We thank Neil Dorosin, Jesse Margolis, and Elizabeth Sciabarra for their expertise and for facilitating access to the data used in this study. A special thanks to Alvin E. Roth for his collaboration, which made this study possible. Jesse Margolis provided extremely helpful detailed comments. We also thank Josh Angrist, Steve Berry, Stefan Bonhomme, Eric Budish, Francesco Decarolis, Matt Gentzkow, Jerry Hausman, Phil Haile, Ali Hortacsu, Derek Neal, Ariel Pakes, Jesse Shapiro, and seminar participants at the University of Chicago, Harvard University, London School of Economics, Yale University and the NBER Market Design conference for input. We received excellent research assistance from Alonso Bucarey, Red Davis, Weiwei Hu, Danielle Wedde, and a hardworking team of MIT undergraduates. Pathak acknowledges support from the Alfred P. Sloan Foundation (grant BR2012-068) and National Science Foundation (grant SES-1056325). The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed a financial relationship of potential relevance for this research. Further information is available online at http://www.nber.org/papers/w21046.ack

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The Welfare Effects of Coordinated Assignment: Evidence from the NYC HS Match
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NBER Working Paper No. 21046
March 2015
JEL No. C78,D47,D50,D61,I21

ABSTRACT

Centralized and coordinated school assignment systems are a growing part of recent education reforms. This paper estimates school demand using rank order lists submitted in New York City's high school assignment system launched in Fall 2003 to study the effects of coordinating admissions in a single-offer mechanism based on the deferred acceptance algorithm. In the previous mechanism, students were allowed to rank five choices and admissions offers were not coordinated across schools. While 18% of students obtained multiple first round offers, the mechanism's uncoordinated offers led more than a third of students to be unassigned after the main round and ultimately administratively assigned. Under the new mechanism, there is a 7.2 percentage point increase in matriculation at assigned school and students are assigned to schools that are on average 0.69 miles further from home. Even though students prefer nearby schools, our estimates suggest substantial heterogeneity in willingness to travel for school. The new mechanism generates higher utility on average and across numerous subgroups of students compared to either a neighborhood school alternative or the previous uncoordinated mechanism. Across a range of estimates, we find that the average student's welfare gain from coordinating assignment is substantially more than the disutility from increased travel. These gains are significantly larger than those from relaxing mechanism design constraints within the coordinated system. Preference heterogeneity implies that choice is far from zero-sum and coordinating admissions offers across schools increases allocative efficiency.

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1 Introduction

Whether transferring to an out-of-zone public school, applying to a charter, magnet or exam school, or using a voucher to attend a private school, a growing number of families can opt out of their neighborhood school. As school choice options have proliferated, so too has the way in which choices are expressed and students are assigned. Ad hoc decentralized assignment systems where students applied separately to schools with uncoordinated admissions offers have been replaced with centralized, coordinated single-offer assignment mechanisms in a number of cities including Denver, London, Newark, New Orleans, New York City, and Washington DC.¹ In these cities, student demand is matched with the school seat supply using algorithms inspired by the theory of matching markets (Gale and Shapley 1962, Shapley and Scarf 1974, Abdulkadiroğlu and Sönmez 2003). Changes in assignment protocols are inevitably controversial, however, since allocating school seats may result in some students assigned to high quality schools while others are assigned to less desirable alternatives.

In this paper, we exploit a large-scale policy change in New York City to study the effects of moving from an uncoordinated assignment system to a coordinated single-offer system on the allocation of students to schools. Prior to 2003, roughly 80,000 aspiring high school students applied to five out of more than 600 school programs; they could receive multiple offers and be placed on wait lists. Students in turn were allowed to accept only one school and one wait list offer, and the cycle of offers and acceptances repeated two more times. We refer to the old mechanism as uncoordinated because students initially expressed their preferences on a common application, but admissions offers were not coordinated across schools. A total of three rounds of offer processing proved insufficient to allocate all students to schools, and more than 25,000 applicants were assigned to schools not on their original application list through an administrative process, which manually placed students to schools in their neighborhood.

In Fall 2003, the system was replaced by a single-offer assignment system, based on the student-proposing deferred acceptance algorithm for the main round. Applicants were allowed to rank up to 12 programs for enrollment in 2004-05 and a supplementary round placed those unassigned in the main round. Since the central office coordinated offers across schools into a single offer, we refer to this new mechanism as a coordinated mechanism. The Department of Education (DOE) publicized that the new mechanism provides greater voice to student’s choices, utilizes school places more efficiently, and reduces gaming involved to obtain a school seat (Kerr 2003). Parents were also provided with additional information on school options through an expanded set of workshops and high school fairs. At the conclusion of the first year, 82% of students were assigned in the main round and the size of the administrative round shrunk by over two-thirds.

The sudden, large-scale policy change in New York City provides a unique opportunity to investigate the consequences of centralized and coordinated school assignment. Changes were advertised widely beginning only in September 2003, and students submitted their preferences in November 2003. This brief timeline limits the scope for participants to react by moving or

¹Many other U.S. cities are in the process of adopting unified enrollment systems using similar matching algorithms, including Chicago and Philadelphia (Darville 2013).
for schools to change curriculum and resources in response to the new mechanism. Moreover, rich micro-level data from both systems allow us to surmount many of the usual challenges associated with evaluating allocative aspects of assignment mechanisms. A key strength of our study is that we observe submitted preferences, round-by-round placements, and subsequent school enrollment of all students in both the old and new mechanism. Using student preferences together with detailed information on student and school attributes, we estimate the factors that account for school demand. The estimated preference distribution is then used to assess mechanism design choices and measure the welfare from the new mechanism.

Our approach focuses on the allocative and distributional aspects of the new assignment mechanism and its comparison to other mechanisms that have been the focus of the large theoretical literature on matching market design. While the longer-term effects on residential choices and school productivity are other possible effects of the new mechanism, their study likely requires understanding allocative issues. Moreover, changes in NYC’s school assignment system have been a template for replacing uncoordinated procedures with coordinated ones. For instance, in England, where the 2003 Admissions Code mandated coordination of admissions nationwide, a number of local education authorities, governing bodies similar to U.S. school districts, subsequently adopted coordinated assignment mechanisms like New York. Furthermore, our results quantifying mechanism design tradeoffs are relevant for districts like Denver, Recovery School District in New Orleans, and Washington DC each of which faced similar design decisions.

Coordination of admissions offers may affect allocations for many reasons. First, the new mechanism allows students to rank up to 12 choices, whereas the old mechanism only allowed for five. Second, the limited number of rounds of offers and acceptances in the old mechanism can lead to situations where students hold on to less preferred choices waiting to be offered seats at more preferred schools once others decline. Few rounds of offer processing and a limited number of applications make it difficult for schools to make enough offers to assign to clear the market, a phenomenon termed congestion by Roth and Xing (1997). Congestion can result in particularly large inefficiencies since in the old mechanism, the district administratively placed unassigned students to the nearest schools with capacity. Third, in New York’s old mechanism, schools were able to see the entire rank ordering of applicants, and some advertised they would only consider those who ranked them first, creating strategic pressure on student ranking decisions. The computerized rounds of offers and acceptances in a centralized single-offer system reduces congestion and associated strategic issues.

On the other hand, coordinated assignment may encourage movement throughout the district.
for little benefit. Ravitch (2011), for instance, argues that the elevation of choice in NYC “destroyed the concept of neighborhood schools” as “children scattered across the city in response to the lure of new, unknown small schools with catchy names, or were assigned to schools far from home.” Critics of the new system believed that denying principals’ information on students’ ranking of the school restricts a principal’s ability to attract “students who want them most” (Herszenhorn 2004). This logic implies that removing a school’s ability to know whether they were ranked first could lead fewer of their offered students to enroll. It also may be in a district’s interest to advantage certain student groups to prevent them from leaving the district (Engberg, Epple, Imbrogno, Sieg, and Zimmer 2014). Providing students with multiple offers and letting them decide upon receiving offers may make it easier for some students to decide what school is best for them. Taken together, arguments against NYC’s choice system suggest that the allocative effects of the mechanism may have mostly been redistributive or even harmful to students.

To quantify the welfare effects of coordinated school assignment in New York City, we estimate the distribution of student preferences for schools using flexible discrete choice demand models allowing for rich student heterogeneity and unobserved school quality. Modelling this heterogeneity is essential for our analysis since otherwise changes in assignments are zero-sum by construction when the same number are assigned. Our empirical approach is motivated by the fact that the new mechanism is a variant of the student-proposing deferred acceptance mechanism, where truth-telling is a weakly dominant strategy for participants (Dubins and Freedman 1981, Roth 1982). While the new mechanism’s incentive properties make treating stated rankings as true preferences a natural starting point, this assumption bears scrutiny for several reasons. First, the new mechanism only allows participants to rank at most 12 choices. Although 80% of applicants in our sample rank fewer than 12, this constraint interferes with the dominant strategy property of the mechanism for students who find more than 12 schools acceptable. Second, it’s possible that the sudden adoption of the new mechanism led some families to use heuristics developed from the old mechanism, despite the DOE’s advice to rank schools truthfully. For instance, some families might have ranked a “safety” school as their last choice, even though the new mechanism’s properties make this unnecessary if ranking fewer than 12 choices. We start with benchmark models that use all ranked choices in the coordinated mechanism and assume the ranking is truthful, before turning to several alternatives to assess the robustness of our conclusions to these concerns. We do not directly model strategic choices because we anticipate strategic considerations to be less important in mechanisms based on deferred acceptance than in other more manipulable mechanisms.

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4 Williams (2004) profiles families who had to pay a deposit to reserve a spot in a private or parochial school while awaiting their school placement in the first year of the new mechanism.

5 Similar arguments for the flexibility provided by multiple offer systems have been raised by critics of the National Residency Matching Program. See, e.g., Bulow and Levin (2006) and Agarwal (2015).


7 The DOE’s school brochure advised: “You must now rank your 12 choices according to your true preferences” (DOE 2003).

8 Hastings and Weinstein (2008) report that in Charlotte-Mecklenburg’s highly manipulable immediate acceptance mechanism, student rankings respond to information on historical odds of assignment. Pathak and
Offer processing and matriculation patterns provide compelling evidence that the new mechanism is an improvement relative to the old one. In the old mechanism, 18.6% of students matriculated at different schools from their assignment at the end of the match compared to 11.4% under the new mechanism. Multiple offers and short rank order lists in the old mechanism advantage few students, but leave many without offers. Roughly one-fifth of students obtained multiple offers, while half of applicants obtain no first round offer and 59% of these applicants are administratively assigned. The take-up rates for students assigned administratively are similar across mechanisms, but the number of students assigned in that round is three times larger in the old mechanism. In addition, 8.5% of applicants left the district after assignment in the old mechanism, while only 6.4% left under the new mechanism.

