

Voter Turnout and Preference Aggregation*

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Abstract

We study how voter turnout affects the aggregation of preferences in elections. Under voluntary voting, election outcomes disproportionately aggregate the preferences of voters with low voting cost and high preference intensity. We show identification of the correlation structure among preferences, costs, and perceptions of voting efficacy, and explore how the correlation affects preference aggregation. Using 2004 U.S. presidential election data, we find that young, low-income, less-educated, and minority voters are underrepresented. All of these groups tend to prefer Democrats, except for the less-educated. Democrats would have won the majority of the electoral votes if all eligible voters had turned out.

keyword: *voter turnout, preference aggregation, election, gerrymandering*

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1 Introduction

Democracies rely on elections to aggregate the preferences of their citizens. Elections, however, aggregate the preferences of only those that participate. The importance of participation for preference aggregation is documented by studies of suffrage expansion in various contexts, such as the abolition of apartheid in South Africa (Kroth et al., 2013), the passage of the Voting Rights Act of 1965 (Husted and Kenny, 1997; Cascio and Washington, 2013), and the passage of women’s suffrage laws (Miller, 2008). Less dramatic measures that have reduced the voting costs of certain groups of voters have also been found to affect policy in important ways (Fujiwara, 2015).

While most democracies now enjoy universal suffrage, participation in elections is far from perfect, given the voluntary nature of voting. To the extent that the preferences of those that turn out are systematically different from those that do not, election outcomes may poorly aggregate the preferences of all citizens. Thus, how well elections aggregate the preferences of citizens and whose preferences are underrepresented are open questions, even in mature democracies.

The issues of preference aggregation and underrepresentation are also relevant from a policy perspective. The concern that the preferences of certain groups of voters are underrepresented has led some to argue for compulsory voting (see, e.g., Lijphart, 1997). More moderate policy proposals, such as introducing Internet voting, relaxing registration requirements, and making election day a holiday, are motivated by similar concerns. Understanding how voter turnout affects preference aggregation can provide a basis for more informed discussions of these policy proposals.

Preference aggregation is also central to the policy debate on partisan redistricting, or gerrymandering. Gerrymandering can be considered as an intentional attempt by one party to aggregate preferences disproportionately in its favor through redistricting. Much of the recent discussion on gerrymandering focuses on how well actual votes are translated into seat shares, but ignores how preferences map to seat shares.¹ Given that turnout is endogenous, it is possible for sophisticated planners to design redistricting plans that map vote shares to seat shares well, but map preferences to seat shares poorly.² Studying how underlying preferences, rather than votes, are aggregated into

¹See Best et al. (2018) for a discussion of several standards that have recently been proposed as tests of gerrymandering.

²In Table 5, we illustrate this point using a numerical example.

election outcomes provides a coherent alternative even when turnout is endogenous.

In this study, we explore the extent to which preferences are aggregated in elections, which hinges on how the preferences, voting costs, and perceptions of voting efficacy are correlated. We show identification of the joint distribution of these three terms, and estimate it using county-level voting data from the 2004 U.S. presidential election. We find that young, low-income, less-educated, and minority voters have a high cost of voting and that all of these groups tend to prefer the Democrats, except for the less-educated. We then simulate the counterfactual election outcome when we set voting costs to zero. The difference between the simulated and actual outcomes allows us to quantify the degree to which preferences are aggregated. In our counterfactual, the two-party vote share of the Democrats increases by about 3.7%, and the Democrats win the plurality of the electoral votes. In our second counterfactual, we compute the election outcome when we eliminate endogeneity in turnout. This allows us to gauge the sensitivity of the efficiency gap, an influential measure of gerrymandering, to endogeneity of turnout.

The key challenge in studying the effect of turnout on preference aggregation is to identify the correlation between preferences and voting costs in the population. In particular, we need to identify how voter characteristics such as race and income simultaneously determine preferences and costs. However, this is not a straightforward task because high preference intensity and high voting cost may predict a similar voting pattern as low preference intensity and low voting cost.

