to rationalize recent developments in the global economic environment. The first question we address, is whether our model is consistent with the fact that Japan appears to have been in a secular stagnation with zero interest rates since the mid/late 1990's while, in the US, the nominal interest rate only fell to zero following the economic crisis of 2008. Here we consider, for simplicity, a world in which Japan and US are the only two countries. While our analysis can in principle apply to any country at any time, by explicitly labeling countries, we can better organize our discussion and pick sensible parameters when characterizing our model to make our numerical example more illustrative.

With imperfect capital integration, it is possible to model a scenario in which Japan is in a secular stagnation, while US remains at full employment (asymmetric stagnation). The fall in interest rates and output in Japan, has clear spillover effects to the US in terms of the interest rate, current account, and credit conditions. We argue that asymmetric stagnation is a plausible description for the state of the global economy in the run-up to the Great Recession. In particular, while Japan was stuck at the zero lower bound and experiencing deflation, interest rates in the US remained unusually low, the US experienced a boom in household/consumer credit, and Japan accumulated a large net foreign asset position in US debt.

In steady state, an asymmetric stagnation can be summarized by the US natural rate of interest under imperfect integration:

$$1 + r^n = \frac{1+\beta}{\beta} \frac{(1+g)\,D}{Y_f - D + \frac{1-\omega}{\omega}\frac{1+\beta}{\beta} K^*} \tag{39}$$

The US economy remains relatively strong. The natural rate of interest remains sufficiently high that the equilibrium real interest rate is able to track the natural rate of interest. Output is at full-employment. In particular, the US inflation rate equals the inflation target: $\Pi = \bar{\Pi}$ and the nominal interest rate tracks the natural rate: $i = (1 + r^n)\,\bar{\Pi}$. The US real rate may be negative in an asymmetric stagnation so long as it exceeds the lower bound imposed by the domestic inflation target.

Japan, by contrast, has a natural rate of interest that violates the bound imposed by its inflation target. Thus, Japan falls into a secular stagnation characterized by a binding zero lower bound, inflation below target, and an output gap as real wages exceed the level needed to attain full employment. In steady state, inflation and output in Japan satisfy the following equilibrium conditions:

$$\frac{1}{\Pi^*} = \frac{1+\beta}{\beta} \frac{(1+g^*)\,D^* + \frac{1+r^n}{1+\beta} K^*}{Y^* - D^* - K^*} \tag{40}$$

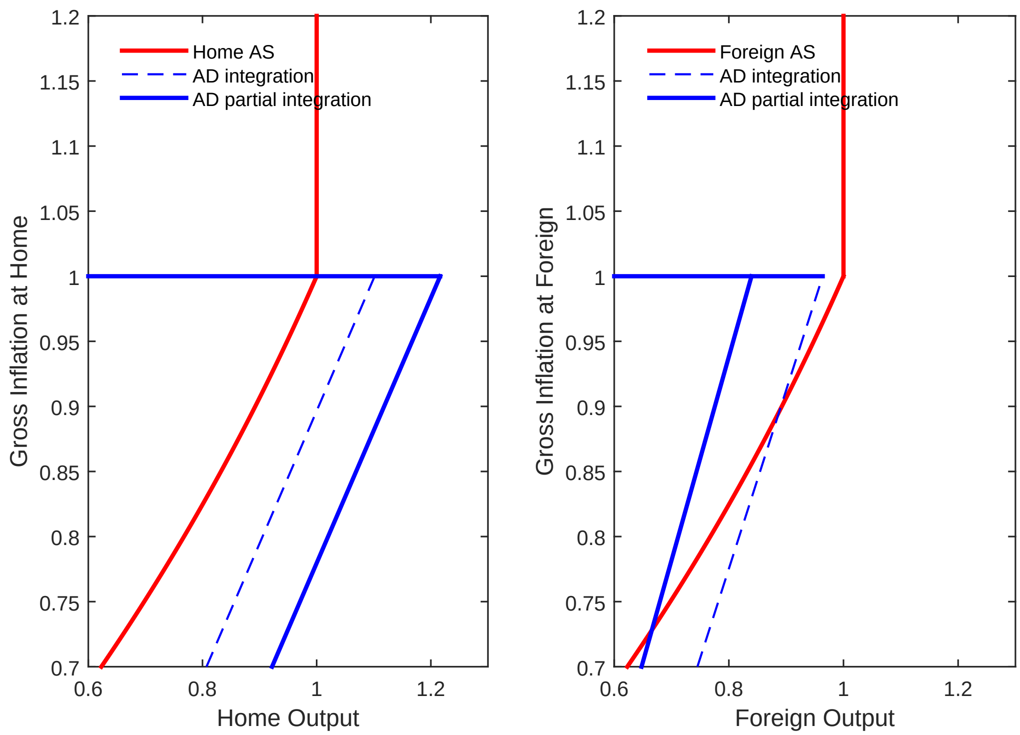
$$Y^* = \left(\frac{1 - \frac{\gamma^*}{\Pi^*}}{1 - \gamma^*} \right)^{\frac{\alpha}{1-\alpha}} Y_f^* \tag{41}$$

Together, equations (39), (40), and (41) determine r^n, Π^*, and Y^*. The latter two equations can be thought of as the aggregate demand and aggregate supply equations for Japan.[9]

The natural rate in equation (39) clearly exists and is unique for suitable parameter values. A solution exists and is unique for the creditor AD and AS curves as established in Proposition 2. As in Eggertsson and Mehrotra (2014), this secular stagnation equilibrium is locally determinate - for small shocks, the economy returns to secular stagnation steady state.

[9] For any given set of parameter values, we must verify that the relevant inequality constraints are satisfied: $r^n > 0$ and $r > r^*$.

Figure 5: Effect of an increase in international lending



Proposition 2. *If the international lending constraint is binding,* $r^n > 0$, $r^{n*} < 0$, $\bar{\Pi} = \bar{\Pi}^* = 1$ *and* $\gamma^* > 0$, *there exists a unique, locally determinate secular stagnation equilibrium in the creditor country with* $i^* = 0$, $\Pi^* < 1$, *and* $Y^* < Y_f^*$.

Proof. See Appendix A □

The net foreign asset position, which is determined by the international lending constraint K, reduces the severity of the output shortfall in Japan. As the lending constraint increases, the natural rate of interest in the debtor country falls absorbing the surplus saving of the creditor country. This fall in the natural rate is matched in the debtor country by a fall in the nominal interest rate keeping output and full-employment as seen in the left-hand panel of Figure 5. However, if there is no corresponding change in the inflation target, this fall in the debtor country's natural rate will make that country more vulnerable to disturbances that would cause the zero lower bound to bind. In other words, the debtor country would have less headroom to lower nominal rates in the face of unfavorable demand shocks.

In Japan an easing of the international lending constraint raises output by raising the rate of inflation as seen on the right-hand panel of Figure 5. A slackening of the international lending constraint reduces the output shortfall by lowering equilibrium real interest rates. This reduction in real interest rates comes from an increase in inflation with the nominal rate held constant at the zero lower bound. As can be seen in equation (40), an increase in K raises output Y^* when holding foreign inflation Π^* constant. In output/inflation space, an increase in K manifests itself as a shift out in the creditor country demand curve. Importantly, the natural rate of interest in the creditor country is rising with an increase in K despite the fall in the equilibrium real rate. The output shortfall falls as the gap between the equilibrium and natural rate closes.