The most significant difference between assignments under the two mechanisms is the distance between students and their assigned school. In the old mechanism, students travelled on average 3.36 miles to their school assignment. Students travel 0.69 miles further in the new mechanism. All else equal, students prefer schools that are closer to home according to their preference rankings in the new mechanism. However, preferences are substantially heterogeneous, and our demand model quantifies the trade-off between distance and measures of school performance across student types. Asian and white students and those with high baseline scores exhibit a greater willingness to travel for their most preferred schools than black and Hispanics and those with low baselines scores, respectively. Variations on our baseline assumptions to assess the robustness of our estimates show broadly consistent patterns. This fact together with evidence on both in and out-of-sample model fit reassure us about the suitability of using the estimated preferences to compare student welfare across allocations.

Since the uncoordinated mechanism administratively assigned a large fraction to neighborhood schools, our investigation of counterfactual assignments starts with a neighborhood assignment. We compute the allocation produced by placing each student to the closest school subject to capacity constraints. Our estimates imply that the average student welfare from the coordinated mechanism is 15 miles in distance-equivalent utility greater than this alternative. The other extreme is bracketed by the utilitarian optimal assignment, which is the assignment maximizing an equally weighted average of distance-equivalent utility given school capacities. The new mechanism is 3.7 miles in distance-equivalent utility below this idealized benchmark.

Next, we empirically examine issues in the mechanism design literature (Erdil and Ergin 2008, Abdulkadiroğlu, Pathak, and Roth 2009, Kesten 2010, Kesten and Kurino 2012). Had the mechanism produced a student-optimal stable matching, average student welfare improves by an equivalent of 0.11 miles. An ordinally Pareto efficient matching, which abandons the stability constraints in the current mechanism, is equivalent to an improvement of 0.62 miles, which in turn is still more than 3.1 miles in distance-equivalent utility below the utilitarian benchmark. Unfortunately, none of these assignments can be implemented without discarding the mechanism’s incentive properties and mechanisms that implement these allocations as equilibrium outcomes.

Sönmez (2013) formalize how constrained deferred acceptance is less manipulable than the immediate acceptance (or Boston) mechanism. Agarwal and Somaini (2014) examine how to recover preferences from strategic reports in the context of school choice.

Even though the mechanism is based on student-proposing deferred acceptance, the matching is not student-optimal because school-side priorities are not strict.
are not known. Nonetheless, our estimates show that the magnitude of any potential gains are swamped by the comparison between neighborhood assignment and the coordinated mechanism.

Finally, we use our demand estimates to evaluate the transition from an uncoordinated to coordinated mechanism. This exercise requires a model of student ranking behavior in the uncoordinated mechanism since the submitted reports may be correlated with (unobserved) preferences. Since students could only rank five choices and some schools advertised they would only consider those ranking them first, modeling perceptions and behavior of applicants under this mechanism is not straightforward. Our approach is to consider three assumptions about rankings under the old mechanism: 1) ranked schools are preferred to unranked schools, 2) rankings under the uncoordinated mechanism are not selected on unobservables, and 3) schools are ranked truthfully. Fortunately, the qualitative conclusions of our comparison between the two mechanisms are similar across these specifications. Under each assumption, the new mechanism has made it easier for students to obtain a choice they want. Even though school assignments are further away, based on our estimated distribution of preferences, the amount that students prefer these schools more than compensates for this difference. Students across all demographic groups, boroughs, baseline achievement levels obtain a more preferred assignment on average from the new mechanism.

The largest gains are for student groups who were more likely to be unassigned after the old mechanism’s main round, suggesting that congestion and ad-hoc placement of unassigned students in the old mechanism are primarily responsible for misallocation. These gains are also seen if utility is measured based on subsequent school enrollment, indicating that the adverse consequences of congestion were not immediately undone following the match. Compared to relaxing algorithmic constraints within the single-offer mechanism, the welfare advantage of the coordinated mechanism over the uncoordinated mechanism is roughly half the difference between neighborhood assignment and the utilitarian optimum.

The literature on school choice is immense and not easily summarized. We share a focus with papers interested in understanding how choice affects assignment and sorting of students (Epple and Romano 1998, Urquiola 2005), rather than the competitive effects of choice on student achievement (Hoxby 2003, Rothstein 2006). We also concentrate our study on allocative efficiency, rather than effects of choice options on subsequent achievement (Abdulkadiroğlu, Angrist, Dynarski, Kane, and Pathak 2011, Deming, Hastings, Kane, and Staiger 2014, Walters 2014, Neilson 2014). This study contributes to a growing theoretical literature on centralized admissions to college (Balinski and Sönmez 1999) and K-12 public schools (Abdulkadiroğlu and Sönmez 2003). A number of recent papers use micro data from assignment mechanisms to understand school demand (Hastings, Kane, and Staiger 2009, He 2012, Ajayi 2013, Agarwal and Somaini 2014, Calsamiglia, Fu, and Guell 2014, Hwang 2014), but these datasets comes from single-offer mechanisms that are manipulable such as the Boston mechanism, under which parents have a strong incentive to rank schools strategically. Another study that examines the effects of coordination, though in the labor market context, is the Niederle and Roth (2003) study of the introduction of a centralized match for gastroenterologists. Finally, our work complements theoretical papers investigating other assignment mechanisms using simulated data (Erdi̇l and Ergin 2008, Kesten 2010, Abdulkadiroğlu, Che, and Yasuda 2015) or those that use

The paper proceeds as follows. Section 2 provides additional details on high school assignment in New York City. Section 3 describes our data. Section 4 reports on descriptive features of offer processing in the two mechanisms. In Section 5 we outline our empirical strategy, and report on estimates of the student preference distribution. Section 6 uses the estimates to quantitatively assess design choices, while in Section 7 we compare student welfare across the uncoordinated and coordinated mechanisms. Section 8 assesses model fit and robustness, and the last section concludes.

2 High School Choice in NYC

2.1 School Options

Aspiring high school students are eligible to apply to any school or program throughout New York City. Programs exist within schools, and have curricula ranging from the arts to sciences to vocational training. The 2002-03 High School directory describes seven types of programs, categorized according to their admissions criteria. Specialized High Schools, such as Stuyvesant and Bronx Science, have only one type of program, which admits students by admissions test performance on the Specialized High Schools Admissions Test (SHSAT).\(^\text{10}\) Audition programs interview students for proficiency in specific performing or visual arts, music, or dance. Screened programs evaluate students individually using an assortment of criteria including a student’s 7th grade report card, reading and math standardized scores, attendance and punctuality, interview, essay or additional diagnostic tests. Educational Option programs also evaluate students individually, but for half their seats. The other half is allocated by lottery. Allocation of seats in each half targets a distribution of student ability: 16 percent of seats should be allocated to high performing readers, 68 percent to middle performers, and 16 percent to low performers. Unscreened programs admit students by random lottery, while Zoned programs give priority to students who apply and live in the geographic zoned area of the high school. Limited Unscreened programs allocate seats randomly, but give priority to students who attend an information session or visit the school’s exhibit at high school fairs or open houses conducted each admissions season. We categorize programs into three main groups: Screened programs which also include Testing and Audition programs, Unscreened programs, which also include limited Unscreened and Zoned programs, and the rest are Educational Option programs.

Throughout the last decade, the NYC DOE closed and opened new small high schools throughout the city, each with roughly 400 students. A big push for these small high schools came as part of the New Century High Schools Initiative launched by Mayor Bloomberg and Chancellor Klein. Eleven new small high schools were opened in 2002, 23 new small schools were opened in 2003, and the peak year of small high school openings was in 2004 (Abulkadiroğlu, Hu, and Pathak 2013). Most of these schools are small and have about 100 students per entering class. As a result, the new small high schools have a relatively small effect on overall enroll-

\(^{10}\)Abdulkadiroğlu, Angrist, and Pathak (2014) describe their admissions process in more detail.
ment patterns during our study period, which focuses on school options available in 2002-03 and 2003-04.

2.2 Uncoordinated Admissions in 2002-03

Forms of high school choice have existed in New York City for decades. Before 2002, high school assignment in New York City featured a hodgepodge of choice options mostly controlled by borough-wide high school superintendents. Significant admissions power resided with school administrators who could directly enroll students. Admissions to the Specialized High Schools and the LaGuardia High School of Music & Art and Performing Arts, however, have been traditionally administered as a separate process from regular high schools and did not change with the new mechanism.\(^{11}\) Our study therefore focuses on admissions to regular public schools.

About 80,000 students interested in regular high schools visit schools and attend city-wide high school fairs before submitting their preference in the fall. In the main round in 2002-03, students could apply to at most five regular programs in addition to the Specialized High Schools. Programs receiving a student’s application were able to see the applicant’s entire preference list, including where their program was ranked. Programs then decided whom to accept, place on a waiting list, or reject. Applicants were sent a decision letter from each program they had applied to and some obtained more than one offer. Students were allowed to accept at most one admission and one wait-list offer. After receiving responses to the first letters, programs with vacant seats could make new offers to students from waiting lists. After the second round, students who do not have a zoned high school are allowed to participate in a supplementary round known as the variable assignment (VAS) process. In the supplementary round, students could rank up to eight choices, and students were assigned based on negotiation on seat availability between the enrollment office and high school superintendents. After replies to the second letter were received, a third round of letters were mailed. New offers did not necessarily go to wait-listed students in a predetermined order. Remaining unassigned students were assigned their zoned programs or administratively, when the central office tried to place kids as close to home as possible, ignoring their preferences. We refer to this final stage as the administrative round.\(^{12}\)

Three features of this assignment scheme motivated the NYC DOE to abandon it in favor of a new mechanism. First, there was inadequate time for offers, wait list decisions, and acceptances to clear the market for school seats. DOE officials reported that in some cases high achieving students received acceptances from all of the schools they applied to, while many received none (Herszenhorn 2004). Comments by the Deputy Schools Chancellor summarized the frustration: “Parents are told a school is full, then in two months, miracles of miracles, seats open up, but other kids get them. Something is wrong” (Gendar 2000).

\(^{11}\)The 1972 Hecht-Calandra Act is a New York State law that governs admissions to the original four Specialized High Schools: Stuyvesant, Bronx High School of Science, Brooklyn Technical, and Fiorello H. LaGuardia High School of Music and Performance Arts. City officials indicated that this law prohibits including these schools within the common application system without an act of the state legislature.