To illustrate, consider a plurality rule election in which voters have private values and choose to vote for candidate A or candidate B or not to turn out. Applying a discrete choice framework to the voter's decision, let $u_A(\mathbf{x})$ and $u_B(\mathbf{x})$ denote the utility of voting for candidates A and B , respectively, and $c(\mathbf{x})$ denote the cost of voting (relative to not voting), where \mathbf{x} is a vector of voter characteristics. Then, the voter's mean utilities are as follows:

$$\begin{aligned} V_A(\mathbf{x}) &= u_A(\mathbf{x}) - c(\mathbf{x}), \\ V_B(\mathbf{x}) &= u_B(\mathbf{x}) - c(\mathbf{x}), \text{ and} \\ V_0(\mathbf{x}) &= 0, \end{aligned}$$

where V_0 represents the mean utility of not turning out. While one can identify $V_A(\mathbf{x}) = u_A(\mathbf{x}) - c(\mathbf{x})$ and $V_B(\mathbf{x}) = u_B(\mathbf{x}) - c(\mathbf{x})$ by using vote share and turnout data (see

Berry, 1994; Hotz and Miller, 1993), $u_A(\cdot)$, $u_B(\cdot)$, and $c(\cdot)$ are not separately identified without further restrictions. This is because making a voter care more about the election outcome (say, by adding an arbitrary function, $g(\mathbf{x})$, to both $u_A(\mathbf{x})$ and $u_B(\mathbf{x})$) is observationally equivalent to lowering the voting cost (by subtracting $g(\mathbf{x})$ from $c(\mathbf{x})$). Even if there are exogenous cost shifters \mathbf{z} (e.g., rainfall), they do not help separately identify $u_A(\cdot)$, $u_B(\cdot)$, and $c(\cdot)$.³ Thus, most existing studies impose ad-hoc exclusion restrictions on the way that \mathbf{x} enters $u_A(\cdot)$, $u_B(\cdot)$, and $c(\cdot)$, assuming that \mathbf{x} is excluded from either $u_k(\cdot)$ or $c(\cdot)$. Imposing such exclusion restrictions assumes away the correlation structure among these terms and precludes the possibility that the preferences of those with high voting costs are different from those with low voting costs. Note that this identification challenge exists regardless of whether the data are available at the individual level or at the aggregate level.

In this paper, we uncover the correlation structure between preferences and costs in a setting in which \mathbf{x} is allowed to enter both $u_k(\cdot)$ and $c(\cdot)$. Our identification is based on the simple observation that, unlike consumer choice problems in which choosing not to buy results in the outcome of not obtaining the good, choosing not to turn out still results in either A or B winning the election. This observation implies that the voter's choice is determined by the utility difference between the two election outcomes rather than by the levels of utility associated with each outcome.⁴ Barkume (1976) first used this observation to separately identify $u_k(\cdot)$ and $c(\cdot)$ in the context of property tax referenda for school districts.

To illustrate how this observation leads to the identification of $u_k(\cdot)$ and $c(\cdot)$, consider the calculus of voting models of Downs (1957) and Riker and Ordeshook (1968). In these models, the utility of voting for candidate k can be expressed as $u_k = pb_k$, where p is the voter's beliefs that she is pivotal; b_A is the utility difference between having candidate A in office and candidate B in office; and b_B is defined similarly.⁵

³Suppose that the cost function is separated into two parts as $c = c_{\mathbf{x}}(\mathbf{x}) + c_{\mathbf{z}}(\mathbf{z})$, where \mathbf{z} is a vector of cost shifters that is excluded from $u_A(\cdot)$ and $u_B(\cdot)$. Then, $u_A(\cdot) - c_{\mathbf{x}}(\cdot)$, $u_B(\cdot) - c_{\mathbf{x}}(\cdot)$ and $c_{\mathbf{z}}(\cdot)$ are all separately identified. However, $u_A(\cdot)$, $u_B(\cdot)$ and $c_{\mathbf{x}}(\cdot)$ are not separately identified. See the subsection titled "Exogenous Cost Shifters" towards the end of Section 4 for details.