The fall in real interest rate in the debtor country from an increase in capital flows from the country in secular stagnation raises the level of household indebtedness by easing the collateral constraint. The household debt to GDP ratio in the debtor country is given by:

$$\frac{B_t^y}{Y_t} = \frac{1+g_t}{1+r_t^n}\frac{D_t}{Y_t}$$

Since output in the debtor country remains at full-employment, the decline in the real interest rate and increase in the debt to GDP ratio implies that consumption by the young generation increases as a share of total consumption. These patterns qualitatively fit the rise in household debt and easing of collateral constraints experienced in the US during the credit boom between 2001 and 2008.[10]

Though the model considered here is fairly stylized, we calibrate the model to get a sense of the magnitudes of various parameters and the implications of capital flows for interest rates, output, and the external balance. In Table 1, we fit the model to match several targets for the US and Japan between 2002 and 2008. Typical parameters such as the rate of time preference and labor share are set to conventional values: $\beta = 0.96$, $\alpha = 0.7$. We must choose population growth rates, inflation targets, and collateral constraints for each country. Additionally, we must choose the wage rigidity parameter γ^* for Japan. We must also set the international lending constraint K.

For the US, the population growth rate is set at 1%, the US inflation target is set at 2% - the unofficial target of the Federal Reserve, and the nominal interest is set at 3% to match the average real interest rate from 2002-2007. The real interest rate pins down the level of the collateral constraint D. For Japan, the population growth rate is set at 0, the inflation target is set at 0% given that the Bank of Japan has only recently announced a 2% target. The rate of inflation is set at -0.5% to match the average real interest rate in Japan from 2002-2007. Given the OLG structure, periods last 20 years and all rates are converted accordingly.

Measuring the output gap is somewhat more difficult. We set the output gap at 10% based on the discussion in Hausman and Wieland (2014). US potential output is normalized to one, while potential output in Japan is set at 0.35 based on Japanese GDP (as a percentage of US GDP in 2007) at market exchange rates. The output gap and inflation rate in Japan pin down the foreign collateral constraint D^* and γ^*. The international lending constraint K is set to match the bilateral net foreign asset position. Based on TIC data from the Department of the Treasury, net Japanese holdings of debt securities in the US were approximately \$2 trillion in June 2008. The K value in

[10]This mechanism is also at work in the quantitative lifecycle model of Favilukis, Ludvigson and Van Nieuwerburgh (2015) who consider the effect of the global savings glut on US house prices and asset prices.

Table 1: Parameterization: US and Japan, 2002-2008

Panel A: Common parameters	*Symbol*	*Value*	
Labor share	α	0.7	
Discount rate	β	0.96^{20}	
Int'l lending constraint	K	0.14	
Panel B: Country-specific parameters	*Symbol*	*US*	*Japan*
Inflation target	$\bar{\Pi}, \bar{\Pi}^*$	2%	0%
Population growth	g, g^*	1%	0%
Potential output	Y_f, Y_f^*	1	0.34
Wage adjustment	γ, γ^*	N/A	0.296
Collateral constraint	D, D^*	0.237	0.071

Table 1 is the net foreign asset position as a percentage of 20-year GDP ($2 trillion/$14.5 trillion x 1/20).

These targets imply a modest degree of wage rigidity with $\gamma^* = 0.3$ - when $\gamma^* = 1$, wages are fully rigid. The collateral constraint is looser for the US but comparable across both countries. Given this calibration, we can consider the implications of autarky for secular stagnation in Japan. If $K = 0$, Japan's inflation rate would fall to -1.38% per year and the output gap would rise drastically from 10% to 28.6%. Based on this numerical example, the $2 trillion net asset position in the US significantly ameliorated Japan's output gap. Conversely, full capital market integration between Japan and US would pull Japan out of a secular stagnation with the world natural rate of interest. However, equilibrium world real interest rates would be quite low in this scenario at 0.87%.

By contrast, the effects of this large negative position were fairly modest for the US. Setting $K = 0$, the US nominal (and real) interest rate would be 7 basis points higher and the household debt to GDP ratio would be 1.5% percent lower relative to the baseline. However, these calculation ignore substantial capital inflows into the US during this time for other emerging market and oil-producing countries.

In short, in a world with incomplete financial integration, it is easy to construct numerical examples that match the broad patters in the data seen between the US and Japan. Japan gained significantly from capital market integration, as allowed it to export some of its excess savings to the US. In the US, this capital flow reduced interest rates, easing lending constraints, and boosted household debt - albeit by a modest amount. One interesting implication of this is that both countries would have benefitted from full financial integration, as this would have pulled Japan out of a secular stagnation, a conclusion that can easily be overturned in different setting as we now shall see.

6 Stagnation under Perfect Integration: Europe and US, 2008-2015

Since 2008, the zero lower bound problem is no longer limited to Japan, it has also spread to Europe and the US. While there surely exists some restrictions on capital flows across these trading partners, we do not need lending constraints to make sense of that last few years. In this section, we study the case of symmetric secular stagnation under perfect capital integration, and for concreteness, we consider the US and the Eurozone as our examples. This calibration help us understand why one region has fared worse than the other and shows that, while Europe may gain from financial integration, just as Japan in the last section, that no longer necessarily the case for the US.

We now consider a scenario in which the international lending constraints are slack and the world economy finds itself in stagnation. In this equilibrium, real interest rates are equalized across countries either because the international lending constraint is sufficiently large to allow for capital flows to equalize or autarky interest rates are not too far apart for each country (i.e. demand conditions are symmetric across countries). This scenario qualitatively fits several aspects of the global economy since 2008 with nominal interest rates at the zero lower bound across major economies.

Consider the US and Europe under a symmetric stagnation. Interest rates are equalized across countries and asset market clearing implies a single expression relating the world real interest rate to world output in steady state:

$$1+r=(1+g)\frac{1+\beta}{\beta}\frac{\omega D+(1-\omega)D^{*}}{\omega(Y-D)+(1-\omega)(Y^{*}-D^{*})} \tag{42}$$

where ω is the share of the domestic population in world population.[11] Equation (42) is a global aggregate demand equation.

It is helpful to label the world real interest rate when both countries are at full-employment as the world natural rate of interest:

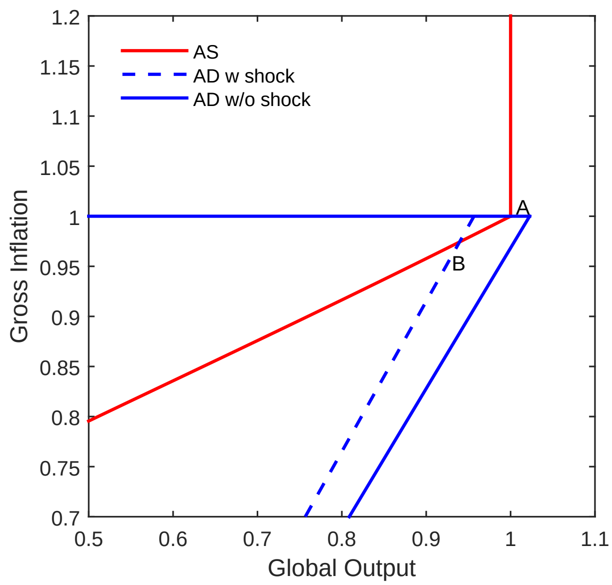
Definition 2. *The world natural rate of interest is given by:*

$$1+r^{W,Nat}=(1+g)\frac{1+\beta}{\beta}\frac{\omega D+(1-\omega)D^{*}}{\omega(Y_f-D)+(1-\omega)\left(Y_f^{*}-D^{*}\right)}$$

The zero lower bound $i, i^{*}\geq 0$ and the inflation target in each country place a lower bound on the *equilibrium* world real interest rate. The world real interest rate cannot fall below the inverse of the inflation target: $1+r^{W}\geq\frac{1}{\bar{\Pi}^{W}}$ where $\bar{\Pi}^{W}=\max\{\bar{\Pi},\bar{\Pi}^{*}\}$. As we show, if the world natural rate falls

[11]Existence of a steady state requires population growth to be equalized across countries.

Figure 6: Global stagnation



below this bound, then there exists a symmetric stagnation equilibrium with output shortfalls in both countries.