\(^{12}\)Students who are new to New York City or did not submit an application participate in an “over the counter” round over the summer. Our analysis follows applicants through to assignment and therefore does not consider students who arrived to the process after the high school match. Arvidsson, Frucht, and Mokhtar (2013) provide further details on the over-the-counter round.
Second, some schools awarded priority in admissions to students who ranked them first on their application form. The high school directory advises that when ranking schools, students should “... determine what your competition is for a seat in this program” (DOE 2002). This recommendation puts strategic pressure on ranking decisions. Listing such a school first would improve the likelihood of an offer at the risk of rejection from other schools, which take rankings into account.

Third, a number of schools managed to conceal capacity to fill seats later on with better students. For example, the Deputy Chancellor stated, “before you might have a situation where a school was going to take 100 new children for ninth grade, they might have declared only 40 seats, and then placed the other 60 outside the process” (Herszenhorn 2004). Overall, critics alleged that the old mechanism disadvantaged low-achieving students and those without sophisticated parents (Hemphill and Nauer 2009).

2.3 Coordinated Admissions in 2003-04

The new mechanism was designed with input from economists (see Abdulkadiroğlu, Pathak, and Roth (2005) and Abdulkadiroğlu, Pathak, and Roth (2009)). When publicizing the new mechanism, the DOE explained that its goals were to utilize school places more efficiently and to reduce the gaming involved in obtaining school seats (Kerr 2003). In the first round, students apply to Specialized High Schools when they take the SHSAT, just as in previous years. Offers are produced according to a serial dictatorship with priority given by SHSAT scores.

In the main round, students can rank up to twelve regular school programs in their application, which are due in November. The DOE advised parents: “You must now rank your 12 choices according to your true preferences” because this round is built on Gale and Shapley (1962)’s student-proposing deferred acceptance algorithm. Schools with programs that prioritize applicants based on auditions, test scores or other criteria are sent lists of students who ranked the school, but these lists do not reveal where in the preference lists they were ranked. Schools return orderings of applicants to the central enrollment office. Schools that prioritize applicants using geographic or other criteria have those criteria applied by the central office. That office uses a single lottery to break ties amongst students with the same priority, generating a strict ordering of students at each school.

Assignment is determined by the student-proposing deferred acceptance algorithm with student preferences over the schools, school capacities, and schools’ strict ordering of students as parameters. The algorithm is run with all students in February. In this first round, only students who receive a Specialized High School offer receive a letter indicating their regular school assignment, and are asked to choose one. After they respond, students who accept an offer are removed, school capacities are adjusted and the algorithm is re-run with the remaining students. All students receive a letter notifying them of their assignment or whether they are unassigned after the main round.

Unassigned students from the main round are provided a list of programs with vacancies and are asked to rank up to twelve of these programs. In 2003-04, the admissions criteria at the

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13There is very limited overlap between the specialized round and subsequent rounds. In 2003-04, 4,175 out of 4,553 of those offered a specialized high in our sample accepted that offer.
remaining school seats were ignored in this supplementary round. Students are ordered by their random number, and the student-proposing deferred acceptance algorithm is run with this ordering in place at each school. Students who remain unassigned in the supplementary round are assigned administratively. These students and any appealing students are processed on a case-by-case basis in the administrative round.

3 Data and Sample Definitions

3.1 Students

The NYC DOE provided us with several data sets for this study each linked by a unique student identification number: information on student choices and assignments, student demographics, and October student enrollment. For 2002-03, the assignment files record a student’s main round rank order list, their offers and rejections for each round, whether they participate in the supplementary round, and their final assignment at the conclusion of the assignment process as of July 2003. For 2003-04, the assignment files contain students’ choice schools in order of preference, priority information for each school, and assignments at the end of each of the rounds, and final assignment as of early August 2004. The student demographic file for both years contains information on home address, gender, race, limited English proficiency, special education status, and performance on 7th grade citywide tests. We use addresses to compute the road distance between each student and school, and to place each student in a census block group.\\footnote{Though we use road distance, we also computed subway distance using the Metropolitan Transportation Authority GIS files; the overall correlation between driving distance and subway commuting distance for all student-program pairs is 0.96.}

We also have access to similar files for 2004-05. Further details are in the Data Appendix.

Our analysis sample makes three restrictions. First, since we do not have demographic information for private school applicants, we restrict the analysis to students in NYC’s public middle schools in the year prior to application. Second, we focus on students who are not assigned to Specialized High Schools because that part of the assignment process did not change with the new mechanism. Third, we consider applicants who are given an assignment at the conclusion of the process (so have not left midway). Given these restrictions, we have two main analysis files: the mechanism comparison sample and the demand estimation sample.

The mechanism comparison sample is used for comparisons of the assignment across the two mechanisms. This sample is the largest set of students assigned through the high school assignment mechanism to a school that exists as of the time of the printing of the high school directory. A key property of the mechanism comparison sample is that every student has an assignment. Columns 1 and 2 of Table 1 summarize student characteristics in the mechanism comparison samples across years. 3,500 fewer students are involved in the mechanism comparison 2003-04 sample, a difference mainly due to the students assigned to schools created after the printing of the high school directory or to closed schools (as shown in Appendix Table C2).

New York City is the nation’s largest school district, and like many urban districts it is majority low-income and non-white. Nearly three-quarters of students are black or Hispanic, and about 10% of students are Asian. Brooklyn is home to the largest number of applicants, followed
by Bronx and Queens, which each account for roughly one quarter of students. Manhattan and
Staten Island account for a considerably smaller share of students at about 13 and six percent,
respectively. Consistent with the sudden announcement of the new mechanism, characteristics
of applicants are similar across years.

The demand sample contains participants in the main round of the new mechanism in the
assignment files. The school choices expressed by these students represent the overwhelming
majority of students. Among the set of main round participants, we exclude a small fraction
of students who are classified as the top 2 percent because these students are guaranteed a
school only if they rank it first and this may distort their incentives to rank schools truthfully.
Additional details on the sample restrictions are in the Data Appendix.

3.2 Schools

Data on schools were taken from New York State report card files provided by NYC DOE.
Information on programs come from the official NYC High School Directories made available to
students before the application process. Table 2 summarizes school and program characteristics
across years. There is an increase in the number of schools from 215 to 235, and a corresponding
decrease in the average number of students enrolled per school of about 40 students. This
fact is driven by the replacement of some large schools with smaller schools that took place
concurrently in 2003-04, as described above. Despite this increase, there is little change in the
average achievement levels of schools and school demographic composition as measured by report
card data.

Students in New York can choose among roughly 600 programs throughout the city. Pro-
grams vary substantially in focus, post-graduate orientation, and educational philosophy. For
instance, the Heritage School in Manhattan is an Educational Option program where the arts
play a substantial role in the curriculum, while Townsend Harris High School in Queens is a
Screened program with a rigorous humanities program making it among the most competitive
in the city. Using information from high school directories, we identify each program’s type, lan-
guage orientation, and specialty. With the new mechanism, there are more Unscreened programs
and fewer Educational Option programs, a change driven by the conversion of many Educational
Option programs to Unscreened programs. This change in labelling was due to overlapping
admissions criteria and similarity of educational programming. We code language-focused pro-
gr a m s a s S p a n i s h , A s i a n , o r O t h e r , a n dc a t e g o r i z e p r o g r a m s p e c i a l i t i e s i n t o A r t s , H u m a n i t i e s ,
Math and Science, Vocational, or Other. Not all programs have specialties, though about 70%
fall into one of these classes. (Details on our classification scheme are in the Data Appendix).
The menu of language program offerings or program specialties changes little across years.

4 Congestion and Changes in Assignments

The similarity of student and school attributes in Tables 1 and 2 suggest that there were not
systematic changes in participant attributes and school supply across years. Moreover, there
does not appear to be a large-scale change in student locations across years, as shown in Figure
1, which maps both student and school locations. These facts motivate attributing differences
in allocations between 2002-03 and 2003-04 primarily to the assignment mechanism rather than changes in student participation or the menu of school options.

4.1 Congestion in the Main Round

Table 3 reports the number of students assigned across rounds of the uncoordinated and coordinated mechanisms. The most noteworthy pattern is that more students obtain their final assignment in the administrative round of the uncoordinated mechanism than in the first round. Panel A of the Table shows that 37% of students are assigned administratively compared to 34% in the first round. Since 33,894 students obtain one or more first round offers (shown in Panel B), but only 23,867 students were finalized in the first round, 10,027 students with a first round offer were finalized with offers made in subsequent rounds. The processing of these students took place as schools revised offers based on first round rejections and made offers anew in the second and third rounds. However, the relatively small number of students placed in the second and third round implies that three rounds were insufficient to process all students. That only half of the students were placed in the main round of the old mechanism contrasts sharply with new mechanism, where 82% of students were placed in the main round.\(^{15}\)

These observations about the old mechanism are characteristic of congestion, as described in Roth and Xing (1997)’s study of the labor market for entry-level clinical psychologists. Offers for training positions in that market were made in an uncoordinated fashion during a 7-hour window, and Roth and Xing (1997) argue that uncoordinated processing and a small market-clearing window led to mismatch. In NYC, serial-processing of batches of offers, whereby programs waited for previous offers to be rejected before making new offers together with a few rounds had a similar effect. In addition to insufficient processing of offers, the small number of applications allowed in the old mechanism also led to situations where students fell through the cracks if they applied to oversubscribed schools. Since rank order lists were short, the mechanism considered a smaller number of alternate choices for these students compared to a mechanism which allowed students to rank more choices. Schools where these students were ultimately placed may have been assigned in the main round had more applications been allowed.

The new mechanism appears to have relieved the market of congestion by increasing the number of choices students can rank and the number of rounds of offer processing. To investigate the role of these two forces – short rank order lists and limited offer processing – in producing administrative assignments, we used data from the coordinated mechanism to simulate two variations: 1) the main round where only the top five choices are considered and there is no restriction in the number of rounds and 2) the main round with twelve choices, but only three sets of proposals from the deferred acceptance algorithm.\(^{16}\) The first is intended to isolate the role of five choices, while the second isolates the role of few offer processing rounds. Since we do not model behavioral responses by students, we only intend this exercise to shed light on mechanical features

\(^{15}\)The marked shift in the number assigned in the main round also appears in the second year of the coordinated mechanism where even more students, 87.3%, were placed in the main round (shown in Appendix Table B1).