⁴This implication holds as long as voters care about the ultimate outcome of the election. However, it may not hold for models in which voters gain utility from the act of voting for a candidate, such as models of expressive voting.

⁵More precisely, the utility of voting for candidate k relative to not turning out can be expressed as $u_k = pb_k$, by normalizing the utility of not turning out to be zero. See Appendix A for details.

Hence, we have $b_A = -b_B$. The mean utilities can now be expressed as

$$\begin{aligned} V_A(\mathbf{x}) &= pb_A(\mathbf{x}) - c(\mathbf{x}), \\ V_B(\mathbf{x}) &= -pb_A(\mathbf{x}) - c(\mathbf{x}), \text{ and} \\ V_0(\mathbf{x}) &= 0. \end{aligned}$$

The property $b_A = -b_B$ allows us to separately identify preferences and costs. By adding the first two expressions above, we have $V_A(\mathbf{x}) + V_B(\mathbf{x}) = -2c(\mathbf{x})$ because $pb_A(\mathbf{x})$ cancels out. Given that $V_A(\mathbf{x})$ and $V_B(\mathbf{x})$ are both identified from the vote share and turnout data, $c(\cdot)$ is identified. Similarly, we can identify $pb_A(\cdot)$ because we have $V_A(\mathbf{x}) - V_B(\mathbf{x}) = 2pb_A(\mathbf{x})$, and $V_A(\mathbf{x}) - V_B(\mathbf{x})$ is identified. As we discuss below, $V_A(x) - V_B(x)$ is identified primarily by the vote share margin and $V_A(x) + V_B(x)$ is identified primarily by voter turnout. Intuitively, changes in preference intensity in this model necessarily changes the vote share margin, while changes in costs have a similar effect on the vote shares of the two parties. This implies that changes in b_A cannot be undone by changes in c .⁶

In this paper, we retain the basic structure of the calculus of voting model but do not place additional restrictions on p , such as rational expectations, in which p equals the actual pivot probability. In our model, we interpret p more broadly as the voter's perception of voting efficacy, which is allowed to differ across individuals and to be correlated with the true pivot probability in a general manner. In particular, we let p be a function of individual characteristics and the state in which the voter lives, as $p = p_s \times \tilde{p}(\mathbf{x})$, where p_s is a state-specific coefficient and $\tilde{p}(\cdot)$ is a function of voter characteristics, \mathbf{x} . By letting p depend on each state, we can take into account the nature of the electoral college system.⁷ We show that the ratios of the state-specific components of efficacy, $p_s/p_{s'}$ ($\forall s, s'$), are identified directly from the data. Moreover, we show that $\tilde{p}(\cdot)$, $b_A(\cdot)$ and $c(\cdot)$ are identified up to a scalar normalization.⁸ Our

⁶See Merlo and de Paula (2016) for identification of voter preferences in a spatial voting model with full turnout.

⁷Under the electoral college system, perceptions of voting efficacy may differ significantly across states. For example, electoral outcomes in battleground states such as Ohio were predicted to be much closer than outcomes in party strongholds such as Texas. Hence, we allow for the possibility that p is higher for voters in Ohio than for voters in Texas.

⁸More precisely, we can identify $p(\cdot)b_A(\cdot)$ state by state given that we have many counties within each state. Assuming that $\tilde{p}(\cdot)$ and $b_A(\cdot)$ are common across states, we can identify $p_s/p_{s'}$. We also show that $\tilde{p}(\cdot)$ and $b_A(\cdot)$ are separately identified up to a scalar multiple in our full specification with

identification discussion does not depend on equilibrium restrictions on p , such as rational expectations. Therefore, our identification and estimation results are agnostic about how voters formulate p .

Given the debate over how to model voter turnout, we briefly review the literature on turnout to situate our model.⁹ The model that we estimate in this paper is based on the decision theoretic model of voter turnout introduced by Downs (1957) and Riker and Ordeshook (1968). In their models, a voter turns out and votes for the preferred candidate if $pb - c + d > 0$, where p is the voter's beliefs over the pivot probability; b is the utility difference from having one's preferred candidate in office relative to the other; c is the physical and psychological costs of voting; and d is the benefit from fulfilling one's civic duty to vote. While the original studies do not endogenize any of these terms, the decision theoretic model has provided a basic conceptual framework for much of the subsequent work on voting and turnout.