The output gap in each country is determined by the deviation of inflation below the inflation target. Assuming a symmetric inflation target of $\bar{\Pi}$, the output gap in each country is given by the following equations:

$$Y = \left(\frac{1 - \frac{\bar{\Pi}\gamma}{\Pi}}{1 - \gamma} \right)^{\frac{\alpha}{1-\alpha}} Y^{fe} \tag{43}$$

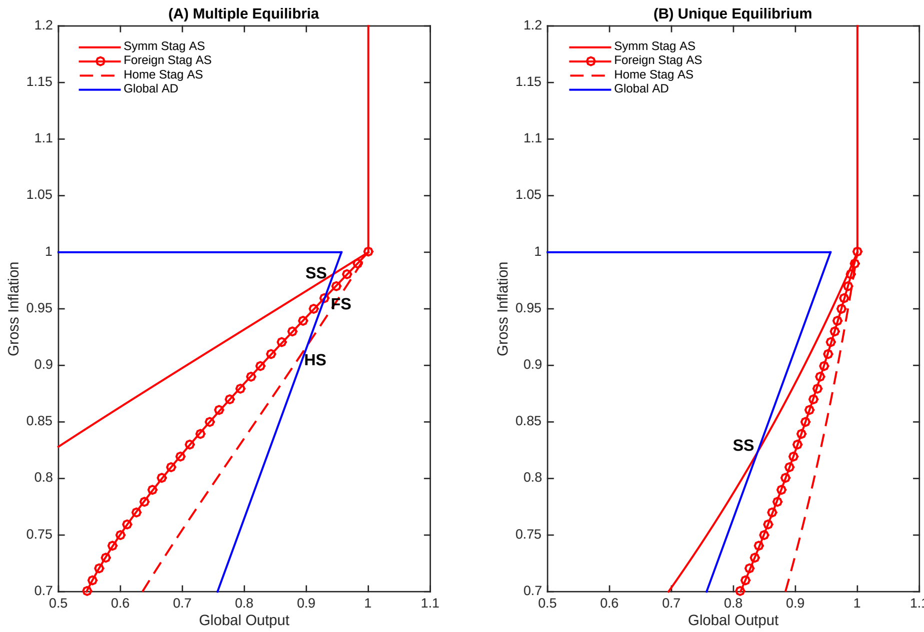
$$Y^* = \left(\frac{1 - \frac{\bar{\Pi}\gamma^*}{\Pi}}{1 - \gamma^*} \right)^{\frac{\alpha}{1-\alpha}} Y^{fe*} \tag{44}$$

$$1 + r = \frac{1}{\Pi} \tag{45}$$

where equation (43) and equation (44) the domestic and foreign AS curves, and (45) is the Fisher equation defining the real interest rate when the zero lower bound is binding in both the home and foreign country. Equations (42) - (45) jointly determine the endogenous variables r, Π, Y, and Y^* in a symmetric stagnation equilibrium.

If the world natural rate of interest is negative, then a symmetric stagnation equilibrium exists. This is essentially an application of Proposition 1 in Eggertsson and Mehrotra (2014) where the AD curve is now the global AD curve in equation (42) and the global AS curve is obtained by the population weighted average of the domestic and foreign AS equations. Moreover, the symmetric stagnation equilibrium is locally determinate in contrast to the deflation steady state considered in Benhabib, Schmitt-Grohé and Uribe (2001). The symmetric stagnation equilibrium is graphically

Figure 7: Asymmetric stagnation



depicted in Figure 6. If we assume a symmetric inflation target for both countries, then we can formally establish these results in Proposition 3.

Proposition 3. *If* $r^{W,Nat} < \bar{\Pi}^{-1}$*, there exists a locally determinate secular stagnation equilibrium with* $Y < Y_f$*,* $Y^* < Y_f^*$*,* $i = i^* = 0$ *and* $\Pi < \bar{\Pi}$*.*

Proof. See Appendix A □

6.1 Multiple Equilibria

When the international lending constraint is slack and global rates are equalized, there may exist additional equilibria in addition to the symmetric stagnation equilibria. It is possible for the domestic (foreign) country to be in stagnation with inflation below target while the foreign (domestic) economy has positive nominal rates and output at its full-employment level. Real rates are still equalized because the rate of deflation in the domestic economy equals the nominal interest rate in the foreign economy (without loss of generality, assume that the inflation target $\bar{\Pi} = 1$ for both the domestic and foreign economies). The left panel of Figure 7 displays the asymmetric equilibria. When $\gamma \neq \gamma^*$, then there exist two distinct asymmetric equilibria (FS and HS). The symmetric stagnation equilibria is the third intersection point (SS).

The equilibrium conditions for asymmetric stagnation are given below. The world real interest rate and output in the country in stagnation are determined by:

$$1+r=(1+g)\frac{1+\beta}{\beta}\frac{\omega D+(1-\omega)D^{*}}{\omega(Y-D)+(1-\omega)\left(Y_{f}^{*}-D^{*}\right)} \tag{46}$$

$$Y=\left(\frac{1-\frac{\bar{\Pi}\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}Y_{f} \tag{47}$$

where the assumption is that the foreign country is at full-employment and the domestic economy absorb the full shortfall in demand. Equations (46) - (47) and equation (45) jointly determine r, Π, and Y. An analogous set of equilibrium conditions determine foreign output Y^{*} and Π^{*} if the domestic economy is at full employment.

It can be shown that the asymmetric stagnation equilibria exists if the global collateral constraint exceeds the level of output of the country not in stagnation.[12] Likewise, relatively high degrees of wage rigidity make it more likely for multiple equilibria to emerge. The right-hand panel of Figure 7, illustrates the case where wage rigidity is sufficiently low that only a symmetric stagnation exists. In an asymmetric stagnation, only one country must absorb the entire shortfall in world output. Intuitively, supply exceeds demand and if the higher interest rates drive down global demand faster than global supply no equilibria exists. The failure of the AD and AS curves to cross is due to the fact that global supply in an asymmetric stagnation is bounded below by the full-employment level of output in the country not in stagnation. In a symmetric stagnation, there always exists a sufficiently high rate of deflation that drives global output to zero while demand remains bounded away from zero.

We can formalize the case of an asymmetric stagnation when the world real interest rate is negative. In this case the home country absorbs the world shortfall, while the foreign country remains at full employment. Interest rates are equalized across countries. Proposition 4 establishes conditions and properties of this equilibrium. The analogous conditions establish when the mirror case occurs: home country at full employment, foreign country in stagnation. Depending on parameter values, both, one or neither of these asymmetric stagnation equilibria may emerge.

Proposition 4. *If $r^{W,Nat}<\bar{\Pi}^{-1}$, $D^{W}>(1-\omega)Y_{f}^{*}$, $\gamma>0$, there exists a unique, locally determinate asymmetric secular stagnation with $r=r^{*}$, $Y<Y_{f}$, $Y^{*}=Y_{f}^{*}$, $i=0$, and $\Pi<\bar{\Pi}$.*

Proof. See Appendix A □

The world real interest rate under asymmetric stagnation exceeds the interest rate under symmetric stagnation. Since the output shortfall must be borne by just one country, a higher rate of deflation is required raising the world interest rate. This higher world interest rate also likely worsens welfare for the country not in secular stagnation since a higher real interest rate tightens the collateral constraint for the young limiting the degree of consumption smoothing.

[12]This is a sufficient, not a necessary condition

Table 2: Parameterization: US and Eurozone, 2008-2015

Panel A: Common parameters	*Symbol*	*Value*	
Labor share	α	0.7	
Discount rate	β	0.96	
Inflation target	$\bar{\Pi}$	1.75%	
Population growth	g	1%	

Panel B: Country-specific parameters	*Symbol*	*US*	*Eurozone*
Potential output	Y_f, Y_f^*	1	0.96
Wage adjustment	γ, γ^*	0.217	0.297
Collateral constraint	D, D^*	0.157	0.136

Panel C: Counterfactual under autarky	*Symbol*	*US*	*Eurozone*
Output gap	Y, Y^*	0%	21.3%
Nominal rate	i, i^*	0.25%	0%
Welfare (rel. to integration)	U, U^*	$+7.5\%$	-4.2%

6.2 US and Eurozone: 2008-2015

We can calibrate our model to generate a symmetric stagnation that matches aspects of the global economic environment in the wake of the Great Recession. To calibrate the model, we choose the wage rigidity parameters in the US and the Eurozone to match output gaps in each region and chose the collateral constraints to match global interest rates and the net foreign asset position of the Eurozone in the US. Standard parameters - the rate of time preference β and the labor share α are set as before. The growth rate g is set at 1% annually in accordance with the average population growth rate across the regions.