\(^{16}\)Even though there are multiple possible implementations of deferred acceptance, our simulation considers the simultaneous-proposing version, where a round is defined by a set of proposals by students who are not tentatively held or have not exhausted their rank order list.
generating administrative assignments in the uncoordinated mechanism. With that caveat in mind, we find that the five choice constraint with unlimited number of rounds leaves about one quarter of applicants unassigned, while the unconstrained mechanism with three proposal rounds leaves roughly half of applicants unassigned. Relative to the uncoordinated mechanism, the new coordinated mechanism appears to reduce administrative assignments by computerizing offer processing, and avoiding the need for active student and school participation once preferences are submitted. Short rank-order lists also generate administrative assignments, but perhaps less so than few offer processing rounds.

4.2 Distance, Exit, and Matriculation

Across mechanisms, there are stark differences in distance to assigned school and offer take-up. Figure 2 reports the distribution of distance between students’ residence and their assigned school in both mechanisms. New York City spans a large geographic range, with nearly 45 miles separating the southern tip of Staten Island with the northernmost areas the Bronx, and 25 miles traveling from the western edge of Manhattan near Washington Heights to Far Rockaway at the easternmost tip of Brooklyn. The closest school for a typical student is 0.82 miles from home, and students in the uncoordinated mechanism on average travelled 3.36 miles to their assignment. In the coordinated mechanism, the average distance is 4.05 miles. The medians (unreported) also increase from 2.25 to 3.04 miles. Panels A and C of Table 3 shows that students in the administrative round of the uncoordinated mechanism are assigned close to home (an average of 1.64 miles) compared to those in the coordinated mechanism or those in the main round of the uncoordinated mechanism.

The increase in distance to assigned school suggests that the coordinated mechanism expanded the scope of the market. This finding parallels Niederle and Roth (2003)’s study of the gastroentrology labor market, where physician mobility increased following a centralized match. However, the increase in distance need not directly imply a statement about student welfare without knowing how students value proximity relative to other aspects of their school choices.

Student enrollment patterns documented in Table 3, however, indicate that student assignments in the uncoordinated mechanism, particularly those made in the administrative round, are undesirable relative to those in the coordinated mechanism. After receiving an assignment, a student may opt for a private school, leave New York, or even drop out. Families may switch schools after their final assignments are announced, but before the school year starts. In the uncoordinated mechanism, principals had greater discretion to enroll students and the DOE officials quoted above alleged that students with sophisticated parents might just show up at a school in the fall and ask for a seat at the school. The exit rate is higher in the uncoordinated mechanism (8.5% compared to 6.4%), and fraction who enroll at a school other than their assignment is higher (18.6% compared to 11.4%).

In the uncoordinated mechanism, students assigned in earlier rounds appear more satisfied with their assignment than those assigned in later rounds. The fraction of students who exit NYC public schools is 13.3% among administrative placements, compared to 5.2% among those

\(^{17}\)Our analysis focuses on road distance, which is highly correlated with subway distance. Appendix B presents a detailed comparison of both measures.
assigned in the first round. More than a quarter of students assigned in the administrative round who are still in NYC public schools matriculate at schools other than where they were assigned. By comparison, the take-up of offered assignments is much higher for those assigned in the first three rounds. Based on exit and matriculation, students with multiple offers in the first round are more satisfied with their assignment than students with zero or one offer. These students also travel further to their final assignment or enrolled school. In contrast, the majority of students with no offers are assigned through the administrative process, and this likely accounts for their higher rates of exit and enrollment at a school other than their assignment. Even though the coordinated mechanism has substantially fewer administratively assigned, the exit rates are highest and the matriculation rates are lowest for the participants of that round.  

4.3 Mismatch in Administrative Round

To further evaluate the assignments of students processed in the administrative round, we compare the attributes of schools that students wanted (or ranked) to the attributes of schools to which they were assigned. Students processed in earlier rounds are assigned to schools that have attributes more similar to the schools they ranked than students processed in later rounds. Table 4 shows that for those assigned in the main round, with the exception of distance, ranked schools tend to have similar or slightly better attributes than assignments in both mechanisms. For instance, ranked schools have higher Math and English performance, more four-year college-going, and higher attendance rates. They are similar in terms of teacher experience, poverty (as measured by percent subsidized lunch), and racial make-up.

For students placed in the supplementary round, assigned schools are also less desirable than ranked schools, and many of the gaps are wider compared to the main round. In the uncoordinated mechanism, for instance, the gap in Math performance between ranked and assigned schools is 0.7 percentage points for those assigned in the main round, while it is 2.5 points in the supplementary round. The gap between ranked and assigned alternatives for 9th grade size is quite pronounced under both mechanisms. For example, in the uncoordinated mechanism, ranked schools have about 200 fewer 9th graders than the schools where students are assigned. Since students participate in the supplementary round when they did not obtain a main round assignment, it is not surprising that the difference what students wanted and received widens.

The most striking pattern in Table 4, however, is for students who are administratively assigned. We've already seen that there are three times more students assigned in this round in the uncoordinated mechanism. Panel C shows that students who are assigned in that round ranked schools on average 5.1 miles away from home, and were assigned to schools only 1.6 miles away in the uncoordinated mechanism, a much larger gap than either the main or supplementary round. For other school characteristics, the difference between what students wanted and what they were assigned widens relative to the supplementary round, suggesting that mismatch is greatest for students in the administrative round. For instance, the 2.5 point spread in Math achievement is now 4.4 points and there is a similar widening in the fraction going on to four-

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18 In the second year of the mechanism, the average distance to the assigned school is 4.07 miles and the average exit rate is 6.4% (shown in Table B1). The take-up rate is higher than the first year and the fraction in the administrative round decreases to about 5%.
year colleges. The difference in 9th grade size is also considerable: in the supplementary round, students wanted schools with on average with roughly 700 9th graders and they were assigned to schools with more than 900 9th graders. In the uncoordinated mechanism's administrative round, they are assigned to schools with nearly 1,200 9th graders.

The differences between ranked and assigned schools are also large in the coordinated mechanism, though in some cases, they are not as stark. Differences between ranked and assigned schools on Math and English achievement or four-year college going are narrower for the administrative round of the coordinated mechanism than the uncoordinated one. On the other hand, assigned schools are not as close to home in the coordinated mechanism. Therefore, it is not possible to assert which mechanism's administrative process generates better matches. What is clear is that being processed in the administrative round is undesirable for students in both mechanisms. As a result, it's reasonable to expect that a significant fraction of the changes in student welfare will be driven by the reduction in the number of students who enter this round in the coordinated mechanism.

4.4 Offer Processing by Student Characteristics

Table 5 reports the attributes of students across rounds compared to the overall population of applicants in the uncoordinated mechanism. Students from Manhattan, those with high math baseline scores, and those who have applied to exam schools (indicated as taking the SHSAT) tend to obtain offers earlier in the uncoordinated mechanism. They are also overrepresented among students who receive multiple first round offers (not shown). Students from Staten Island, students who are white, and those from high income neighborhoods tend to systematically obtain offers later in the process. Compared to the overall population, these groups are overrepresented in the administrative round. About a quarter of students in the administrative process are white, compared to 15% of participants overall.

The coordinated mechanism distributed school access more evenly across rounds. That is, the differences in the types of students assigned in each round are not as pronounced under the coordinated mechanism. This can be seen by comparing students across boroughs or racial groups in column 4 of Table 5. The fraction of students assigned in the main round is similar across all five boroughs, as is the racial composition of students. Higher baseline applicants are more likely to be assigned in the main round in the new mechanism than low baseline applicants, but they are not as overrepresented as in the old mechanism.

The coordinated mechanism also reduced the number of students assigned to in-demand schools relative to the uncoordinated mechanism. Figure 3 reports the change in the number of students assigned to a school compared to a measure of how oversubscribed the school was in the uncoordinated mechanism. For example, the 1,455 students were assigned to the Louis Brandeis High School, a struggling Manhattan high school with among the lowest four-year graduation rates in the city, in 2002-03, but only 911 students were assigned there in 2003-04. The upward sloping line means that if a school is more oversubscribed in the old mechanism, the new mecha-

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19 The NYC DOE announced the closure of this school in 2009. The largest size reduction is the Evander Childs High School in the Bronx, which went from 1,739 to 453 9th graders. This high school had a longstanding reputation for violence and disorder, and was eventually closed in 2008.
anism tended to assign more students to that school. Conversely, the new mechanism assigned fewer students to schools that were undersubscribed in the old mechanism. This phenomenon suggests that the coordinated mechanism was able to use the submitted preferences more effectively to place children into schools that they wanted. The extent to which this represents an improvement in student welfare depends on the heterogeneity of student preferences, an issue we turn to next.

5 Estimating Student Preferences

5.1 Student Choices

What makes a families rank particular schools and where do they obtain information about school options? In surveys, parents state that academic achievement, school and teacher quality are the most important school characteristics (Schneider, Teske, and Marschall 2002). In NYC, families obtain information about high schools and programs from many sources. Guidance counselors, teachers, and other families in the neighborhood all can provide input and recommendations. The official repository of information on high schools is the NYC High School directory, a booklet that includes information about school size, advanced course offerings, Regents and graduation performance, the school’s address, the closest bus and subway, and a description of each program together with a list of extracurricular activities and sports teams. Families also learn about schools at High school fairs, school open houses, rankings in local newspapers, online school guides and from books about high schools (e.g., Hemphill (2007)).

Even though the school ranking decision involves evaluating many different alternatives and features that we do not observe, the aggregate distribution of preferences displays some consistent regularities: students prefer closer and higher quality schools as measured by student achievement levels, shown in Table 6.20 The first row of the table shows that 20% of applicants rank 12 school choices; the majority rank nine or fewer choices and nearly 90% rank at least three choices. The average student’s top choice is 4.43 miles away from home. Given that the closest school is on average 0.82 miles away, this means that students do not simply rank the school closest to home first. For the typical student, the first choice is 0.44 miles closer than her second choice, and her second choice is 0.25 miles closer than her third choice. Even though other school characteristics change with distance, the distance to ranked school increases monotonically until the 9th choice, which is 5.65 miles away on average.

Lower ranked schools are not only farther away, but they also less desirable on other measures of school quality. Math performance decreases going down rank order lists (English performance exhibits the same trends as Math and is therefore not reported). Other measures of performance (also not reported) such as the percent of students attending a four year college and the fraction of teachers classified as inexperienced change monotonically going down rank order lists. Schools enrolling lower shares of poor students or a higher share of white students tend to be ranked higher.