Studies subsequent to Riker and Ordeshook (1968) endogenize or micro-found each of the terms in the calculus of voting model in various ways. Ledyard (1984) and Palfrey and Rosenthal (1983, 1985) introduce the pivotal voter model, in which the pivot probability p is endogenized in a rational expectations equilibrium. They show that there exists an equilibrium with positive turnout in which voters have consistent beliefs about the pivot probability. Coate et al. (2008), however, point out that the rational expectations pivotal voter model has difficulties matching the data on either the level of turnout or the winning margin.¹⁰ Moreover, using laboratory experiments, Duffy and Tavits (2008) find that voters' subjective pivot probabilities are much higher than the actual pivot probability, which is at odds with the rational expectations assumption.

More recently, there have been attempts at endogenizing p in ways other than rational expectations. For example, Minozzi (2013) proposes a model based on cognitive dissonance in the spirit of Akerlof and Dickens (1982) and Brunnermeier and Parker (2005). In his model, voters jointly choose p and whether or not to turn out in or-

county-level shocks to preferences and costs.

⁹For a survey of the literature, see, e.g., Dhillon and Peralta (2002), Feddersen (2004), and Merlo (2006).

¹⁰Note, however, that with aggregate uncertainty, Myatt (2012) shows that the level of turnout can still be high with rational expectations. Levine and Palfrey (2007) also show that combining the quantal response equilibrium with the pivotal voter model can generate high turnout and finds that the results of laboratory experiments are consistent with the model prediction.

der to maximize subjective expected utility. Kanazawa (1998) introduces a model of reinforcement learning in which boundedly rational voters, who cannot compute the equilibrium pivot probabilities, form expectations about p from the correlation between their own past voting behavior and past election outcomes (see, also, Bendor et al., 2003; Esponda and Pouzo, 2016, for similar approaches). While these models are based on the basic calculus of voting model, the p term in them no longer carries the interpretation of the actual pivot probability.

Another strand of the literature endogenizes the c and d terms. Harsanyi (1980) and Feddersen and Sandroni (2006) endogenize the d term by proposing a rule-utilitarian model in which voters receive a warm-glow payoff from voting ethically. Based on their approach, Coate and Conlin (2004) estimate a group-utilitarian model of turnout. Shachar and Nalebuff (1999) also endogenize the d term by considering a follow-the-leader model in which elites persuade voters to turn out. In a paper studying split-ticket voting and selective abstention in multiple elections, Degan and Merlo (2011) consider a model that endogenizes c to reflect the voter’s psychological cost of making mistakes.

In our paper, we bring the calculus of voting model to the data without taking a particular stance on how the p , b , c , or d terms are endogenized. Specifically, our identification and estimation do not use the restriction that p is equal to the actual pivot probability, as in the rational expectations model. The p term that we recover can be broadly interpreted as the voter’s perception of voting efficacy. We purposely aim to be agnostic about the different ways of modeling voter turnout so that our estimates of voter preferences and costs are robust to how the p , b , c , or d terms are endogenized. Instead of imposing equilibrium restrictions of a particular model a priori, we let the data directly identify the p , b , and $c - d$ terms.

Relatedly, our study does not impose a priori restrictions on how the covariates enter the p , b , or $c - d$ terms, allowing, instead, the same set of covariates to affect all three terms. This is important because the way in which covariates enter the p , b , and $c - d$ terms determines the correlation structure among them, which, in turn, determines how well preferences are aggregated. In most existing studies, the sets of covariates that enter the p , b and $c - d$ terms are disjoint, precluding the possibility that preferences and costs are correlated. For example, Coate and Conlin (2004) and Coate et al. (2008) include demographic characteristics only in the b term,¹¹ while

¹¹To be more precise, Coate and Conlin (2004) and Coate et al. (2008) use demographic characteristics as covariates for the fraction of the population supporting one side.