Both the US and Eurozone nominal rates are set at zero given the zero lower bound has remained binding in each region over this period. The inflation target is set at 1.75% to reflect the Eurozone's somewhat lower desired inflation target. The inflation rate is set at 1% in both regions to equate the world real interest rate at -1% - approximately consistent with US and Eurozone short-term real rates between 2008-2015. The full-employment level of output is normalized to unity in the US and 0.96 in the Eurozone based on GDP relative to the US evaluated at market exchange rates in 2008.

We target an output gap in the US and Eurozone at 10% and 15% respectively reflecting the deviation of real GDP per capita in the US and Eurozone relative to pre-recession trend. The average output gap and global real interest rate pin down the average collateral constraint. In

June of 2013, Eurozone holdings of US debt securities net of US holding of Euro debt equaled $2 trillion. The net foreign asset target is computed as a percentage of 20-year GDP ($2 trillion/$16.5 trillion x 1/20). The foreign asset position determines the difference between the US and Eurozone collateral constraints: D and D^*.

In Table 2, we show the implied parameter values that match the targets described above. Wage rigidity in the US and Eurozone are comparable and, for the Eurozone, imply a somewhat greater degree of wage adjustment than found in Schmitt-Grohé and Uribe (2011). In particular, the US display more flexible wages than the Eurozone implying that, for a given level of inflation below target, the shortfall in output is less in the US than the Eurozone. These wage rigidity values are also consistent with the more prominent role of unions in wage-setting in the Eurozone.

The collateral constraints are also comparable across regions and somewhat tighter in the Eurozone to reflect the fact that net capital flows towards the US. Interestingly, in steady state, the US runs a trade deficit with the Eurozone despite the fact that the US has a negative net foreign asset position. Since the equilibrium world interest rate is negative, the US is, in effect, paid to borrow from the Eurozone. This permanent trade deficit is however quite small - only 0.22% of GDP in steady state.

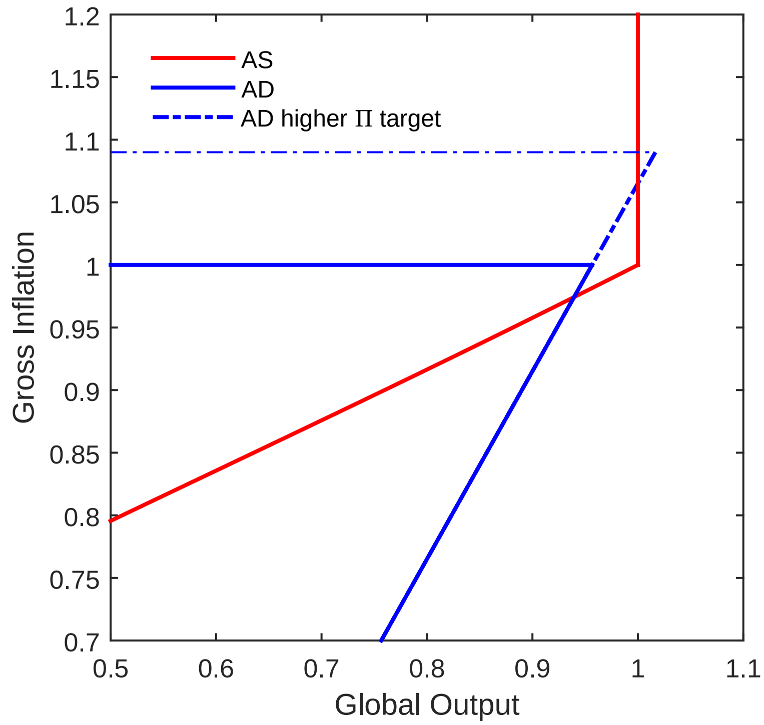
Table 2, Panel C displays the counterfactual case of financial autarky. In the absence of capital integration, the natural rate of interest would be -1.5% in the US and -2.1% in the Eurozone. At the assumed inflation target, the US would be able to remain at full-employment. In other words, net capital flows from the Eurozone pushed the US into a secular stagnation. These values also suggest that only a modest increase in inflation expectations is needed to attain the world natural rate of interest. Under autarky, the output gap in the Eurozone would worsen by 6 percentage points and the inflation rate would fall to 0.7%. As Table 2 shows, US welfare increases under autarky while Eurozone welfare worsens. Any gains for further consumption smoothing under integration are offset by the gains from a smaller output gap in the US.

7 Monetary Policy

7.1 Increasing the Inflation Target

Imagine that both countries have an inflation target of $\Pi = \Pi^* = 1$ (net inflation of zero) and that the world natural rate of interest is negative. Now imagine that the domestic economy announces an inflation target that is higher than 0 while the foreign economy does not. If the domestic economy reaches its new inflation target, the only equilibrium is an asymmetric secular stagnation where the foreign economy is trapped and the domestic economy is not. This will be important when we analyze exchange rates. Note, however, there is no guarantee that the domestic economy reaches its new target - one cannot exclude the possibility of the domestic economy finding itself

Figure 8: Effect of raising the inflation target



in a secular stagnation as well.

Consider now a symmetric equilibria where both countries are in secular stagnation, and both countries increase their inflation target. Our model does not have much new to day about this thought experiment. In essence it is equivalent to the experiment examined in Eggertsson and Mehrotra (2014) in a closed economy. The experiment is depicted in Figure 8. An increase in both inflation targets $\bar{\Pi}^* = \bar{\Pi}$ shifts up the kink point in the AD curve. This allows for the possibility of a new equilibrium at the intersection of the two curves at the new inflation target. While the higher inflation target allows for a full employment equilibria, it does not by itself exclude the secular stagnation equilibria. This multiplicity provides one motivation for considering other policy options including fiscal policy. Before getting there, however, it is worth highlighting that the model does have something new to say about about how the relative monetary policies of the two countries affects equilibrium.

7.2 Currency Wars

Our key assumption was that the central bank set policy using the short term nominal interest rate. The logic for this assumption traditionally given is that the government controls the money supply, and the short-term risk-free nominal interest rate is the opportunity cost of holding money. Thus the nominal rate can be set from a money demand equation, i.e. for a particular value of the nominal stock of money one can determine the nominal interest rate. The only complication is that the return on money can never go below zero (as then people would hold money as an asset) and hence the zero bound. At the zero bound, any supply of money above the satiation level is consistent with a zero nominal interest rate (see Eggertsson and Woodford (2003)).

Strictly speaking, the government does not control the nominal exchange rate in the model. The exchange rate is the relative price of the currencies (one unit of dollar in terms of euro) of the two countries which is given by the relative price level $\frac{P_t}{P_t^*}$. Once the zero bound become binding, the central bank no longer has direct control of the price level due to the zero bound. Accordingly, it does not have control of the exchange rate either.

Many authors have suggested that the exchange rate can act as an additional instrument of policy and that the central bank can affect the exchange rate by printing money and buying foreign currencies (see e.g. Bernanke (2000)). In the model, however, the exchange rate is the price of interest rate differentials between the two countries (see equation (28)). This equation does not directly depend on the relative asset position of either central bank beyond the effect the money supply has on the nominal interest.