Table B3 provides additional information on school assignments. 31.9% of students receive their top choice, 15.0% receive their second choice, and 2.4% receive a choice ranked 10th, 11th, or 12th. 17.5% of students are asked to participate in the supplementary round because they are unassigned in the main round.
Using requests for individual teachers, Jacob and Lefgren (2007) find that parents in low income and minority schools value a teacher’s ability to raise student achievement more than in high income and non-minority schools. In contrast, Hastings, Kane, and Staiger (2009) report that higher SES families are more likely to choose higher-performing schools than lower-SES ones based on stated reports under Charlotte’s school choice plan. This difference across groups motivates our investigation of ranking behavior by baseline ability and neighborhood income. High achieving students tend to rank schools with high Math achievement relative to low achievers, though both groups place less emphasis on achievement with further down their preference list. Similarly, students from low income neighborhoods tend to put less weight on Math achievement than students from high income neighborhoods, but both groups rank higher achieving schools higher. These differences suggest the importance of allowing for tastes for school achievement to differ by baseline achievement and income groups.

The characteristics of schools ranked in the uncoordinated mechanism, also shown in Table 6, are decreasing in a school’s average achievement and income. However, there is a compression in the preferences relative to the coordinated mechanism. For instance, the distance to a students’ first choice is 4.80, while it is 4.79 miles for their fifth choice. In the coordinated mechanism, the fifth choice is about one mile further than the first choice. Such a pattern is consistent with students being more expressive with their choices in the new mechanism, which would be expected given its incentive properties and the fact that more choices can be ranked.

Based on their submitted preferences, all else equal, students prefer attending a school closer to home. The fact that students in the new mechanism are assigned to schools further from home might suggest that the new mechanism led to assignments that are worse on average than the old mechanism. On the other hand, students may prefer schools outside of their neighborhood because they are a better fit. We must therefore weigh the greater travel distance in the new mechanism against changes in the assigned school. Students do not rank the closest school to their home and instead trade-off school attributes with proximity. Our next task is to quantify how students evaluate distance relative to school attributes such as average achievement levels, demographic composition, and size based on their submitted preferences.

5.2 Model and Estimation

The comparison of the attributes of choices ranked higher to those ranked lower or not at all provides rich information to identify how a student trades-off school features with distance. To quantify these tradeoffs, we work with a random utility model, which allows for factors which are not observed in our dataset to influence ranking decisions. Let i index students, j index programs, and let sj denote the school in which program j is located. Since all of the schools we study are free, we treat distance as our numeraire for our welfare analysis. Specifically, we model student i’s indirect utility for program j using the following specification:

\[ u_{ij} = \delta_j + \sum_{l} x_l \alpha_l + \sum_{k} y_k x_k^j - d_{ij} + \epsilon_{ij} \]  
(1)

with \( \delta_j = x_j \beta + \xi_j \)
where $z_i$ is a vector of student characteristics, $x_j$ is a vector of program $j$’s characteristics, $d_{ij}$ is distance between student $i$’s home address and the address of program $j$, $\xi_j$ is a program-specific unobserved vertical characteristic, $\gamma_i$ is a vector of random-coefficients that capture idiosyncratic tastes for program characteristics and $\epsilon_{ij}$ captures idiosyncratic tastes for programs.

Since we’d like the model to capture heterogenous preferences, but still be computationally tractable, we employ an ordered version of the choice model in Rossi, McCulloch, and Allenby (1996). They describe a class of parametric distributions for unobserved characteristics and idiosyncratic preferences that allows for estimation via Gibbs’ sampling.\textsuperscript{21} Specifically, we assume that

$$
\gamma_i \sim N(0, \Sigma_{\gamma}) \\
\xi_j \sim N(0, \sigma_{\xi_j}^2) \\
\epsilon_{ij} \sim N(0, \sigma_{\epsilon}^2)
$$

We assume conjugate priors for $\beta$, $\alpha$, $\Sigma_{\gamma}$, $\sigma_{\xi_j}^2$, and $\sigma_{\epsilon}^2$. The specific distributions and additional details are described in the Computation appendix.

Students may have idiosyncratic tastes for schools that are not captured by the observable students characteristics in our dataset. This specification allows for arbitrary correlation between the $k$ dimensions of $\gamma_i$, which exploits the richness rank-ordered data. Berry, Levinsohn, and Pakes (2004) argue that data on top and second choices can be used to estimate these parameters by exploiting common characteristics between subsequent rankings for a given student. Since our rank order list data includes up to 12 choices, we do not restrict the covariance between school size, percent White, percent subsidized lunch, and Math performance.

The parametric assumptions are made for computational tractability as the distribution of indirect utilities is non-parametrically identified in our setting.\textsuperscript{22} Given the independence of idiosyncratic tastes, the two primary restrictions in our specification is that the distance to school is additively separable in the indirect utility function and that taste shocks are independently distributed. The coefficient of $-1$ is an alternative scale normalization to the common practice of setting the variance of $\epsilon$ if, all else equal, students dislike traveling to further away schools.

We do not explicitly include an outside option and instead normalize, without loss of generality, the value of $\delta$ for an arbitrarily chosen school to zero. This assumption is motivated by our primary interest in studying the allocation within inside options rather than substitution outside of the NYC public school system. Moreover, the commonly used model of the outside option, which infers that a school is unacceptable if not ranked, would require us to assume that students who have not ranked all 12 choices prefer their outside option to a NYC high school. However, Table B4 shows that roughly three-quarters of supplementary round students enroll in their offer for that round, and the majority enroll in some other NYC high school.

The demand sample contains rankings of 69,907 participants in 2003-04 over 497 programs in 235 schools, representing a total of 542,666 school choices. Let $r_i$ denote the rank order list of

\textsuperscript{21}We use Gibbs sampling rather than simulated maximum likelihood because of biases in datasets with a large number of choices (Train 2009). The posterior means we report have the same asymptotic distribution as maximum likelihood estimates (see chapter 10.1 in van der Vaart (2000)).

\textsuperscript{22}Assumptions needed to identify preferences for choice characteristics in binary and multinominal settings have been examined in Ichimura and Thompson (1998), Lewbel (2000) and Briesch, Chintagunta, and Matzkin (2002), though our data includes ordered choices, though ordered choice data contains additional information. Agarwal and Somaini (2014) study identification in the school choice context with a potential manipulable mechanism. Non-parametric identification results in these settings carry over to our case.
Our specifications follow other models of school demand and include average school test scores and racial attributes as characteristics (see, e.g., Hastings, Kane, and Staiger (2009)). In each specification, we also include three program type dummies and ten program speciality dummies. The categorization of programs into specialities is described in the Data Appendix.

We start by assuming that rankings reflect true preferences. This benchmark is natural because of the straightforward incentive properties of the mechanism and the advice that the NYC DOE provides in the 2003-04 High School Directory and in their information and outreach campaign. For instance, the DOE guide advises participants to “rank your twelve selections in order of your true preferences” with the knowledge that “schools will no longer know your rankings.” Nonetheless, truthful behavior is a strong assumption that we revisit in Section 8.

5.3 Preference Estimates

We report select estimates for six specifications of our demand model in Table 7 (the full set of estimates is in Table A1). The first specification includes school characteristics (high math achievement, percent subsidized lunch, percent white, and 9th grade size), but does not include interactions of student characteristics with student characteristics. We do not include additional achievement characteristics examined in Table 4 such high English achievement and percent attending four college because they are closely related to high math achievement. The second specification includes student-school interactions. Both of these models do not include student-specific random coefficients. The next four specifications add in student-specific random coefficients to the second specification and vary in how they use the choice data: using all choices, using the top three choices, dropping the last choice, and dropping students who rank 12 choices. Each specification with student interactions include dummies for Spanish, Asian and Other Language Program, interacted with a student’s English proficiency status and whether they are Hispanic or Asian. Computational constraints prevent us from estimating all of these models on the full sample, so we only estimate using the full sample for the specification with student-school interactions and student-specific random coefficients; the rest of these models are estimated on a 10% random sample of the data which contains about 7,000 students ranking several schools.

There are three main patterns in Table 7. First, student-school interactions are often estimated precisely. For instance, high baseline math students tend to prefer higher achieving schools and minority students tend not to prefer schools with high white percentages. Second, the estimates are broadly similar across the four specifications in column 3-6, with random coefficients. Third, many of the random coefficients are significantly estimated, suggesting the importance of a flexible specification in accounting for the underlying heterogeneity in student preferences. We report further on measures of model fit in Section 8. Since it fully exploits all of the choice data in the most flexible model, the third column specification is our preferred specification.

The program type dummies are Unscreened, Screened, and Educational Option. The program specialty dummies are Arts, Humanities/Interdisciplinary, Business/Accounting, Math/Science, Career, Vocational, Government/Law, Other, Zoned, and General.

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23 The program type dummies are Unscreened, Screened, and Educational Option. The program specialty dummies are Arts, Humanities/Interdisciplinary, Business/Accounting, Math/Science, Career, Vocational, Government/Law, Other, Zoned, and General.
6 Comparing Alternative Allocation Schemes

6.1 Measuring Welfare

The preference estimates allow us to compute measures of welfare across assignments. Under the assumption that distance is the numeraire, we can measure the difference between two assignments for a student in terms of the distance-equivalent utility or the amount a student is willing to travel for the more preferred school assignment. To compare the welfare associated with two assignments, we compute the average welfare in distance-equivalent utility. Consider a group of students in set $G$ and a matching $\mu$, which specifies the program for each student as $(i)$. Define average welfare for students in $G$ as a function of parameters $\theta$ as

$$W^G_\mu(\theta) = \frac{1}{|G|} \sum_{i \in G} u_{i\mu(i)}(\theta)$$

where $u_{ij}$ is the utility student $i$ associated with assignment to program $j$. For two matchings, and $\mu'$, and students in group $G(\mu)$ and $G(\mu')$, the difference in welfare between the two matchings is given by:

$$W^G_\mu(\theta) - W^G_{\mu'}(\theta) = \frac{1}{|G(\mu)|} \sum_{i \in G(\mu)} u_{i\mu(i)}(\theta) - \frac{1}{|G(\mu')|} \sum_{i \in G(\mu')} u_{i\mu'(i)}(\theta)$$

In the coordinated mechanism, we observe the rankings submitted by students. Under the assumption that these reports are truthful, the $k^{th}$ ranked program yields the $k^{th}$ highest utility. For a given student and estimate of $\theta$, this fact contains information about her unobserved tastes $\epsilon_{ij}$. For students in the coordinated mechanism who submit rank order list $r_i$, we calculate the expected utility for each ranked and unranked choice by simulating the utility from the estimated preference distribution, conditional on the relationships implied by the submitted ranks.

