Targeted Government Intervention in the Housing Market

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Abstract

This paper estimates the effect of GSE elimination on the housing market. I solve a model for housing demand with borrowing constraints. GSE activity is modeled as an option for households to obtain low-interest mortgages that are smaller than the conforming limit. Households that benefit most from GSEs have either low life-time incomes or high wealth-to-income ratios. Elimination of GSE's leads to a decrease in homeownership rate from 69% to 48% and a decrease in average homeowner leverage from 35% to 15%.

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1 Introduction

Housing is the most important asset for U.S. households, and homeownership is a major focus for the government. One particular example of government housing policy is Government-Sponsored Enterprises (GSE’s). By effectively insuring banks against defaults on mortgages, GSE’s allow banks to lend at lower rates to households. The recent sub-prime mortgage crisis, however, has shown that such a policy comes at a significant cost. Unexpectedly large number of defaults on GSE-insured mortgages has essentially bankrupted two of the most well known GSE’s, Fannie Mae and Freddie Mac. Since 2008, they have been put under government conservatorship and have relied on taxpayer money to stay operational. This episode has called into question whether the benefits of GSE’s are worth the costs. To help answer that question, this paper quantifies the distributional and overall effects of GSE’s on ownership rate, house price, housing demand, and mortgage demand.

Banks can sell to GSE’s mortgages that conform to certain requirements. The implicit government guarantees on GSE-issued securities, which after 2008 became explicit, allow GSE’s to obtain capital cheaply either through debt or mortgage-backed securities. Since GSE’s are willing to buy large amounts of conforming mortgages at high prices, banks are also willing to issue large amounts of conforming mortgages at low interest rates. There are many requirements for a loan to be conforming, but my model considers only the main one, which is that the loan size must be below a conforming limit. I model the presence of GSE’s as a constant spread in interest rates between conforming and jumbo (non-conforming) loans. Households that choose a mortgage larger than the conforming limit must pay the higher jumbo rate.

To capture home purchase decisions of households parsimoniously, I solve a deterministic model for housing demand. The cost of buying vs. renting a house depends on the amount of mortgage used in the buying case. Heterogeneity in income and wealth leads to different borrowing needs, which leads to different home purchase decisions. In particular, for households with low wealth relative to income, the need to borrow heavily to buy a home makes renting the more attractive option. GSE’s have the potential to shift the decision in favor of buying by lowering the mortgage rate which lowers the cost of ownership.

The first contribution of this paper is providing a theoretical framework to analyze the effects of conforming/jumbo spread and conforming limit on household decisions across wealth-income groups. My model simultaneously determines homeownership, home size, and mortgage size at the individual household level. In addition, I show that the conforming loan limit makes income level, and not just wealth-to-income ratio, a factor in the home purchase decision. I find that the conforming limit does not apply to low income households because their optimal mortgage choices are below the limit anyways. For middle income households, some choose loan sizes exactly at the limit to obtain the lower rate. Finally for high income households, some actually choose the jumbo rate. The definitions of low, middle, and high income groups are determined by the conforming limit as well as the conforming/jumbo spread.

The second contribution of this paper is incorporating the assignment model of housing into the analysis of GSE’s. I show that GSE subsidy has differential effects
across housing quality markets. The mortgage subsidy to low income households encourages those households to buy higher quality homes. To the extent that housing supply at all quality levels cannot adjust immediately to demand, prices increase more for higher quality homes.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the setup of the model. Section 4 shows the theoretical results. Section 5 explains the data preparation and calibration. Section 6 presents the calibrated results, and Section 7 concludes.

\section{Related Literature}

The paper most similar to mine in terms of goal and purpose is Jeske, Krueger, and Mitman (2011), which also analyzes the distributional as well as overall effects of GSE’s on the mortgage market. They conclude that GSE subsidy provides more benefits to high income households. While they model the subsidy as a decrease in mortgage rates for all agents, my model incorporates the conforming limit. This additional element allows targeting of subsidy benefits to low income households.

My paper adds to the literature that studies the effect of mortgage rates on the housing market. Stroebel and Floetotto (2011) considers the effect of mortgage interest tax deduction in a general equilibrium overlapping generation model. They find that removing the deduction, which effectively increases the mortgage rate, has significant effects on homeownership and home prices. Sommer (2012) finds mortgage rates to have significant effects on house prices but not on homeownership rates. Ling (1998) and Follain (1997) use heterogeneity in mortgage interest deduction across different income groups to measure the effect of interest rate on mortgage demand. None of these papers consider the effect of a decrease in mortgage rates for loans below a conforming limit.

There is empirical evidence that the conforming loan limit affects household behavior. DeFusco (2014) estimates that there are 3.78 times more loans at the limit than otherwise. Kaufman (2013) also finds bunching of mortgages at the limit. Adelino (2012) finds that loan to value ratios of mortgages around the conforming limit to differ significantly from the norm. They also show that houses that can be financed by conforming loans are more expensive.