Shachar and Nalebuff (1999) include them in the $c - d$ term. In contrast, we let each demographic characteristic enter all three terms, allowing us to study the effects of turnout on preference aggregation.¹²

We use county-level data on voting outcomes from the 2004 U.S. presidential election to estimate the model.¹³ A benefit of using actual voting data over survey data is that we can avoid serious misreporting issues often associated with survey data, such as the overreporting of turnout and reporting bias in vote choice (see, e.g., Atkeson, 1999; DellaVigna et al., 2015).¹⁴ Our data on turnout and vote share incorporate the number of non-citizens and felons to account for the difference between the voting-eligible population and the voting-age population (McDonald and Popkin, 2001). We construct the joint distribution of demographic characteristics within each county from the 5% Public Use Microdata Sample of the Census.

We find that young, less-educated, low-income, and religious voters have high voting costs, as do African Americans, Hispanics, and other minorities. Moreover, young voters have low perception of voting efficacy, which further depresses turnout among this group. Overall, young, less-educated, and low-income voters are particularly underrepresented. In terms of preferences, minority, young, highly educated, low-income, and non-religious voters are more likely to prefer Democrats.

Our results show that, overall, there is a positive correlation between voting cost and preference for Democrats that can be accounted for through observable characteristics. Except for two voter characteristics—years of schooling and being religious—we find that demographic characteristics that are associated with a higher cost of voting are also associated with preferring Democrats. We also find that unobservable cost shocks are positively correlated with unobservable preference shocks for Democrats. These correlations result in fewer Democratic votes relative to the preferences of the underlying population. Our estimate of turnout is significantly lower among the elec-

¹²One possible exception is Degan and Merlo (2011). They consider a model based on the theories of regret in which the cost term is endogenized in a way that captures voters' preferences over candidates. They include the same set of covariates in the c and d terms. In one of their counterfactual analyses, they consider the effect of increasing voter turnout, focusing on split-ticket voting and selective abstention across presidential and congressional elections.

¹³Although we use aggregate data, we account for the issue of ecological fallacy by computing the behavior of individual voters and aggregating them at the county level.

¹⁴There is a set of survey-based studies that investigate the differences in preferences between voters and non-voters (see, e.g., Citrin et al., 2003; Brunell and DiNardo, 2004; Martinez and Gill, 2005; Leighley and Nagler, 2013).

torate who prefer Democrats to Republicans, at 55.7%, compared with turnout among those who prefer Republicans to Democrats, at 64.5%.¹⁵ Moreover, we find that voters who have a strong preference for one of the parties are more likely to turn out, suggesting that preference *intensity* affects preference aggregation (see Campbell, 1999; Casella, 2005; Lallely and Weyl, 2015).

Regarding our results on the perception of voting efficacy, we find substantial across-state variation in our estimates of p_s , the state-specific coefficient in p . Furthermore, the estimates are correlated with the ex-post closeness of the election: Battleground states such as Ohio and Wisconsin tend to have high estimates of p_s , while party strongholds such as New Jersey and California have low estimates, which is consistent with the comparative statics of the pivotal voter model with rational expectations. However, the ratios of the estimated p_s are, at most, three for any pair of states. This is in contrast to a much larger variation in the ratio implied by the pivotal voter model.¹⁶ Our results are more consistent with models of turnout in which voters' perception of efficacy is only weakly correlated with the actual pivot probabilities.

In our first counterfactual experiment, we simulate the voting outcome when we set voting costs to zero. We find that the vote share of the Democrats increases in all states. Overall, the increase in the Democrats' two-party vote share is about 3.7%. We also find that the increase in the Democratic vote share would overturn the election results in nine states, including key states such as Florida and Ohio, resulting in the Democrats winning a plurality of the electoral votes.