In particular, we compute the expected utility of a program ranked $k^{th}$ by student $i$ as

$$E_i u_{ij} \mid u_{ij}^{(k+1)} < u_{ij} < u_{ij}^{(k-1)} \ r_{ik} = j$$ (2)

and the expected utility for all unranked schools as

$$E_i u_{ij} \mid u_{ij} < u_{ij}^{(|r_i|)} \ j \in \{r_{ik}\}$$ (3)

where $u_{ij}^{(k)}$ is the $k^{th}$ highest value of $\{u_{ij}\}_{j=1}^J$ and $|r_i|$ is the number of ranks submitted by student $i$. With these definitions, the welfare difference for group $G$ from assignments and $\mu'$ is

$$W^G_\mu - W^G_{\mu'} = \frac{1}{|G(\mu)|} \sum_{i \in G(\mu)} E[u_{i\mu(i)} \mid r_i] - \frac{1}{|G(\mu')|} \sum_{i \in G(\mu')} E[u_{i\mu'(i)} \mid r_i]$$

where $E[u_{ij} \mid r_i]$ denotes the conditional expectations in equations (2) and (3) above.

24 For students who are in the mechanism comparison sample, but not the demand sample (so we don’t observe their rank order lists), we use the mean utility conditional on observables alone in the welfare calculations. We follow a similar approach for students assigned to choices not ranked in the main round.
6.2 Evaluating Mechanism Design Features

Even though many of the new mechanism’s features were designed to address issues in the old mechanism, the coordinated mechanism still involves several design constraints. Here, we ask: how far is the allocation produced by the coordinated mechanism from the best possible one for students and what are the quantitative effects of particular design decisions? The alternatives we consider vary the mechanism, holding fixed the set of schools, students, and residential choices, and therefore are best interpreted as short-run effects of alternative mechanisms.

We begin by using the demand model estimates to assess two benchmark allocations intended to capture the range of what’s achievable with a choice system. The first is a neighborhood assignment, computed by running the deferred acceptance algorithm with applicants simply ranking schools in order of distance. Since the scope of the market is smaller under the uncoordinated mechanism, this allocation further restricts the market’s scope, as in the administrative round of the uncoordinated mechanism.

The second benchmark allocation, the utilitarian assignment, maximizes the sum of student utility subject the feasibility constraints of the assignment. Given the estimated distribution of student preferences, no other allocation can yield higher average utility. This allocation therefore represents the other extreme where a planner implements the best possible allocation taking full advantage of the cardinal distribution of student preferences.

To compare the coordinated mechanism to these two benchmarks, in Table 8 we report the difference with the utilitarian outcome. The first column of Table 8 shows that the difference in distance-equivalent utility between the neighborhood and utilitarian assignment is 18.96 miles. This difference is a theoretical upper bound on the gains from school choice. The coordinated mechanism achieves about 80% of the possible gains from a choice system, since the difference in distance-equivalent utility with the utilitarian assignment for the average student is 3.73 miles.

The utilitarian optimal assignment is an idealized benchmark, but is difficult to achieve for two reasons. First, there are 208 screened programs in New York City in 2003-04, so implementing this allocation would ignore school-side rankings at those programs. This allocation may also allow for situations of justified envy at programs that only use coarse priorities. Second, this assignment uses cardinal information, which is not solicited by the coordinated mechanism. Therefore, we consider a more efficient outcomes for students that do not completely abandon school priorities and only uses student rank-order lists.

The deferred acceptance algorithm in the coordinated mechanism need not produce a student-optimal stable matching because it must resolves ties between students when they have identical priorities at a school. This tie-breaking can lead DA to stable outcomes, which are not student-

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We solve for this allocation following Shapley and Shubik (1971). The utilitarian allocation implies equal weights on students; in quasi-linear problems, such an allocation would be synonymous with the Pareto efficient allocation, but need not be here.

Using illustrative examples, Pycia (2014) argues that the welfare loss of ordinal mechanisms relative to cardinal ones can be arbitrary large.

We implement the coordinated mechanism by drawing 100 sets of lottery numbers and re-run the student-proposing deferred acceptance algorithm given student’s choices. For students unassigned after the main round, we implement NYC’s supplementary round by using preferences from the demand model and assigning students under a serial dictatorship according to the lottery number.
optimal. Deferred acceptance cannot be Pareto-improved upon without sacrificing strategy-proofness for students (Erdil and Ergin 2008, Abdulkadiroğlu, Pathak, and Roth 2009, Kesten 2010, Kesten and Kurino 2012). We quantify the cost of providing straightforward incentives for students by computing a student-optimal stable assignment that improves the DA assignment by assigning students higher in their choice lists, while respecting stability for strict school priorities. Such an assignment can be computed by the stable improvement cycles algorithm developed by Erdil and Ergin (2008), which iteratively finds Pareto improving swaps for students, while still respecting the stability requirement for underlying weak priority ordering of schools (i.e., prior to tie-breaking). A total of 2,348 students in the demand sample can obtain a better assignment in a student-optimal stable matching. The difference in distance-equivalent utility is 0.11 miles compared to the assignment produced by the coordinated mechanism. The cost of this change is that the underlying mechanism is not based on a strategy-proof algorithm.28

Another limitation of the coordinated mechanism is that it constrains student welfare due to its treatment of school priorities and preferences. To quantify the importance of this constraint, we compute the welfare of students under the Pareto efficient assignment found by transferring students from their assigned schools to their higher ranked choices via the Gale’s Top Trading Cycles algorithm (Shapley and Scarf 1974). Since this mechanism does not produce a stable outcome, it is possible that schools benefit by offering students seats outside of the assignment process. The difference in aggregate student welfare under this Pareto efficient assignment and the student optimal stable matching may therefore be viewed as the cost of incentives for schools to participate in the system.29

We calculate a Pareto efficient matching which dominates each student-optimal stable matching we simulate and estimate students’ utilities from this Pareto efficient matching in column 4 of Table 8. A total of 10,882 students obtain a more preferred assignment at a Pareto efficient matching. An ordinal Pareto efficient allocation still represents a substantial difference between the utilitarian optimum. The utility difference for the average student 3.11 miles. Relative to the current mechanism, the cost of limiting the scope for strategizing by schools (by imposing stability) is 0.62 miles.

In summary, these comparisons show the difference between having a choice system with the coordinated mechanism and not having a choice system at all represents a much larger difference in student welfare than fine-tuning the coordinated mechanism. That is, the ability to choose schools generates substantial welfare when preferences are heterogeneous. Within a coordinated matching system, further optimizing the matching algorithm produces relatively little gain in the best case, and even if it were possible to implement cardinal allocation schemes. This conclusion does not imply that the matching scheme is not important since we’ve seen the large number of those processed administratively in the uncoordinated mechanism. To see where in the spectrum the uncoordinated mechanism lies, we next turn to analyzing its properties.

28Azevedo and Leshno (2011) provide an example where the equilibrium assignment of the stable improvement cycles mechanism is Pareto inferior to the assignment from deferred acceptance when students are strategic.

29Balinski and Sönmez (1999) and Abdulkadiroğlu and Sönmez (2003) provide an alternative equity rationale for stability. Note that no stable mechanism eliminates strategic maneuvers by schools (Sönmez 1997), although this may not be an issue in markets with many participants (Kojima and Pathak 2009).
7 Comparing the Coordinated and Uncoordinated Mechanisms

Unlike the variations on the coordinated mechanism we’ve just examined, there is no simple way to model behavior in the uncoordinated mechanism. To evaluate it, we adopt an alternative approach that takes advantage of the fact that we observe the assignments of students in that mechanism. This approach requires two important assumptions.

First, since we do not observe the same student in both the uncoordinated and coordinated mechanism, we have to assume that we can use preference estimates from the coordinated mechanism to make statements about the previous year. One potential concern is that schools have changed significantly because of the mechanism so a student’s valuation of the school likely has changed alongside the mechanism. To probe this issue, in Figure A1, we plot the average characteristics of the schools, as measured by the attributes of peers, in both mechanisms. The Figure shows that for measures of poverty, racial make-up, and baseline math scores, there is relatively little change in school attributes despite the change in the mechanism. Consistent with the increased travel distance in Table 3, the last panel of the Figure shows that schools do appear different when measuring the average travel distance of enrolled students.

The second assumption involves our interpretation of the rankings submitted in the uncoordinated mechanism. For the counterfactuals in Table 8, to compute the utility associated with an assignment for the new mechanism, we condition on the rank order list submitted by the student, adjusting for what that rank order list implies about unobserved tastes under the assumption that the student reported her true preferences. The old mechanism’s uncoordinated nature makes the relationship between preferences and the final assignments less straightforward. Computing the expected utility conditional on the assignments produced by the uncoordinated mechanism requires strong assumptions about that mechanism, its properties, and agent behavior and expectations. This difficulty is not unique to our setting and is likely a challenge in evaluating the welfare effects of other assignment systems where the incentive properties of the mechanism are not well understood and agent behavior is difficult to model. Instead, our approach exploits our data on observed rankings from the uncoordinated mechanism’s first round.

The approach we start with, unordered applications, assumes that choices submitted in the uncoordinated mechanism have the property that any ranked choice is preferred to an unranked choice, but does not assume that higher ranked choices are preferred to lower ranked ones. This assumption has the benefit of not assuming rank order lists submitted in the uncoordinated mechanism are truthful, while still using some of the information contained in the preference submission. A weakness of this assumption is that it excludes the possibility that students have omitted choices from their rank order list that they prefer, but do not expect to be admitted. However, in Table 6, we saw that less than half of participants rank all five choices in the old mechanism, suggesting that if these participants wanted to rank more schools, the mechanism does not constrain them. Nonetheless, we will also consider two other assumptions.

Budish and Cantillon (2012) utilize survey data from a manipulable mechanism to make statements about changes in mechanism design. Unfortunately, similar survey data does not exist in our setting. Agarwal and Somaini (2014) present an approach to uncovering preferences from a class of single-offer manipulable mechanisms that involve lotteries; because the uncoordinated mechanism is a multiple-offer system, it falls outside of this class, and their methods are not applicable in our setting.
Under the unordered applications selection assumption from the uncoordinated mechanism and assuming true preferences from the coordinated mechanism, we find that most students are better off under the coordinated mechanism. Figure 4 plots the overall distribution of student welfare from the two mechanisms. The average student improvement in welfare is equivalent to 10.96 miles. The figure shows a bimodal pattern of utility in the uncoordinated mechanism due to the students who are assigned in the administrative round. Most of the mass in the first mode shifts rightward in the coordinated mechanism, a phenomenon driven by the sharp reduction in the number of students assigned administratively.