The major driver for homeownership and mortgage in my model is wealth-to-income ratio. A vast literature supports this claim that only borrowing-constrained households rent. On the theoretical side, Henderson (1983) shows that low wealth households rent because buying forces them to save when borrowing is preferred. Jones (1993) and Brueckner (1994) also show that mortgage demand is driven by the lack of current period wealth. In later literature that considers housing as part of the portfolio choice, such as Yao and Zhang (2005), wealth-to-income ratio continues to be the main determinant of homeownership. On the empirical side, Zorn (1993) has shown that 46% of all households are wealth constrained and Barakova (2003) identifies wealth as the most important constraint over income and credit.

The assignment model of housing has been used in Maattanen and Tervio (2014)
to study how income distribution affects house price distribution. Landvoigt, Piazzesi, and Schneider (2013) provides empirical evidence using San Diego housing data to show how cheaper credit for a subpopulation affects house price distribution. My paper incorporates this idea of housing as an indivisible good to estimate how the price impact of GSE’s differs across house qualities.

3 Model Setup

Each household in this model lives until the age of 85, but as shown in the section that follows, the household problem reduces down to one with only two periods. Households differ in income and wealth. As a result of consumption smoothing, low wealth households want to borrow while high wealth households want to save.

3.1 Households

Each household in the economy starts in period \( t = 0 \) with wealth \( W \) at age \( a \) and planning horizon \( T \). I focus my analysis on factors that drive the homeownership decision in period 0 and therefore simplify modeling of periods \( t > 0 \). At the beginning of period 1, the household receives an income \( Y \), which represents the present value of lifetime income of a real household. The household can either buy or rent a house in period 0, but can only rent in periods \( t > 0 \).

For \( t = 0 \), the household chooses the consumption of numeraire good \( C \), housing services \( H \), and bonds \( B \) at the gross interest rate \( R \). I impose the non-negative constraint that bond holdings cannot be negative (\( B \geq 0 \)). For any amount of housing services \( H \), the household can obtain it by renting at rate \( k \) or buying it at price \( p \).

Let \( X \) be the rent/buy decision where \( X = 1 \) represents buying and \( X = 0 \) represents renting. If the household chooses to buy, it must also pay for depreciation that is a fraction \( \delta \) of the price and expected moving cost that is a fraction \( \psi \) of the price. Buying gives households the option to borrow in the form of a mortgage \( M \) at rate \( R_M \), which may be a function of \( M \). I impose the constraint \( M \geq 0 \). The budget constraint for period 0 is:

\[
W + XM = C + B + H((1 - X)k + Xp(1 + \delta + \psi))
\] (1)

The starting wealth for period 1 is:

\[
W_1 = Y + BR + X(Hp - MR_M)
\] (2)

For \( t > 0 \), the household makes consumption and investment decisions with the following budget constraint

\[
W_t = C_t + B_t + H_t k
\] (3)

The evolution of wealth is then

\[
W_{t+1} = B_t R
\] (4)
3.2 Banks

The household’s utility over numeraire good and housing services is assumed to be Cobb-Douglas with housing share $\alpha$, discount rate $\beta$, and inter-temporal elasticity of substitution $\sigma$:

$$U = \sum_{t=0}^{T} \beta^t \frac{(C_t^{1-\alpha}H_t^\alpha)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$  \hspace{1cm} (5)

3.2 Banks

I model the banks similar to Jeske, Krueger, and Mitman (2011). Banks issue mortgages that involve a certain cost of capital to finance. The cost of capital is the sum of two components. First component is the time value of money, which is equal to the bond rate $R$. Second component is a risk premium $s$ for undiversifiable market risk. Assuming the banks are competitive, the mortgage rate is $R_M = R + s$ in the absence of GSE’s.

3.3 GSE’s

GSE’s pay banks an interest spread $d$ for all mortgages less than the conforming limit $\bar{M}$. For example, if a bank issues a mortgage of size $M$, the GSE’s pay the bank $dM$. In reality the GSE’s buy the mortgages at a price higher than what the market would have demanded by the amount $dM$. We also assume for simplicity that mortgages larger than $\bar{M}$ cannot be broken down into multiple mortgages to satisfy the conforming limit requirement. The result is that households will face mortgage rate $R + s - d$ if $M \leq \bar{M}$ and $R + s$ if $M > \bar{M}$.

4 Theoretical Results

In this section I show the driving forces that determine homeownership and mortgage decisions. I first consider an economy without GSE’s to show that only wealth-to-income ratio matters. Next I show how the results change with the addition of GSE’s. The conforming loan limit makes both income and wealth-to-income ratio relevant in household decisions.