As has been argued by Eggertsson (2006), one way in which foreign asset market purchases could have an effect is by signaling something about future monetary policy, for example making future interest rate policy "credible". Without trying to model that mechanism here it is interesting to ask the following question: Suppose the government could commit to a particular path for the nominal exchange rate (without modeling exactly how and through what mechanism it can achieve that nominal peg). What is the implication for the equilibrium we have studied?

Consider first the case of an asymmetric secular stagnation. A key implication of the asymmetric stagnation is that one country is producing at full capacity, while the other is not. As a result, the nominal exchange rate is continuously appreciating for the country in stagnation. Consider the example in which the domestic economy is in stagnation, so that $i = 0$ but the foreign economy achieves its inflation target of $\bar{\Pi}^* = 1$. Then:

$$\Pi^D = \left(\frac{S_{t+1}}{S_t}\right)^{-1}$$

i.e. the nominal exchange rate of the stagnated domestic economy is continuously appreciating at the rate of deflation of the domestic economy.

Consider now a policy in which the domestic economy manages to peg its nominal exchange rate so that $S_{t+1} = S_t$. In this case a straightforward proposition follows:

Proposition 5. *Suppose $r^{W,Nat} < 0$, the domestic and foreign inflation targets are given by $\bar{\Pi} = \bar{\Pi}^* = 1$, and the nominal exchange rate is pegged at $\bar{S}_{t+1} = S_t = \bar{S}$. Then the global symmetric secular stagnation equilibria is the unique solution of the model.*

The proof of this proposition follows directly from the fact that if nominal peg is constant, the inflation rates of the two countries must be the same. Since their inflation target of $\bar{\Pi} = \bar{\Pi}^* = 1$ cannot be achieved due to the negative world natural interest rate, the only equilibria is the symmetric secular stagnation equilibria.

An interesting implication of this proposition is that if a given country finds itself in a secular stagnation (and the other does not) and reacts by pegging its exchange rate, it does not escape stagnation. Instead, it exports deflation to its trading partner.

Can a country escape a secular stagnation all together? Denote the rate deflation of the foreign country if it finds itself in a asymmetric secular stagnation by $\Pi^{FS*} < 1$. Suppose now that the nominal exchange rate of the domestic economy is such that $\frac{S_{t+1}}{S_t} < \Pi^{FS*}$. We then obtain the following proposition:

Proposition 6. *Suppose $\frac{S_{t+1}}{S_t} < \Pi^{FS*}$ and the world natural rate of interest is negative. Then there exists no equilibrium in which the domestic economy is in a secular stagnation, but, if $\bar{\Pi}^* = 1$, the foreign economy must always be secular stagnation.*

The proof of this proposition follows directly from the fact that $\frac{S_{t+1}}{S_t} < \Pi^{FS*}$ implies that the inflation rate in domestic economy has to be higher than in the foreign economy. Since the inflation target of the foreign economy is $\Pi^* = 1$, we can exclude the possibility that neither one is in a secular stagnation, and since the inflation rate of the domestic economy is higher, the only feasible equilibria is one in which the domestic economy is not in stagnation so it is achieving its inflation target $\bar{\Pi}$ (which can be any number equal to or greater than 1), while the foreign economy is trapped.

An interesting element of the last two propositions, is that a policy commitment that is framed in terms of the nominal exchange rate when the world natural rate of interest is negative, is always going to come at the expense of the trading partner. This is in contrast to the higher inflation target of both countries we studied first in which case both countries win. A problem there, however, was that we could not exclude the secular stagnation equilibria. Can this be done? The answer is yes. That bring us to fiscal policy.

8 Fiscal Policy

8.1 Government Spending

One natural policy to consider is an increase in government spending. However, a key consideration is how an increase in government spending is financed. For example, if it is financed via a tax on the credit-constrained young, a fiscal expansion has no effect, since then every government spending increase will be met by a corresponding cut in the spending by the young.[13] For now, let

[13] See Eggertsson and Mehrotra (2014) for elaboration of this point.

Table 3: Government spending multipliers

Output Multiplier	Symmetric Stag	Asymmetric Stag	Autarky
$\frac{\mathrm{d}Y}{\mathrm{d}G}$	1.39	2.20	1.35
$\frac{\mathrm{d}Y^*}{\mathrm{d}G}$	1.39	0	0
$\frac{\mathrm{d}Y^{world}}{\mathrm{d}G}$	2.78	2.20	1.35

us consider a balanced budget. The most natural balanced budget assumption is that the spending is financed via tax on the working population which is the middle generation.

Adding taxes T_t^m to the budget constraint of the middle generation, and assuming $G_t = T_t^m$ we can once again derive the world natural interest rate under perfect integration:

$$1 + r_t^W = (1 + g_t) \frac{1+\beta}{\beta} \frac{\omega D_t + (1-\omega) D_t^*}{\omega (Y_t - G_t - D_{t-1}) + (1-\omega) \left(Y_t^* - G_t^* - D_{t-1}^*\right)} \tag{48}$$

Replacing (38)) with and (48) we can define the equilibrium in the same way as at the end of Section 4. Meanwhile, the real interest rate under autarky, can similarly be derived to yield:

$$1 + r_t^A = \frac{1+\beta}{\beta} \frac{(1+g_t) D_t}{Y_t - G_t - D_{t-1}} \tag{49}$$

To start with, let us summarize the effect of government spending by computing the multiplier of government spending. This value says by how much output increases for every dollar in government spending. For domestic government spending (the foreign is symmetric) we compute both the world output multiplier and the domestic output multipliers. Observe that this is a local statistic, that is, we are computing the effect of output for small increases in spending. We summarize the multipliers in the next proposition

This proposition reveals several interesting insights. The first thing to notice is that because labor supply is inelastic in our model, government spending has no effect under normal circumstances, it just reallocates output from private to public consumption. It is only at zero interest rate that it matters. There, we have several results. First, the domestic government spending multiplier is greater than one. In our particular numerical example from previous section it is 1.35. Another interesting aspect is that this is only a part of the story. If the entire world is in secular stagnation then the world output multiplier is two times larger, e.g. 2.78 in our numerical example.

This suggest that there are positive demand externalities of government spending, i.e. the foreign country will also benefit from the increase in demand. In fact, we see that the foreign country benefits exactly by the same amount as the home country in our model.

One implication of this proposition is that, if government spending stimulus is costly, then each of the government under provides it if it only cares about the welfare of its own citizens and does not coordinate policy with its trading partner. To formalize this insight, let us assume that

Table 4: Increase in government spending under different regimes

	ΔG (as % of steady state Output)
Non-Cooperative Regime	5.28 %
Cooperative Regime	10.57 %

each government has a period utility function given by:

$$L_t = (\Pi_t - 1)^2 + (Y_t - Y^f)^2 + (G_t - G^{\text{target}})^2$$

and the utility function for the foreign government is the same but in terms of the foreign variables. Let us call a solution that selects G and G^* jointly to maximize both countries objective function simultaneously the cooperative solution. Let us call the solution if each country maximizes its own objective, taking the other countries spending as given the non-cooperative solution. In the Appendix, we prove the following proposition:

Proposition 7. *Government spending in the noncooperative solution is smaller than in the cooperative solution, given the conditions stated in the Appendix.*

Proof. See Appendix B □

The logic of this proposition is pretty straightforward. Because the fiscal stimulus is costly (government is larger than its optimal size in absence of frictions) each country supplies too little stimulus to stabilize world output, because the benefits are equally borne by foreigners. Table 4 shows the numerical solutions to the game for the parameters discussed before. For a given secular stagnation regime, each government increases its expenditure by about 5% when it *unilaterally* makes the decision as compared to the cooperative regime, where each government increases the expenditure by twice as much. This suggests that the scope for international coordination in a secular stagnation is considerable.

8.2 Debt Policy

We have already shown, in Section 3, how the model can be generalized to include government debt. As we showed, foreign reserve accumulation puts downward pressure on the natural rate of interest, as can be seen in equation (24).