For each student group shown in Table 9, there is a positive gain from the new mechanism. Across racial groups, whites and Asians gain more than blacks and Hispanics. There are more pronounced differences across boroughs. Students from Staten Island and Queens gain the most, while the smallest gain of any student group in the table comes from Manhattan residents. Low baseline math students gain more than high baseline math students or SHSAT test takers.

It's worthwhile to compare the difference in utility to the difference in travel distance, to decompose the welfare effects of the improved school match. On average, the improved match ignoring distance is equal to 11.65 miles. This implies that the improved school match is worth thirteen times the costs associated with increased travel distance. The lowest gains are for Manhattan residents, a group which experiences no increase in travel distance. However, the welfare gains are not solely driven by changes in distance. For instance, across boroughs, Staten Island pupils only travel 0.34 miles further, but they experience the largest improvement of any borough at 22.56 miles. This suggests that mismatch was particularly severe in Staten Island, and is consistent with the substantially larger fraction of Staten Island residents who are administratively assigned in the uncoordinated mechanism. The magnitude of our estimates of improved school matches suggest that even if there are not enough good schools to assign everyone their top choice, preference heterogeneity generates an important role for a coordinated mechanism.

The welfare gains in the coordinated mechanism are larger for many disadvantaged groups, a pattern consistent with Hemphill and Nauer (2009)'s claim that the uncoordinated mechanism advantaged high-achieving students and those with sophisticated parents. For instance, welfare gains are larger for low baseline math students than high baseline math students. They are also higher for limited English proficient students than SHSAT test-takers. However, the difference for Staten Island, which has a larger white population and weather neighborhoods, plays a large role in the fact that whites and rich neighborhoods experience larger welfare gains than blacks and Hispanics and poorer neighborhoods.

Differences across student groups closely track the rounds in which students were processed in the uncoordinated mechanism. For instance, substantially more high baseline students were assigned in the main round of the old mechanism compared to low baseline students. Student groups with higher fractions assigned in the administrative round, shown in column 7, tend to experience the largest gains. Figure 5 reports on the relationship between likelihood to be assigned

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\(^{31}\)In the uncoordinated mechanism, there are 1,054 students who ranked Staten Island Technical, a highly sought-after screened school. Only 16% are assigned there, and about 75% do not obtain a main round offer and are subsequently administratively assigned.
administratively and student welfare. We fit a probit of whether a student is administratively assigned on all student characteristics in the demand model for the uncoordinated mechanism sample. We also include census tract dummies to account for neighborhoods that may or may not have local high schools, which is a guaranteed fallback option for some students. We then use this estimated relationship to compute each student’s propensity to be assigned administratively in both the uncoordinated and coordinated mechanism. For each decile of this propensity, we compare the utility from assignment across both mechanisms. However, there is a clear monotonic pattern between administrative assignment propensity and the welfare improvement, whether comparing utility differences either including distance or net of distance.

A possible threat to our welfare calculation is that an initially bad allocation was subsequently undone post-assignment through the aftermarket, the period between when offers where made and the school year starts. Table 3 showed that students enrolled in schools further away on average than where they were assigned in the uncoordinated mechanism, but the opposite pattern is true in the coordinated mechanism. The coordination of admissions occurred with greater central control of enrollment, which may have made it possible that the more rigid aftermarket in the coordinated mechanism is actually worse for students. To examine this possibility, we also compute the utility associated with the schools students enroll at in October of the following school year. Compared to assignments, the gains from the coordinated mechanism measured by enrollment are somewhat smaller, but are still large. For instance, the distance-equivalent utility for the average student is 9.67 miles, which is 88% of the gain from the assignment. The change in travel distance to enrolled school is also lower than the change in travel distance to assignment. Though a smaller gain from enrollment suggests that some of the old mechanism’s mismatch was undone in its aftermarket, these facts weigh against the argument that post-market reallocation has undone a large fraction of misallocation.

Finally, it’s worth noting how the mechanism design matters when we compare the uncoordinated and coordinated mechanisms. That difference represents nearly 60% of the total range between the neighborhood and utilitarian assignments, which is much larger than the range associated with tweaks to the matching algorithm. This finding informs a broader debate in the market design literature about the importance of sophisticated market clearing mechanisms. In the context of auctions, Klemperer (2002) argued that “most of the extensive auction literature is of second-order importance for practical auction design,” and that “good auction design is mostly good elementary economics.” Consistent with this point of view, for school matching market design, coordinating admissions producing much larger gains than algorithm refinements within the coordinated system.

To see how robust our conclusions are, in Table 10, we repeat the analysis in Table 9 for three alternative assumptions. In each case, we break the asymmetric treatment of selection rules in Table 9, and use the same assumption about rank order lists for both mechanisms. In the first variation, we do not adjust the utility for based on rank order lists submitted in either the uncoordinated or coordinated mechanism. This unselected applications (on unobservables) selection rule simply assumes that the idiosyncratic component of tastes has no influence on utility. While this assumption may be unappealing given the importance of unobserved determinants of applicant preferences, examining welfare differences under it indicates how much
our estimates are driven by observable dimensions of school and student characteristics. Column 1 of Table 10 reports that the difference in welfare is more than one half of that implied by unordered applications selection rule. The average student is going to a school that is about eight times more valuable compared to the increased travel distance. The qualitative patterns are similar to those under unordered applications selection rule: all student subgroups benefit and the gains are larger for groups that were more likely to be administratively assigned. For instance, gains are larger for Asians and Whites; they are largest for Staten Island residents and lowest for Manhattan. One difference with the unordered applications selection rule is for comparisons across baseline achievement groups: high baseline math students benefit more than low baseline math students, though low baseline students still gain slightly more than SHSAT test-takers.

Columns 3 and 4 present estimates using the unordered application selection rule for both the coordinated and uncoordinated mechanism. The average welfare difference of 9.02 miles is smaller than that reported in Table 9, but the patterns across student groups is similar. Finally, in columns 5 and 6, we report estimates from the truthful applications selection rule, which assumes that the rank order list submitted in the either the uncoordinated or coordinated mechanism reflect true applicant preferences. Under this assumption, the estimated welfare gains are slightly smaller than those reported in Table 9 at 10.62 miles on average. The qualitative patterns mirror those from the unordered application rule, though tend to be slightly larger.

It is reassuring that the estimates we obtain under different selection rules using information from submitted rank order lists are similar between Table 9 and 10. The smaller estimate under the unselected applications selection rule in Table 10 imply that unobserved tastes play a role for our quantitative conclusions. Except for comparisons across baseline achievement groups, the qualitative comparisons between student groups are also similar under each of the assumptions.

8 Model Fit and Alternate Behavioral Assumptions

8.1 Model fit

Since our goal is to make statements about welfare, it is important to examine how well our econometric model matches the data. We first investigate within-sample fit is to see what our estimates imply for the aggregate patterns by rank in Table 6. Figure A2 reports on measures of fit using the specification with student-level random coefficients and student-school interactions. We plot the observed versus predicted pattern of three school characteristics - high math achievement, percent subsidized lunch, and percent white – as we go down a student’s choice list. The panels include three pairs of lines for the entire sample, and for the low and high baseline math applicants. For these three characteristics, our estimates capture the broad pattern of the choices, matching both the level and slope of these characteristics. For instance, the average high math achievement is 10.0, and the range from the top choice to the 12th choice is 16.7 to 10.4. Our estimates imply that for first choices, school’s fraction high math achievement is 16.7, while it drops to 11.8 for last choices. Moreover, the average percent white in New York’s schools is 10.8, but first choices are 19.1 white, while we predict them to be 19.8. Relative to the average attributes of schools, the model fit is much closer to the actual ranked distribution.
In the last panel of Figure A2, we report the fit for distance. Here, we find that the increase in distance observed for lower ranked choices mirrors that predicted by our model, there is a greater divergence in the level of distance. This pattern appears in all of the models we’ve estimated with random coefficients. It’s worth noting that the average distance to a high school in New York is 12.7 miles from home, while the closest school is under a mile.

Berry, Levinsohn, and Pakes (2004) emphasize the importance of mixture models in the context of rank data for automobiles. In particular, they emphasize that when examining the within-consumer relationship between the attributes of alternatives ranked first and second, models without random coefficients do a poor job. This concern may be particularly important in our context. For instance, a high correlation between the size of the first and second ranked school’s size may be indicative of taste for large schools. In Table A2, we report on the correlation between the first and second choice, the first and third choice, and the second and third choice. Consistent with earlier work, we see that the observed correlation between choices is much closer in our preferred specification than in the simpler model within sample. When we examine a more demanding out-of-sample test, comparing the 2003-04 preference estimates to examine the correlation pattern of choice made in 2004-05, we also see that the correlation pattern in our main specification is closer to the observed pattern than from a demand model without student interactions.

8.2 Assessing the Behavioral Assumptions on Ranking

Motivated by the incentive properties of the deferred acceptance algorithm, the preference estimates that we reported come from models assuming that students are truthful. We revisit some potential objections to this assumption in this subsection.

In an influential experiment, Hastings and Weinstein (2008) show how malleable parent preferences are when provided with direct information on school test-scores in Charlotte-Mecklenberg’s choice system. Such a finding suggests that students may be overwhelmed by the prospect of evaluating over 500 school programs and preferences are an unreliable guide for welfare analysis. However, if preferences were generated without much of a systematic component, then we’d expect that most of our point estimates to be imprecisely estimated or have unintuitive patterns, contrary to what we’ve seen. Moreover, in Figure A3, we plot school market shares between the first year of the coordinated mechanism and its second year. This plot illustrates the extent of variation in market shares, which may occur with new information such as expanded high school fairs across time. The market shares of most programs are very similar across both years of the new system. This fact suggests that preferences are persistent.

Another way to see that choices are consequential for participant welfare is to examine enrollment decisions by choice received. Table B4 reports assignment and enrollment decisions for students who are assigned in the main round. The table shows that 92.7% of students enroll in their assigned choice, and this number varies from 88.4% to 94.5% depending on which choice a student receives. Interestingly, take-up is higher for students who receive lower ranked choices, while the fraction of students who exit is highest among students who obtain one of their top three choices. This fact suggests that either families are indifferent between later choices and simply enroll where they obtain an offer or families have deliberately investigated later choices.
and are therefore willing to enroll in lower ranked schools. If families are more uncertain about lower ranked choices, then using all submitted ranks may provide a misleading account of student preferences. To examine how sensitive our conclusions are to this assumption, we fit a demand model that consider only the top three choices of applicants in column 4 of Table 7.