4.1 Base Case

The value function for any period after $t = T$ is 0. For any period $1 \leq t \leq T$, the value function of the household can be written recursively as

$$V_t = \max_{C_t,H_t} \left\{ \frac{(C_t^{1-\alpha}H_t^\alpha)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \beta V_{t+1}(W_{t+1}) \right\} \hspace{1cm} (6)$$

s.t. \hspace{1cm} (3) and (4)
Using backwards induction, the value function in period 1 is

\[ \beta V_1 = A \frac{\zeta k^{\alpha(1-\frac{1}{\sigma})}}{1 - \frac{1}{\sigma}} \frac{W_1^{\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \]  \hspace{1cm} (7)

\[ \zeta = \beta \left( \frac{1 - (\beta^\sigma R^{\sigma-1})^{T-a}}{1 - \beta^\sigma R^{\sigma-1}} \right)^{\frac{1}{\sigma}}, \quad A = (\alpha^\alpha(1 - \alpha)^{1-\alpha})^{1-\frac{1}{\sigma}} \]

See Appendix A.1 for proof.

For \( t = 0 \), I drop the subscripts on variables in that period. The value function is similar to (6) except with additional choice variables \( X \) and \( M \) because the household has the option to buy instead of rent.

\[
V = \max_{C,H,B,M,X} \left\{ \frac{(C^{1-a} H^a)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \beta V_1 \right\}
\]

\[
= \max_{C,H,B,M,X} \left\{ \frac{(C^{1-a} H^a)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \frac{\zeta}{k^{\alpha(1-\frac{1}{\sigma})}} \frac{(W_1^{1-\frac{1}{\sigma}})}{1 - \frac{1}{\sigma}} \right\} \hspace{1cm} (8)
\]

s.t.

(1) and (2)

In what follows I will show how the wealth-to-income ratio, \( w = \frac{W}{Y} \), determines the homeownership and mortgage decisions. Proposition 1 considers the case of renting. Proposition 2 considers the case buying. Proposition 3 compares the two cases.

**Proposition 1.** Consider the optimization in problem (8) with \( X = 0 \). Define the solution as constrained if \( B = 0 \) and unconstrained if \( B > 0 \). Then:

1. The solution is unconstrained if and only if \( w > \frac{1}{\zeta^\sigma R^\sigma} \).

2. Value function for the unconstrained solution is

\[ V_{X=0} = \frac{A}{1 - \frac{1}{\sigma}} (W + Y R^{-1})^{1-\frac{1}{\sigma}} k^{-\alpha(1-\frac{1}{\sigma})} (1 + \zeta^\sigma R^{\sigma-1})^{\frac{1}{\sigma}} \]

See Appendix A.2 for proof.

The household wants to smooth consumption between periods 0 and 1. When wealth in period 0 is small enough relative to income in period 1, the household wants to borrow in period 0. However, the non-negative constraint on \( B \) prevents this from happening, leading to a constrained solution. The two state variables of wealth and income actually reduces down to one state variable, which is the present value of the sum \( W + Y R^{-1} \). I focus my analysis on the unconstrained solution because that will be the one that competes with the buying option.

**Proposition 2.** Consider the optimization in problem (8) with \( X = 1 \). Define the solution as constrained if \( B = M = 0 \) and unconstrained otherwise. Then there exists a decreasing function \( \varpi(r) \) such that:
4.1 Base Case

1. The solution is

\[
\begin{cases}
\text{unconstrained with } B > 0, M = 0 & \text{if } w > \overline{w}(R) \\
\text{unconstrained with } B = 0, M > 0 & \text{if } w < \overline{w}(R + s) \\
\text{constrained} & \text{otherwise}
\end{cases}
\]

2. Value functions for the unconstrained solutions are

\[
V_{X=1} = \frac{A}{1 - \frac{1}{\sigma}} (W + Yr^{-1})^{1 - \frac{1}{\sigma}} \left( \rho(r)^{-\alpha \sigma(1 - \frac{1}{\sigma})} + \zeta r^{\sigma - 1} k^{-\alpha \sigma(1 - \frac{1}{\sigma})} \right)^\frac{1}{\sigma}
\]

where

\[
\rho(r) = p(1 + \delta + \psi - \frac{1}{r})
\]

\[
r = \begin{cases} R & \text{if } w > \overline{w}(R) \\
R + s & \text{if } w < \overline{w}(R + s) \end{cases}
\]

See Appendix A.3 for proof.

Households can be separated into three types based on \( w \). It is important here to point out that the desire to save increases with interest rate \( r \) and \( w \). A household with high \( w \) wants to save so only the interest rate \( R \) is relevant. A household with low \( w \) wants to borrow so only the mortgage rate \( R + s \) is relevant. There is a middle region of \( w \) values for which households want to borrow if the rate is \( R \) and save if the rate is \( R + s \). Being bounded by the non-negative constraints, these households will do neither.