Equation (24) has other important implications. In particular, we see that an increase in government debt will directly increase the natural rate of interest. Thus a straightforward solution to secular stagnation is to increase government debt. That foreign reserve accumulation puts downward pressure on the natural rate of interest is essentially the inverse of this since an increase in reserve accumulation reduced the total amount of government bonds held by the private sector.

But will increasing government debt always work? Our model is silent on what type of limitations may constrain a government's ability to issue debt. It is not very hard to think of some reasonable limitations, however. One possibility is, give a probability that the forces that give rise to secular stagnation ultimately reverse themselves, that the interest rate rises. If the government has accumulated large amounts of debt, the real cost of servicing this debt may be quite high. Higher distortionary taxation may result in welfare losses, and hence put limits on the amount of debt the government is willing to issue. Another possibility is that if debt goes above a certain level, this triggers uncertainty about if debt will get repaid again. To the extent that government debt serves as a collateral in various private transactions, this may even have a negative effect on a broad class of what had been considered ”safe assets”. This is another kind of mechanism we have not modeled, but one could imagine may in principle be important.

To the extent such constraints exist on the government's willingness to issue debt, a reasonable approximation to the government's objective function might take a similar form as we saw in the last section. The government would choose the optimal level of debt to minimize a loss function of the following form (in this case holding G_t constant for simplicity):

$$L_t = (\Pi_t - 1)^2 + \lambda_Y (Y_t - Y_t^f) + \lambda_b (B_t - B^{safe})^2$$

where we denote B^{safe} as the level above which agents start putting some probability on a government default. If the loss function of the government of each country takes this form, then policy will be subject to exactly the same problem as we reviewed in previous section: each government has the incentive to ”free ride” on the fiscal stimulus of the other country. Again, this suggest a considerable gains for policy coordination.

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A Proofs

Here we provide formal proofs for various propositions presented in the body of the text.

Proposition 2 *If the international lending constraint is binding, $r^n > 0$, $r^{n*} < 0$, $\bar{\Pi} = \bar{\Pi}^* = 1$ and $\gamma^* > 0$, there exists a unique, locally determinate secular stagnation equilibrium in the creditor country with $i^* = 0$, $\Pi^* < 1$, and $Y^* < Y_f^*$.*

Proof. Under the assumptions of the proposition and the monetary policy rule, the zero lower bound is binding for the creditor country and the equilibrium real interest rate in steady state is given by $r^* = \frac{1}{\Pi^*}$. Equilibrium inflation and output in steady state in the creditor country solve the following equations:

$$Y^* = D^* + K^* + \psi^* \Pi^* \tag{A.1}$$

$$Y^* = \left(\frac{1 - \frac{\gamma^*}{\Pi^*}}{1 - \gamma^*} \right)^{\frac{\alpha}{1-\alpha}} Y_f^* \tag{A.2}$$

where $\psi^* = \frac{1+\beta}{\beta} (1+g) \left(D^* + \frac{1+r^n}{1+\beta} K^* \right) > 0$. We may define the difference equation $\Delta(\Pi^*)$ by taking the difference between ($A.1$) and ($A.2$) . This function is continuous in Π^* with $\Delta(\gamma^*) > 0$ and $\Delta(1) < 0$. Therefore, there exists a $\gamma^* < \Pi^* < 1$ such that $\Delta(\Pi^*) = 0$. Since $\Pi^* < 1$, it follows that $Y^* < Y_f^*$.

To establish uniqueness, we first assume that their exist multiple distinct values of Π^* at which $\Delta(\Pi^*) = 0$. Graphically, in inflation-output space (output on the x-axis), the AS curve (equation ($A.2$)) lies above the AD curve (equation ($A.2$)) when inflation equals γ^* and the AS curve lies below the AD curve for inflation at unity. Thus, if multiple steady states exist, given that AS is a continuous function, there must exist at least three distinct points at which the AS and AD curve intersect.

At the first intersection point, the slope of AS curve crosses the AD line from above and, therefore, at the second intersection the AS curve crosses the AD curve from below. Since the AD curve is a line, the AS curve is locally convex in output in this region. Similarly, between the second and third intersection, the AS curve is locally concave in output. Thus, as output Y^* increases, the AS curve must first have a positive second derivative followed by a negative second derivative.

We compute the second derivative of inflation with respect output of the AS curve and derive the following expression (we drop the $*$ for simplicity):

$$\frac{d^2\Pi}{dY^2} = G(Y)\left((1+\phi)(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi} + (\phi - 1)\right) \tag{A.3}$$

$$G(Y) = \frac{\phi\gamma(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi}}{Y^2\left(1-(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi}\right)} \tag{A.4}$$

$$\phi = \frac{1-\alpha}{\alpha} \tag{A.5}$$

As can be seen, over the region considered, the function $G(Y)$ is positive and, therefore, the convexity of the AS curve is determined by the second term. This term may be negative if $\phi < 1$, but this expression is increasing in Y between 0 and Y_f. Therefore, the second derivative cannot switch signs from positive to negative. Thus, we have derived a contradiction by assuming multiple steady states. Therefore, there must exist a unique intersection point.

As established before, it must be the case that the AS curve has a lower slope than the AD curve at the point of intersection. The slope of the AS curve is:

$$\frac{d\Pi^*}{dY^*} = \frac{1-\alpha}{\alpha}\frac{1}{\gamma^*}\frac{\Pi^*}{Y^*}\left(\Pi^* - \gamma^*\right) \tag{A.6}$$

If the slope of the AS curve is less than the slope of the AD curve at the intersection point, then it must be the case that:

$$\frac{1-\alpha}{\alpha}\frac{\Pi^*}{Y^*}\left(\frac{\Pi^*}{\gamma^*} - 1\right) < (\psi^*)^{-1}$$

$$\frac{1-\alpha}{\alpha}\frac{\psi^*\Pi^*}{Y^*}\left(\frac{\Pi^*}{\gamma^*} - 1\right) < 1$$

$$\frac{1-\alpha}{\alpha}\frac{Y^* - D^* - K^*}{Y^*}\left(\frac{\Pi^*}{\gamma^*} - 1\right) < 1$$

$$s_y\frac{\alpha}{1-\alpha} + 1 > \frac{\Pi^*}{\gamma^*}$$

$$\frac{\gamma^*}{\Pi^*}\left(s_y\frac{\alpha}{1-\alpha} + 1\right) > 1$$

Linearizing the equilibrium conditions around the secular stagnation steady state, we obtain the following linearized AD and AS equations:

$$0 = E_t\pi_{t+1} - s_y y_t^* + d_t^* + s_d d_{t-1}^* \tag{A.7}$$

$$y_t^* = \gamma_w^* y_{t-1} + \gamma_w^*\frac{\alpha}{1-\alpha}\pi_t^* \tag{A.8}$$

where d_t^* is the collateral shocks and various coefficients are given in terms of their steady state values.

$$\gamma_w^* = \frac{\gamma^*}{\bar{\Pi}^*}$$
$$s_y = \frac{\bar{Y}^*}{\bar{Y}^* - \bar{D}^* - \bar{K}^*}$$
$$s_d = \frac{\bar{D}}{\bar{Y}^* - \bar{D}^* - \bar{K}^*}$$

Substituting ($A.8$) into ($A.7$), we obtain a forward looking difference equation in y_t^*. The local determinacy condition requires the coefficient on $E_t y_{t+1}^*$ to be less than one. This condition is the same as the slope condition. Therefore, the unique secular stagnation steady state is always locally determinate as required. □

Proposition 3 *If $r^{W,Nat} < \bar{\Pi}^{-1}$, there exists a locally determinate secular stagnation equilibrium with $Y < Y_f$, $Y^* < Y_f^*$, $i = i^* = 0$ and $\Pi < \bar{\Pi}$.*