A second concern with treating submitted rankings as truthful is that parents rank schools using heuristics carried over from the previous system. Despite the theoretical motivation and the DOE’s advice, parents might still deviate from truth-telling because of misinformation. Table B4 shows that students are more likely to be assigned their last choice than their penultimate choice. This pattern may be caused by strategic behavior if students apply to schools that they like and, as a safety option, rank a school in which they have a higher chance of admissions last. For instance, Calsamiglia, Haeringer, and Klijn (2010) present laboratory evidence that a constraint on rank order lists encourages students to rank safer options. However, it may also be fully consistent with truth-telling. For example, students usually obtain borough priority or zone priority for schools in their neighborhoods. Ranking these schools improves their likelihood of being assigned to these schools in case they are rejected by their higher choices. If students consider applying and commuting to schools further away from their neighborhood for reasons like math and English achievement, they may as well stop ranking schools below their neighborhood schools once such considerations no longer justify the cost of commute. Alternatively, search costs may induce parents to stop their search for schools before they identify twelve schools for their children and rank their neighborhood school as last choice. This preference pattern would produce the observed assignment pattern. To examine how sensitive our conclusions are to this assumption, we fit demand models that drop the last choice of each student in in column 5 of Table 7.

The third issue with the assumption of truthful preferences is that students can rank at most 12 programs on school applications. When a student is interested in more than 12 schools, she has to carefully reduce her choice set down to at most 12 schools. If a student is only interested in 11 or fewer schools, this constraint in principle should not influence her ranking behavior (Abdulkadiroğlu, Pathak, and Roth 2009, Haeringer and Klijn 2009). It is a weakly dominant strategy to add an acceptable school to a rank order list as long as there is room for additional schools in the application form. However, 20.3% of students in our demand sample rank 12 schools. Some of these students may drop highly sought-after schools from top of their choice lists because of this constraint. To examine how sensitive our conclusions are to this assumption, we fit a demand model that drops students who have ranked all twelve choices in column 6 of Table 7.

In Table A4, we report on our evaluation of mechanism design choices under these three specifications: 1) using only the top 3 choices, 2) excluding applicants who have ranked all 12 programs, and 3) dropping the last choice of applicants. Because of computational constraints, we estimate the models on a 10% random sample, but we use the full sample for the counterfactuals. For all three demand models, the coordinated mechanism in column 2 is roughly 81% of the way from the neighborhood assignment to the utilitarian assignment. It therefore appears that our conclusions on the value of choice relative to changes within the coordinated mechanism are robust to these alternative ways of using the submitted rank order lists in the coordinated
mechanism.

Panel B of Table A4 reports on how the comparison between mechanisms varies with our demand specification using all of the ranking information of participants. Preference heterogeneity generates a larger role for school choice compared to neighborhood assignment. This phenomenon can be seen by comparing the estimates from our main specification to those from specifications without heterogeneity (column 1 in Table A1) and without random coefficients (column 2 in Table A1). The neighborhood assignment is more appealing according to those two demand models, since they are only 15.5 and 16.2 miles away from the utilitarian assignment, compared to 21.5 miles from the main specification in the 10% sample. Moreover, the difference between neighborhood assignment and the coordinated mechanism is smaller in specifications that do not allow for student interactions or random coefficients than in the main specification which includes both student interactions and random coefficients.

9 Conclusion

The reform of NYC’s high school assignment system provides a unique opportunity to study the effects of centralizing and coordinating school admissions with detailed data on preferences, assignments, and enrollment. We find that the new coordinated mechanism is an improvement relative to the old uncoordinated mechanism on a variety dimensions. More than a third of students were assigned through an ad-hoc administrative process in the uncoordinated mechanism after multiple offers with few choices and few rounds of clearing left a large number of students without offers after the main round. Students placed in the administrative round were assigned to schools with considerably worse characteristics than what they ranked. The new mechanism relieved this congestion and assigned more students to schools where they applied.

The coordinated mechanism assigns students 0.69 miles further from home to their assignments. However, the benefit of being assigned through the new mechanism is at least eight times the cost of additional travel, and is often larger depending on the assumption about the information revealed about unobserved tastes from rank order lists submitted under the uncoordinated mechanism. The gains are positive on average for students from all boroughs, demographic groups, and baseline achievement categories. Welfare improvements are also seen whether utility is measured based on assignments made at the end of the high school match or subsequent school enrollment. The largest gains are for students who were more likely to be processed in the administrative round of the uncoordinated mechanism. These conclusions are robust to alternative behavioral assumptions on the preferences submitted in both the uncoordinated and coordinated mechanism.

These gains are measured by a rich specification of student demand that implies significant estimated heterogeneity in willingness to travel for school. Preference heterogeneity is important for measuring the allocative effects of choice when there is a shortage of good schools. Our estimates reveal that the benefits from having coordinated choice are much larger than than those associated with modifications within the coordinated mechanism. This does not imply that the design of the mechanism is not important however, since the gap in average student welfare between the uncoordinated and coordinated mechanism is large.
The increase in student welfare due to the new mechanism illustrates that there are considerable frictions to exercising choice in poorly designed assignment systems. The 2003 change in NYC took place in an environment where participants already had some familiarity with choice since both the uncoordinated and coordinated system had a common application. In other cities, the school choice market is even less well organized, without readily available information on admissions processes and application timelines. For instance, admissions in Boston’s growing charter sector are uncoordinated, and the schools have only recently adopted a standardized application timeline. Recently, there have been calls to unify enrollment across school sectors (Vaznis 2013). The relative value of policies such as common timelines, common applications, single vs. multiple offers, sophisticated matching algorithms, and good information and decision aides is an interesting avenue for future research.

Finally, it’s worth emphasizing that our analysis has focused on the allocative aspects of school choice and different school assignment procedures. An important question is whether allocative changes contribute to changes in the productive dimensions of assignment. That is, do different student assignment protocols affect levels of student achievement post-assignment? This far more difficult question requires understanding the link between choices and the education production function, but is left for future work.
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Table 4. Ranked vs. Assigned Schools by Student Assignment Round

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<td>Assigned Schools (2)</td>
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<tr>
<td>Distance (in miles)</td>
<td>4.82</td>
<td>4.30</td>
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<tr>
<td>High Math Achievement</td>
<td>12.4</td>
<td>11.7</td>
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<tr>
<td>High English Achievement</td>
<td>20.9</td>
<td>20.2</td>
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<tr>
<td>Percent Attending Four Year College</td>
<td>49.1</td>
<td>47.1</td>
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<tr>
<td>Fraction Inexperienced Teachers</td>
<td>45.3</td>
<td>45.6</td>
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<tr>
<td>Attendance Rate (out of 100)</td>
<td>85.1</td>
<td>84.6</td>
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<tr>
<td>Percent Subsidized Lunch</td>
<td>60.0</td>
<td>60.5</td>
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<tr>
<td>Size of 9th grade</td>
<td>694.3</td>
<td>698.8</td>
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<tr>
<td>Percent White</td>
<td>15.1</td>
<td>14.7</td>
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<td><strong>B. Supplementary Round</strong></td>
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<tr>
<td>Distance (in miles)</td>
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<td>4.59</td>
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<tr>
<td>High Math Achievement</td>
<td>11.8</td>
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<td>High English Achievement</td>
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<td>Percent Attending Four Year College</td>
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<td>Size of 9th grade</td>
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<td>Percent White</td>
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Notes: Means unless otherwise noted. Analysis restricts the sample to students from the welfare sample with observed assignments. Uncoordinated mechanism refers to the 2002 03 mechanism and coordinated mechanism refers to the 2003 04 mechanism based on deferred acceptance. Main round in the uncoordinated mechanism corresponds to the first round. Rankings used are those submitted in the main round of the process. Student distance calculated as road distance using ArcGIS. See Table 2 notes for details on school characteristics.
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**Notes:**
- Columns 2 and 4 represent the theoretical values for the given conditions.
- Columns 3 and 5 represent the experimental values for the given conditions.
- The table shows a comparison between theoretical and experimental values for various conditions and their corresponding outputs.
Number Assigned in New Mechanism
- 1-83
- 84-107
- 108-169
- 169-523
- 529-1432

Change in Number Assigned

Oversubscription in Old Mechanism

$\%$ $!122$ $3!04!56$ $!2789$ $!# !$; $7! : 83# , 93! < \ ' 7+ , \ #!$ !

"%$&'$*($)!+, -$$# !. $% 0 &122$-$' 3!04!56$ $%02789$ $!# !$; $7! : 83# , 93! < \ ' 7+ , \ #!$ !

"%$&'$*($)!+, -$$# !. $% 0 &122$-$' 3!04!56$ $%02789$ $!# !$; $7! : 83# , 93! < \ ' 7+ , \ #!$ !

"%$&'$*($)!+, -$$# !. $% 0 &122$-$' 3!04!56$ $%02789$ $!# !$; $7! : 83# , 93! < \ ' 7+ , \ #!$ !

"%$&'$*($)!+, -$$# !. $% 0 &122$-$' 3!04!56$ $%02789$ $!# !$; $7! : 83# , 93! < \ ' 7+ , \ #!$ !
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A<BC% "$CE(1)^*, 1E")$:

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| 4      | 7      | >776     | >779   | >779  | >779 | >779     | >779  | >779  | >779 |
| 6      | 7      | >778     | >777   | >777  | >777 | >777     | >777  | >777  | >777 |

D(1) $(+$P@) (%$ $C

| 4      | 6      | >A@      | >78>   | >78>  | >78> | >78>     | >78>  | >78>  | >78> |
| 4      | 7      | >A@      | >78>   | >78>  | >78> | >78>     | >78>  | >78>  | >78> |
| 6      | 7      | >A@      | >78>   | >78>  | >78> | >78>     | >78>  | >78>  | >78> |

D(1) $(+$P@ C#(1

| 4      | 6      | >A@      | >78>   | >78>  | >78> | >78>     | >78>  | >78>  | >78> |
| 4      | 7      | >A@      | >78>   | >78>  | >78> | >78>     | >78>  | >78>  | >78> |
| 6      | 7      | >A@      | >78>   | >78>  | >78> | >78>     | >78>  | >78>  | >78> |

&@ (%"@C#T)/ (1

| 4      | 6      | >A@      | >78>   | >78>  | >78> | >78>     | >78>  | >78>  | >78> |
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