I now compare the the solutions for \( X = 0 \) and \( X = 1 \). There are two factors driving utility. First is the total present value of wealth, and second is the aggregate price of consumption. Utility is higher when wealth is higher or price is lower. In the case of \( w > \overline{w}(R) \), renting and buying leads to the same wealth, but buying gives a lower usage cost \( \rho(R) \) than rent \( k \). In this region, buying is optimal. In the case of \( w < \overline{w}(R + s) \), wealth is higher for renting because the interest rate is lower. However, the higher interest rate also makes the price of future consumption lower. For low values of \( w \), the wealth component dominates while for high values of \( w \), the price component dominates.

**Proposition 3.** Consider the optimization in problem (8). There exists an increasing function \( \overline{w}(r) \) such that:

1. Households rent if and only if \( w < \overline{w}(R + s) \)
2. \( \frac{1}{\zeta \overline{w}(R)} < \overline{w}(R + s) < \overline{w}(R + s) \) for certain parameter restrictions

See Appendix A.4 for proof.

To summarize, households can be categorized into four types based on homeownership and mortgage behaviors. The only variable that determines the type is \( w \).

1. **Buyer B** (\( \overline{w}(R) < w \)): buys house with positive bond holdings
2. **Buyer N** (\( \overline{w}(R + s) < w < \overline{w}(R) \)): buys house with neither bond nor mortgage
3. **Buyer M** (\( w(R + s) < w < \overline{w}(R + s) \)): buys house with mortgage
4. **Renter** (\( w < \overline{w}(R + s) \)): rents
4.2 Subsidy Case

Consider for now a general decrease in the mortgage rate to $R + s - d$. Based on the results in Section 3.1, only Buyer B’s decisions are unchanged. Since $w(R + s) < w(R + s - d)$ and $w(R + s) > w(R + s - d)$, some Buyer N’s and some renters will become Buyer M’s. The lower mortgage rate not only encourages some renters to buy, it also encourages some current homeowners with no mortgages to take out mortgages.

I now add in the constraint $M \leq \overline{M}$ for loans to qualify for the lower rate. Only the Buyer M’s with optimal mortgage choices larger than $\overline{M}$ are affected. These households have three options: take a loan of $\overline{M}$ at rate $R + s - d$, take any sized loan at rate $R + s$, or rent. The decision depends on how far the optimal $M$ under the subsidized rate is from $\overline{M}$.

The optimal $M$ is an increasing function of $Y$ and decreasing function of $w$. Within Buyer M’s, only those with high $Y$’s or low $w$’s are constrained by the conforming limit. Figure 1 shows the homeownership and mortgage decisions in the $Y - w$ space. For low income households, the conforming limit has no effect because mortgage demand is never high enough. For middle income households, the highest $w$ types are unaffected, but middle $w$ types choose $\overline{M}$ at the subsidized rate. For low $w$ types, choosing $M = \overline{M}$ actually constrains the household so much that renting is a better option. For high income households, there is an additional group that prefers to take out an unsubsidized mortgage over a subsidized one of size $\overline{M}$.

5 Empirical Application

The model uses heterogeneity in wealth and income to create differences in homeownership and mortgage decisions. To apply the model empirically, I first identify the relevant data that corresponds to both the inputs and outputs of the model. Then I calibrate some parameters to certain moments in the data, namely aggregate homeownership rate, leverage, and home-to-income ratio. Other parameters I take from literature.

5.1 Data

The inputs to the model are wealth and income. Both are obtained from the 2007 Survey of Consumer Finances (SCF). SCF is a triennial survey of U.S. families from all economic strata. The 2007 survey contains 6,500 households. The survey was taken before the GSE’s were taken into conservatorship and before major restrictions were put on GSE activities.

I define wealth as net worth excluding income-generating assets (businesses, net equity in non-residential real estate, and other miscellaneous non-financial assets) and quasi-liquid retirement accounts (IRAs and pensions). In terms of SCF code,

\[ W = \text{NETWORTH} - \text{BUS} - \text{NNRESRE} - \text{OTHNFIN} - \text{RETQLIQ} \]
The difference between wealth and income in my model is that wealth is immediately accessible whereas income is only available in the future. I exclude the income-generating assets and retirement accounts because they are both promises of future income and cannot be liquidated without incurring significant costs. I also assume the household’s investment decisions on these assets are exogenous. The literature on life cycle consumption and portfolio choices endogenizes this decision, but I will not go into detail here.

The definition of $Y$ in my model is the total present value of all income during the life-span of the household. I assume the household lives to age 85, which is the average lifespan of Americans. Using the normal income, age, and education level from the SCF, I project the income path for each household in the sample using parameters from Cocco (2005). I then calculate the present value of the sum of this lifetime income using the interest rate $R$. Value of the retirement account is then added to present value to match the definition of $Y$. I assume that income from businesses, non-residential real estate, and other non-financial assets are already included in the SCF normal income data.