Proof. Under the assumptions of the proposition, monetary policy in both countries cannot track the world natural rate of interest and $i = i^* = 0$. Perfect capital market integration requires equalization of the domestic and foreign real interest rate, hence $\Pi = \Pi^*$. Steady state inflation, domestic output, and foreign output jointly satisfy the following equilibrium conditions:

$$\omega Y + (1+\omega) Y^* = D^W + \psi^W \Pi \tag{A.9}$$
$$Y = \left(\frac{1 - \frac{\gamma \bar{\Pi}}{\Pi}}{1-\gamma} \right)^{\frac{\alpha}{1-\alpha}} Y_f \tag{A.10}$$
$$Y^* = \left(\frac{1 - \frac{\gamma^* \bar{\Pi}}{\Pi}}{1-\gamma^*} \right)^{\frac{\alpha}{1-\alpha}} Y_f^* \tag{A.11}$$

where $D^W = \omega D + (1-\omega) D^*$ and $\psi^W = \frac{1+\beta}{\beta} (1+g) D^W > 0$. We may define the difference equation $\Delta(\Pi)$ by taking the difference between ($A.9$) and the weighted sum of ($A.10$) and ($A.11$). Without loss of generality, assume that $\gamma < \gamma^*$. We assume that output is bounded below by zero - that is, if $\Pi < \gamma^*$, then $Y^* = 0$. Given this assumption, the function $\Delta(\Pi)$ is continuous (but not necessarily differentiable), with $\Delta(\gamma) > 0$ and $\Delta(\bar{\Pi}) < 0$. Therefore, there exists a global inflation rate Π_{ss} with $\Pi_{ss} < \bar{\Pi}$ implying that, in steady state, $Y_{ss} < Y_f$ and $Y_{ss}^* < Y_f^*$.

To establish that this steady state is locally determinate, we observe that, graphically, at $\Pi = \gamma$ the global AS curve (weighted sum of equations ($A.10$) and ($A.11$)). At $\Pi = \bar{\Pi}$, the global AD curve lies above the AS curve. Thus, there exists at least one equilibrium in which the AD curve is locally steeper than the AS curve. We first derive the condition for local determinacy. The log-linearized equilibrium conditions for a symmetric stagnation equilibrium are given below:

$$E_t\pi_{t+1} = \bar{\omega} s_y y_t + (1-\bar{\omega})\, y_t^* + shocks \tag{A.12}$$

$$y_t = \gamma_w y_{t-1} + \gamma_w \phi \pi_t \tag{A.13}$$

$$y_t^* = \gamma_w^* y_{t-1}^* + \gamma_w^* \phi \pi_t \tag{A.14}$$

where $\phi = \frac{\alpha}{1-\alpha}$ and the other coefficients are defined below:

$$\gamma_w = \frac{\gamma\bar{\Pi}}{\Pi_{ss}}$$
$$\gamma_w^* = \frac{\gamma\bar{\Pi}}{\Pi_{ss}}$$
$$s_y = \frac{\omega Y_{ss} + (1-\omega)\, Y_{ss}^*}{\omega(Y_{ss} - D) + (1-\omega)\,(Y_{ss}^* - D^*)}$$
$$\bar{\omega} = \frac{\omega Y_{ss}}{\omega Y_{ss} + (1-\omega)\, Y_{ss}^*}$$

This linearized system can be expressed as:

$$AE_t x_{t+1} = Bx_t + shocks$$
$$E_t x_{t+1} = A^{-1}Bx_t + A^{-1}shocks$$

where $x_t = [\pi_t, y_{t-1}, y_{t-1}^*]'$ and the A, B are square matrices with suitably defined coefficients. Local determinacy requires that the matrix $A^{-1}B$ has exactly one eigenvalue outside the unit circle.

Since the matrix B has a row of zeros, one eigenvalue of the system is zero. The characteristic polynomial that determines the remaining eigenvalues is:

$$\lambda^2 - \left(\phi s_y \left(\bar{\omega}\gamma_w - (1-\bar{\omega})\,\gamma_w^*\right) + \gamma_w + \gamma_w^*\right)\lambda + \gamma_w\gamma_w^*\,(1 + s_y\phi) = 0$$

Since the characteristic polynomial is positive at $\lambda = 0$, the condition that ensures local determinacy is that the characteristic polynomial is negative at $\lambda = 1$. This condition requires:

$$1 + \gamma_w\gamma_w^*\,(1 + s_y\phi) < \phi s_y \left(\bar{\omega}\gamma_w - (1-\bar{\omega})\,\gamma_w^*\right) + \gamma_w + \gamma_w^* \tag{A.15}$$

It remains to show that this local determinacy condition is identical to the slope condition that must be satisfied in equilibrium. The slope of the global AS curve and global AD curve is given below:

$$\frac{dY_{AS}^W}{d\Pi} = \phi\left(\omega\gamma_w \frac{Y_{ss}}{\Pi_{ss} - \gamma\bar{\Pi}} + (1-\omega)\,\gamma^* \frac{Y_{ss}^*}{\Pi_{ss} - \gamma^*\bar{\Pi}}\right)$$
$$\frac{dY_{AD}^W}{d\Pi} = \psi^W$$

A steeper slope for the AD curve relative to the AS curve implies:

$$\frac{dY_{AS}^W}{d\Pi} > \frac{dY_{AD}^W}{d\Pi}$$

$$\phi\left(\omega\gamma_w \frac{Y_{ss}}{\Pi_{ss} - \gamma\bar{\Pi}} + (1-\omega)\,\gamma_w^* \frac{Y_{ss}^*}{\Pi_{ss} - \gamma^*\bar{\Pi}}\right) > \psi^W$$

$$\phi\left(\bar{\omega}\frac{\gamma_w}{1-\gamma_w} + (1-\bar{\omega})\,\frac{\gamma_w^*}{1-\gamma_w^*}\right) > \psi^W \frac{\Pi_{ss}}{Y^W}$$

$$\phi s_y\left(\bar{\omega}\gamma_w\left(1-\gamma_w^*\right) + (1-\bar{\omega})\,\gamma_w^*\left(1-\gamma_w\right)\right) > (1-\gamma_w)\left(1-\gamma_w^*\right)$$

$$\phi s_y\left(\bar{\omega}\gamma_w + (1-\bar{\omega})\,\gamma_w^* - \gamma_w\gamma_w^*\right) > 1 - \gamma_w - \gamma_w^* + \gamma_w\gamma_w^*$$

$$\phi s_y\left(\bar{\omega}\gamma_w - (1-\bar{\omega})\,\gamma_w^*\right) + \gamma_w + \gamma_w^* > 1 + \gamma_w\gamma_w^*\left(1 + s_y\phi\right)$$

where the last inequality is identical to the determinacy condition derived in equation ($A.15$).

□

Proposition 4 *If $r^{W,Nat} < \bar{\Pi}^{-1}$, $D^W > (1-\omega)\,Y_f^*$, $\gamma > 0$, there exists a unique, locally determinate asymmetric secular stagnation with $r = r^*$, $Y < Y_f$, $Y^* = Y_f^*$, $i = 0$, and $\Pi < \bar{\Pi}$*

Proof. Under the assumptions of the proposition and the monetary policy rule, the zero lower bound is binding for the home country and not binding for the foreign country. Nevertheless, real interest rates are equalized across both countries: $\frac{1}{\Pi} = r = r^* = \frac{i^*}{\Pi^*}$ where $i^* > 0$. Equilibrium inflation and output in the home country solve the following equations:

$$Y = \frac{1}{\omega}\left(D^W - (1-\omega)\,Y_f^* + \psi^W\Pi\right) \tag{A.16}$$

$$Y = \left(\frac{1 - \frac{\gamma\bar{\Pi}}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y_f \tag{A.17}$$

where $D^W = \omega D + (1-\omega)\,D^*$ and $\psi^W = \frac{1+\beta}{\beta}\,(1+g)\,D^W > 0$. We may define the difference equation $\Delta\,(\Pi)$ by taking the difference between ($A.16$) and ($A.17$) . This function is continuous in Π with $\Delta(\gamma) > 0$ (since $D^W > (1-\omega)\,Y_f$) and $\Delta(\bar{\Pi}) < 0$ since $r^{W,Nat} < \bar{\Pi}^{-1}$. Therefore, there exists a $\gamma < \Pi < \bar{\Pi}$ such that $\Delta(\Pi) = 0$. Since $\Pi < \bar{\Pi}$, it follows that $Y < Y_f$.