5.2 Calibration

Some of the parameters in my model has been extensively studied and estimated in literature. I adopt these commonly used values listed in Table 1. The unit of housing is normalized such that the price of housing is 1. The subsidy of 0.4% is in line with empirical estimates by Ambrose, LaCour-Little, and Sanders (2004) and Passmore, Sherlund, and Burgess (2005). Note that the household’s planning horizon is 15 years longer than the lifespan to account for bequest motives.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Risk free rate</td>
<td>1.02</td>
</tr>
<tr>
<td>$s$</td>
<td>Housing premium</td>
<td>0.01</td>
</tr>
<tr>
<td>$d$</td>
<td>Subsidy</td>
<td>0.004</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal substitution</td>
<td>0.5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$p$</td>
<td>House price</td>
<td>1</td>
</tr>
<tr>
<td>$k$</td>
<td>Rent-price</td>
<td>0.06</td>
</tr>
<tr>
<td>$T$</td>
<td>Planning horizon</td>
<td>100-age</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>Conforming limit</td>
<td>$417,000$</td>
</tr>
</tbody>
</table>

The three parameters I calibrate to data are listed in Table 2. Each moment in the data is listed next to the parameter that has the most effect. The ownership rate is driven mainly by the moving cost since it changes the user cost of buying a house. The discount rate drives the household’s desire to borrow, which affects leverage.
Finally, the housing share parameter drives the size of the home purchase relative to the amount of lifetime income.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Matching Moment</th>
<th>Data Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>Moving cost</td>
<td>0.0075</td>
<td>Ownership rate</td>
<td>0.69</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.92</td>
<td>Leverage</td>
<td>0.35</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Housing share</td>
<td>0.12</td>
<td>House to income</td>
<td>0.13</td>
</tr>
</tbody>
</table>

### 6 Results

Using the parameters from the calibration exercise, I now use the model for two scenarios. The first scenario is one with GSE subsidy and conforming limit. This exercise quantifies the mechanisms described in Section 4. The second scenario is one without GSE subsidy. I compare the results from this scenario to the first to show how elimination of GSE’s affect the housing market.

#### 6.1 Quantifying the Mechanism

In the presence of GSE subsidy, homeownership decision is described in Section 4.2. Figure 1 is an example of households at age 50 with college education. Households with $w < 0.013$ rent, and households with $w > 0.17$ buy without mortgages. These boundaries would be 0.07 and 0.16 without the subsidy. For households with less than $3M$ of lifetime income, the conforming limit does not apply and they enjoy the full benefits of the subsidy.

Figure 1 shows that the situation is more complicated for households with lifetime income greater than $3M$. In the range of $3M$ to $10M$, certain low $w$ households become constrained by the conforming limit. They still choose the subsidized rate but are forced to choose $M = \bar{M}$. For even lower values of $w$ (those close to the lower boundary of 0.013), renting becomes optimal. These households want to borrow so much that a subsidized rate on $\bar{M}$ amount is insignificant. When income is higher than $10M$, there are more households for which the subsidy does nothing. In addition to renters, there are now M Buyers who want to borrow so much that $\bar{M}$ is insignificant. At the limit of infinite income, the household decision is identical to the no subsidy case.

I now look at how the subsidy affects house size and leverage decisions. Figure 2 shows how $Hp/Y$ changes with $w$. The ”fully subsidized” line represents households with $Y < 3M$. The conforming limit does not apply to these households. Comparing this line with the unsubsidized line shows the intuitive result that lowering interest rate increases housing demand. Cheaper mortgages encourage households to buy bigger homes. However, this is not always the case. The ”Low Y” line represents
6.2 Elimination of GSE Subsidy

Figure 1: Homeownership Decision

households with income between $3M to $10M and the "High Y" line represents households with income greater than $10M. As $w$ decreases, the household must borrow more and buy a smaller home. When the mortgage demand is constrained, the borrowing channel is no longer available. Since all adjustments must come from buying a smaller home, the home size decreases much faster in this region. For certain values of $w$, the subsidy actually leads to a smaller home purchase.

Figure 3 shows the leverage decision. It is analogous to the house size decision. In general the subsidy encourages higher leverage. However, certain constrained households may choose lower leverage as a result of the subsidy. These are the same households that choose smaller homes. Since mortgage size remains constrained at the conforming limit, leverage decreases.

6.2 Elimination of GSE Subsidy

To estimate the overall effects of GSE’s, I apply the results of Section 6.1 to the population density in the $Y - w$ space calculated from SCF. I then compare the case of having a subsidy with a conforming limit to the case of no subsidy under the assumption that supply, rather than price, adjusts to changes in housing demand. Figure 4 shows how the population is affected by the subsidy elimination. About 40% of the population takes out mortgages so those are the only affected households. Out of that 40%, 21% are low $w$ types who will choose to rent while 1% are high $w$ types who will buy without a mortgage. The remaining will continue take out a mortgage at the unsubsidized rate. Overall home ownership decreases from 69% to
Figure 2: House Size Choice

![House Size Choice Graph]

Figure 3: Leverage Choice

![Leverage Choice Graph]

48%, and leverage decreases from 35% to 15%.

I define the total subsidy amount as the sum of all conforming mortgages times
6.2 Elimination of GSE Subsidy

Figure 4: Changes in homeownership decision

![Homeownership Decision Chart]

<table>
<thead>
<tr>
<th>Decision</th>
<th>With Subsidy</th>
<th>Without Subsidy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy with bonds</td>
<td>27%</td>
<td>0%</td>
</tr>
<tr>
<td>Buy with unsubsidized</td>
<td>0.3%</td>
<td>18%</td>
</tr>
<tr>
<td>Buy with subsidized</td>
<td>40%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Rent</td>
<td>31%</td>
<td>21%</td>
</tr>
<tr>
<td>Buy with neither</td>
<td>1.6%</td>
<td>0.9%</td>
</tr>
</tbody>
</table>

the subsidy $d$. Figure 5 shows how this total amount is divided amongst different types of households. About 2/3 of the subsidy goes to households that would have otherwise chosen to rent. The remaining 1/3 of the subsidy goes to households that would have chosen to buy anyways. In general this second group buys bigger homes as a result of the subsidy.

There are no particular age groups that receive a disproportionate amount of subsidy. The subsidy distribution by income group is more interesting. The right-side bar of Figure 5 shows how much of the subsidy is received by each of the ten income decile groups. In this categorization, households are separated based on current, and not lifetime, income. I choose this categorization because one of the major goals of GSE’s is to help low-income households buy homes, and their definition is based on current income. The top 20% receives more than 40% of the subsidy, and the top 30% receives more than 60% of the subsidy. This result suggests that while low-income households have been helped, high-income households has benefited more.

In this model I assume that houses are indivisible goods so it is possible to compare house size distributions in the two cases. I first divide all houses, for both renters and owners, in the subsidy case into deciles by size. This creates an upper and lower bound for house size in each decile. I then use those same bounds to sort houses in the no subsidy case into 10 bins. The results are shown in Figure 6. By construction, the distribution is 10% for all bins in the subsidy case. Without the subsidy, demand for larger homes falls while demand for smaller homes rises. The shift in home size distribution leads to a 1% decrease in average home size and a 1% decrease in overall housing stock.
Figure 5: Distribution of Subsidy

Figure 6: Changes in House Size

7 Conclusion

I solve a housing consumption model to demonstrate the distributional effects of GSE subsidy on the housing market. Not all households choose the subsidized rate since
doing so restricts mortgage size to the conforming limit. I show that households with low mortgage demand are those with low incomes and high wealth-to-income ratios, and that they benefit most from the subsidy because they are not restricted by the conforming limit. On the other hand, high income or low wealth-to-income households are not affected by the subsidy. Finally there is a third group in the middle that choose mortgage sizes exactly equal to the conforming limit but would like to borrow more.

The elimination of GSE’s leads to significant decreases in homeownership and leverage without much changes in overall housing stock and price. Low income and high wealth-to-income households stand to lose the most. Half of them become renters and the other half continue to buy with a mortgage at the higher unsubsidized rate. Prices of higher quality homes will drop more than those of lower quality homes in the short run.

References


REFERENCES


Appendices

A Analytical Solutions to the Household Problem

A.1 Solution for $1 \leq t \leq T$

I prove by induction that the general solution to (6) is

$$\beta V_t = A \frac{\zeta_{t-1}}{k^{\alpha(1 - \frac{1}{\sigma})}} \frac{W_t^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$

(9)

$$\zeta_t = \beta \left( \frac{1 - (\beta^{\sigma} R^{\sigma-1})^{T-a-t}}{1 - \beta^{\sigma} R^{\sigma-1}} \right)^{\frac{1}{\sigma}}$$

where $A = \left( \frac{\alpha}{1 - \alpha} \right)^{1 - \frac{1}{\sigma}}$.

Start at $t = T$. $B_T = 0$ is optimal since bonds provide no future utility. Utility maximization given budget constraint (3) gives the following optimal allocation from first order conditions.

$$C_T = (1 - \alpha)W_T$$

$$H_T = \frac{\alpha W_T}{k}$$

The last period value function is then

$$V_T = \frac{(\alpha^\alpha (1 - \alpha)^{1 - \alpha})^{1 - \frac{1}{\sigma}} W_T^{1 - \frac{1}{\sigma}}}{k^{\alpha(1 - \frac{1}{\sigma})} (1 - \frac{1}{\sigma})}$$

which satisfies the functional form in (9).