Uniqueness of an asymmetric stagnation equilibrium under perfect integration is established identically as in Proposition 2. Graphically, the global AD curve (equation ($A.16$) form a line in domestic inflation-output space. The domestic AS curve (equation ($A.17$)) is identical to equation ($A.2$) and cannot cross the AD curve more than once given that the second derivative cannot switch signs from positive to negative.

It must be the case that the AS curve has a lower slope than the AD curve at the point of intersection. The slope of the AS curve is identical to equation ($A.6$) If the slope of the AS curve is less than the slope of the AD curve at the intersection point, then it must be the case that:

$$\frac{1-\alpha}{\alpha}\frac{\Pi}{Y}\left(\frac{\Pi}{\gamma\bar{\Pi}}-1\right) < \left(\frac{\psi^W}{\omega}\right)^{-1}$$

$$\frac{1-\alpha}{\alpha}\frac{\psi^W \Pi}{\omega Y}\left(\frac{\Pi}{\gamma\bar{\Pi}}-1\right) < 1$$

$$\frac{1-\alpha}{\alpha}\frac{\omega Y - D^W - (1-\omega)\,Y_f^*}{\omega Y}\left(\frac{\Pi}{\gamma\bar{\Pi}}-1\right) < 1$$

$$s_y \frac{\alpha}{1-\alpha} + 1 > \frac{\Pi}{\gamma\bar{\Pi}}$$

$$\frac{\gamma\bar{\Pi}}{\Pi}\left(s_y \frac{\alpha}{1-\alpha} + 1\right) > 1$$

The linearization of the global AD curve (equation (*A*.16)) and the domestic AS curve (equation (*A*.17)) around the asymmetric stagnation steady state imply identical expressions to the linearized equilibrium conditions in Proposition 2 where the coefficients are given by:

$$\gamma_w = \frac{\gamma\bar{\Pi}}{\Pi_{ss}}$$

$$s_y = \frac{\omega Y_{ss}}{\omega Y_{ss} - D^W - (1-\omega)\,Y_f^*}$$

where Π_{ss} and Y_{ss} are the solution to steady state equilibrium conditions (*A*.16) and (*A*.17). Substituting the linearized AS curve into the linearized AD curve as in Proposition 2 provides a forward-looking difference equation in y_t. Local determinacy requires the coefficient on $E_t y_{t+1}$ to be less than unity. This condition is identical to slope condition derived above implying that the asymmetric stagnation equilibrium is always locally determinate, as required. □

B Fiscal Policy Coordination

The government in each country *unilaterally* chooses its own G to minimize the deviations of output, inflation and level of government spending from their own respective target levels:

Non-Cooperative Game

$$\min_G \; (Y - Y^f)^2 + (\Pi - 1)^2 + (G - G^{target})^2$$

subject to

$$\text{AD} \quad 2Y - G - G^* = \frac{1+\beta}{\beta}(D + D^*)\Pi + D + D^*$$

$$\text{AS} \quad 2Y = 2Y^f \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}$$

First Order Conditions:

$$2(Y - Y^f) - 2\lambda_1 - 2\lambda_2 = 0$$

$$2(\Pi - 1) + \lambda_1 \frac{1+\beta}{\beta}(D + D^*) + 2\lambda_2 \frac{\gamma}{\Pi^2}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f = 0$$

$$2(G - G^{target}) + \lambda_1 = 0$$

where λ's are the respective Lagrange multipliers. Rearranging, we get:

$$2(\Pi-1)-2(G-G^{target})\frac{1+\beta}{\beta}(D+D^*)+2(Y-Y^f+2(G-G^{target}))\frac{\gamma}{\Pi^2}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f = 0$$

$$G - G^{target} = \frac{(\Pi - 1) + (Y - Y^f)\left[\frac{\gamma}{\Pi^2}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f\right]}{\frac{1+\beta}{\beta}(D + D^*) - 2\left[\frac{\gamma}{\Pi^2}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f\right]}$$

Under a secular stagnation regime, $Y < Y^f$ and $\Pi < 1$, so the numerator is negative. The Government should increase its expenditure above the target if and only if :

$$\frac{1+\beta}{\beta}(D + D^*) < 2\left[\frac{\gamma}{\Pi^2}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f\right] \tag{B.1}$$

Cooperative Game

$$\min_{G,G^*} \; 2(Y - Y^f)^2 + 2(\Pi - 1)^2 + (G - G^{target})^2 + (G^* - G^{target*})^2$$

subject to

$$\text{AD} \quad 2Y - G - G^* = \frac{1+\beta}{\beta}(D + D^*)\Pi + D + D^*$$

$$\text{AS} \quad 2Y = 2Y^f\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}$$

First-Order Conditions :

$$4(Y - Y^f) - 2\lambda_1 - 2\lambda_2 = 0$$

$$4(\Pi - 1) + \lambda_1 \frac{1+\beta}{\beta}(D + D^*) + 2\lambda_2 \left[\frac{\gamma}{\Pi^2}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f\right] = 0$$

$$2(G - G^{target}) + \lambda_1 = 0$$

$$2(G^* - G^{target*}) + \lambda_1 = 0$$

where λ's are the respective Lagrange multipliers. Rearranging the first three equations for the Home country, we get:

$$4(\Pi-1)-2(G-G^{target})\frac{1+\beta}{\beta}(D+D^{*})+4(Y-Y^{f}+G-G^{target})\frac{\gamma}{\Pi^{2}}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}Y^{f}=0$$

$$G-G^{target}=\frac{2(\Pi-1)+2(Y-Y^{f})\left[\frac{\gamma}{\Pi^{2}}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}Y^{f}\right]}{\frac{1+\beta}{\beta}(D+D^{*})-2\left[\frac{\gamma}{\Pi^{2}}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}Y^{f}\right]}$$

Similarly for the Foreign country we get,

$$G^{*}-G^{target*}=\frac{2(\Pi-1)+2(Y^{*}-Y^{f*})\left[\frac{\gamma}{\Pi^{2}}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}Y^{f}\right]}{\frac{1+\beta}{\beta}(D+D^{*})-2\left[\frac{\gamma}{\Pi^{2}}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}Y^{f}\right]}$$

Under a secular stagnation regime, $Y<Y^{f}, Y^{*}<Y^{f*}$ and $\Pi<1$, so the numerator is negative. The Government should increase its expenditure above the target if and only if the denominator is negative:

$$\frac{1+\beta}{\beta}(D+D^{*})<2\left[\frac{\gamma}{\Pi^{2}}\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{\gamma}{\Pi}}\left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}}Y^{f}\right] \tag{B.2}$$

We can easily observe that this condition is same as in the non-cooperative regime. Further, we can show that, given eqn B.1 is satisfied[14], Government expenditure is higher under a cooperative regime as compared to a non-cooperative regime.

Proposition (Proposition 7)**.** *Conditional on the world being in a symmetric secular stagnation regime ($Y<Y^{f}, Y^{*}<Y^{f*}$ and $\Pi<1$),*

$$\text{If eqn B.1 holds, then } G^{cop}>G^{ncop}. \tag{B.3}$$

Proof.

$$\begin{aligned} G^{cop}-G^{ncop} &= \frac{2B+2CX}{A-2X}-\frac{B+CX}{A-2X} \\ &= \frac{B+CX}{A-2X}>0 \end{aligned}$$

where $B<0, C<0, A>0, X>0$ and the last inequality follows from the assumption that eqn B.1 is satisfied, which implies that the denominator is negative. □

[14]The program `fiscalconditions.m` verifies that this condition holds for the parameter values that we discussed earlier.