For $t < T$, I substitute (3), (4), and (9) into (6) so that the only choice variables are $C_t$ and $H_t$.

$$V_t = \max_{C_t, H_t} \left\{ \frac{(C_t^{1 - \alpha} H_t^{\alpha})^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + A \frac{\zeta_t}{k^{\alpha(1 - \frac{1}{\sigma})}} \frac{(R(W_t - C_t - H_t k))^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \right\}$$

First order conditions then give the allocation:

$$C_t = \frac{(1 - \alpha)W_t}{(1 + \zeta_{t-1} R^{\sigma-1})}$$

$$H_t = \frac{\alpha W_t}{k(1 + \zeta_{t-1} R^{\sigma-1})}$$

The value function is then

$$\beta V_t = \beta A \frac{(1 + \zeta_t R^{\sigma-1})^{\frac{1}{\sigma}}}{k^{\alpha(1 - \frac{1}{\sigma})}} \frac{W_t^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$

$$= \frac{A \zeta_{t-1}}{k^{\alpha(1 - \frac{1}{\sigma})}} \frac{W_t^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$
A.2 Solution for $X = 0$

Assuming interior solution, substitute $X = 0$, (1), and (2) into (8) and take first order conditions on $C$, $H$, and $B$ to obtain the optimal allocation

$$C = \frac{(1 - \alpha)(W + YR^{-1})}{1 + \zeta^\sigma R^{\sigma -1}}$$

$$H = \frac{\alpha(W + YR^{-1})}{k(1 + \zeta^\sigma R^{\sigma -1})}$$

$$B = \frac{W\zeta^\sigma R^{\sigma -1} - YR^{-1}}{1 + \zeta^\sigma R^{\sigma -1}}$$

The value function for renting is then

$$V_{X=0} = \frac{A}{1 - \frac{1}{\sigma}}(W + YR^{-1})^{1 - \frac{1}{\sigma}}k^{-\alpha(1 - \frac{1}{\sigma})}(1 + \zeta^\sigma R^{\sigma -1})^{\frac{1}{2}}$$

Note that for certain values of $W$ and $Y$, the condition $B \geq 0$ is violated, in which case the interior solution is no longer valid. The necessary and sufficient condition for which the interior solution applies ($B \geq 0$) is

$$\frac{W}{Y} \geq \frac{1}{\zeta^\sigma R^{\sigma}}$$

A.3 Solution for $X = 1$

Note that any allocation with $B > 0$ and $M > 0$ is not optimal because the household can decrease both by the same marginal amount to earn the mortgage spread. Problem (8) breaks down into two problems: one with restriction $B = 0$ and another with restriction $M = 0$. Both problems, however, can be expressed in a more general form after substituting $X = 1$, (1), and (2) into (8).

$$V = \max_{C,H} \left\{ \frac{(C^{1-\alpha}H^\alpha)^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + A \frac{\zeta}{k^\alpha(1 - \frac{1}{\sigma})(Y + r(W - C - H\rho(r)))^{1 - \frac{1}{\sigma}}} \right\}$$

$$\rho(r) = 1 + \delta + \psi - \frac{1}{r}$$

$$r = \begin{cases} R & \text{if constraint is } M = 0 \\ R + s & \text{if constraint is } B = 0 \end{cases}$$

I assume interior solution. First order conditions yield the optimal allocation

$$C = \frac{(1 - \alpha)(W + Yr^{-1})}{1 + \zeta^\sigma R^{\sigma -1}(\frac{k}{\rho(r)})^{\alpha(1 - \sigma)}}$$

$$H = \frac{\alpha}{\rho(r)} \frac{W + Yr^{-1}}{1 + \zeta^\sigma R^{\sigma -1}(\frac{k}{\rho(r)})^{\alpha(1 - \sigma)}}$$
The value function is
\[ V_{X=1} = \frac{A}{1-\frac{1}{\sigma}} (W + Y r^{-1})^{1-\frac{1}{\sigma}} \left( \rho(r)^{-\alpha \sigma (1-\frac{1}{\sigma})} + \zeta \sigma r^{-\alpha \sigma - 1} k^{-\alpha \sigma (1-\frac{1}{\sigma})} \right)^{\frac{1}{\sigma}} \]

Now I consider conditions under which the interior solutions apply. For the case with constraint \( M = 0 \), the interior solution applies if \( B = W - C - Hp(1 + \delta + \psi) \geq 0 \) which after substituting for the optimal values of \( C \) and \( H \) yields the condition
\[ w \geq \frac{\alpha \frac{p}{\rho(R)} R^{-1} + 1}{\zeta \sigma r^\sigma \left( \frac{k}{\rho(R)} \right)^{\alpha (1-\sigma)} - \alpha \frac{p}{\rho(R)}} \]
Similarly for the case with constraint \( B = 0 \), the interior solution applies if \( M = -(W - C - Hp(1 + \delta + \psi)) \geq 0 \) which is true if and only if
\[ w \leq \frac{\alpha \frac{p}{\rho(R+s)} (R + s)^{-1} + 1}{\zeta \sigma r^\sigma \left( \frac{k}{\rho(R+s)} \right)^{\alpha (1-\sigma)} - \alpha \frac{p}{\rho(R+s)}} \]
The two cutoff values of \( w \) are simply the values of the function
\[ \bar{w}(r) = \frac{\alpha \frac{p}{\rho(r)} r^{-1} + 1}{\zeta \sigma r^\sigma \left( \frac{k}{\rho(r)} \right)^{\alpha (1-\sigma)} - \alpha \frac{p}{\rho(r)}} \] (10)
evaluated at the specific points \( r = R \) for the constraint \( M = 0 \) and \( r = R + s \) for the constraint \( B = 0 \). If \( \bar{w}(r) \) is a decreasing function, then there are three types of solutions. The above analysis has already demonstrated the two regions of \( w \) where the unconstrained solutions apply. The constrained solution applies when \( \bar{w}(R + s) < w < \bar{w}(R) \).

I now prove that \( \bar{w}(r) \) is indeed a decreasing function by showing that for all values of \( r \),
\[ \frac{d\bar{w}}{dr} < 0 \]
The above condition after much algebra and rearrangement simplifies to
\[ \alpha (\alpha - 1) - \zeta \sigma \left( \frac{k}{p} \right)^{\alpha (1-\sigma)} R^{(1-\sigma)(\alpha - 1)} ((1 + \delta + \psi) R - 1)^{1-\alpha(1-\sigma)} \]
\[ \left( ((1 + \delta + \psi) R - 1) \sigma + 2 \alpha \sigma + \alpha \frac{1 - \alpha (1 - \sigma)}{(1 + \delta + \psi) R - 1} \right) < 0 \]
Taking first order condition with respect to \((1 + \delta + \psi) R - 1\) on the term
\[ ((1 + \delta + \psi) R - 1) \sigma + 2 \alpha \sigma + \alpha \frac{1 - \alpha (1 - \sigma)}{(1 + \delta + \psi) R - 1} \]
shows that this term has a minimum of
\[ 2 \alpha \sigma + 2 \sqrt{\alpha \sigma - \alpha^2 \sigma + \alpha^2 \sigma^2} > 0 \]
A.4 Comparing $X = 0$ to $X = 1$

In the case of $w > w(R)$, $V_{X=1}$ is evaluated at $r = R$.

$$
\frac{V_{X=1}}{V_{X=0}} = \left( \frac{\left( \frac{\rho(R)}{k} \right)^{-\alpha \sigma (1 - \frac{1}{\sigma})} + \zeta \sigma R^{\sigma - 1}}{1 + \zeta \sigma R^{\sigma - 1}} \right)^{\frac{1}{\sigma}}
$$

$\rho(R) < k$ implies the ratio is less than 1 so $V_{X=1} > V_{X=0}$. Note that the value functions are negative numbers.

In the case of $w < w(R + s)$, $r = R + s$.

$$
\frac{V_{X=1}}{V_{X=0}} = \left( 1 - \frac{s}{(R + s)(Rw + 1)} \right)^{1 - \frac{1}{\sigma}} \left( \frac{\left( \frac{\rho(R+s)}{k} \right)^{-\alpha \sigma (1 - \frac{1}{\sigma})} + \zeta \sigma (R + s)^{\sigma - 1}}{1 + \zeta \sigma R^{\sigma - 1}} \right)^{\frac{1}{\sigma}}
$$

The ratio is decreasing in $w$ and has a value of 1 when

$$
w = w(R + s)
$$

for some function $w$. This implies that $V_{X=1} > V_{X=0}$ when $w > w(R + s)$ and $V_{X=1} < V_{X=0}$ when $w < w(R + s)$.

I now prove that $w(r)$ is an increasing function. Since $V_{X=1}$ is a decreasing function of $r$ when $w < w(R + s)$, the ratio $\frac{V_{X=1}}{V_{X=0}}$ must be an increasing function of $r$. Under the condition that the ratio is 1, an increase in $r$ implies an increase in $w$.

The necessary and sufficient conditions for $\frac{1}{\zeta \sigma R^\sigma} < w(R + s) < \bar{w}(R + s)$ is

$$
\frac{V_{X=1}}{V_{X=0}}(w = \frac{1}{\zeta \sigma R^\sigma}) > 1 > \frac{V_{X=1}}{V_{X=0}}(w = \bar{w}(R + s))
$$

which becomes

$$
\left( \frac{\rho(R + s)}{k} \right)^{\alpha (1 - \sigma)} \in \left( (1 + \zeta \sigma R^{\sigma - 1})(1 - \frac{s}{(R + s)(\frac{R}{\zeta \sigma R^\sigma} + 1)})^{1 - \sigma} - \zeta \sigma (R + s)^{\sigma - 1}, \right.
$$

$$
(1 + \zeta \sigma R^{\sigma - 1})(1 - \frac{s}{(R + s)(\frac{R}{\zeta \sigma R^\sigma} + 1)})^{1 - \sigma} - \zeta \sigma (R + s)^{\sigma - 1})
$$