In our second counterfactual experiment, we compare the actual election outcome with the counterfactual outcome when we equalize the state-specific component of efficacy across states (set $p_s = p_{s'}$). Equalizing p_s across states can be considered as eliminating the electoral college system, thus removing endogeneity in turnout that results from the way state boundaries are drawn. This counterfactual is motivated by the recent development in measuring gerrymandering, in particular, the use of a metric called the "efficiency gap" (Stephanopoulos and McGhee, 2015) for determining the

¹⁵See DeNardo (1980) and Tucker and DeNardo (1986) for studies that report negative correlation between turnout and the Democratic vote share using aggregate data. For more recent work, see Hansford and Gomez (2010) who use rainfall as an instrument for turnout.

¹⁶The pivotal voter model with rational expectations predicts high variation in the ratio of pivot probabilities across states, given the winner-take-all nature of the electoral college system. Voters in only a handful of swing states have a reasonable probability of being pivotal (see, e.g., Shachar and Nalebuff, 1999).

legality of districting plans. By eliminating state-specific endogeneity in turnout, we gauge the robustness of the efficiency gap to endogenous turnout.¹⁷

We find that equalizing p_s across states while keeping turnout at the actual levels changes the efficiency gap by 2.2 percentage points. We also find that the efficiency gap changes substantially when we vary the value of p_s that we use. For example, a change in the level of p_s corresponding to a change in turnout from 50% to 80% increases the efficiency gap by 14.1 percentage points. To put these numbers in perspective, 2.2 percentage points is comparable to about half of the increase in the efficiency gap for the Republican party over the past 30 years in the U.S. state legislative elections. A change in the efficiency gap of 14.1 percentage points is larger than the proposed threshold value of 8 percentage points above which Stephanopoulos and McGhee (2015) argue that districting plans should be deemed presumptively unlawful. Our results suggest that the efficiency gap is quite sensitive to endogenizing turnout. One natural alternative to comparing actual votes and seat shares, which is what the efficiency gap does as well as other measures such as partisan symmetry (e.g., Grofman and King, 2007), is to compare underlying voter preferences and seat shares. If we think about elections as a way to aggregate preferences into outcomes, evaluating the electoral system in terms of its ability to aggregate preferences seems most coherent.

2 Model

Anticipating the empirical application of the paper, we tailor our model to the U.S. presidential election. Let $s \in \{1, \dots, S\}$ denote a U.S. state and $m \in \{1, \dots, M_s\}$ denote a county in state s .

Preference of Voters We consider a model of voting with two candidates, D and R . Each voter chooses to vote for one of the two candidates or not to vote. We let b_{nk} denote voter n 's utility from having candidate $k \in \{D, R\}$ in office, p_n ($p_n > 0$) denote her perception of voting efficacy, and c_n denote her cost of voting. Given that there are only two possible outcomes (either D wins or R wins the election), the utility of voting for candidate k , U_{nk} depends only on $b_{nD} - b_{nR}$ rather than on b_{nD} and b_{nR}

¹⁷The literature on districting focuses on the mapping from vote share to seat share, and little attention is paid to the fact that turnout is endogenous. See, e.g., Coate and Knight (2007), and Friedman and Holden (2008). For a survey, see Nagle (2015).

individually:

$$U_{nD} = p_n(b_{nD} - b_{nR}) - c_n, \quad (1)$$

$$U_{nR} = p_n(b_{nR} - b_{nD}) - c_n, \quad (2)$$

$$U_{n0} = 0,$$

where U_{n0} is the utility of not turning out, which we normalize to zero (see Appendix A for a derivation).¹⁸ When p_n is the actual pivot probability, the behavior of the voters under our model is the same as the equilibrium play of the voters under the pivotal voter model of Palfrey and Rosenthal (1983, 1985). However, we interpret p_n broadly as the voter's subjective perception of voting efficacy, as we discuss below. The cost of voting, c_n , includes both physical and psychological costs, as well as possible benefits of fulfilling one's civic duty. Hence, c_n can be either positive or negative. When c_n is negative, the voter turns out regardless of the value of p_n and $b_{nD} - b_{nR}$.

We let the utility of voter n in county m of state s depend on her demographic characteristics, \mathbf{x}_n , as follows:

$$b_{nk} = b_k(\mathbf{x}_n) + \lambda_{sk} + \xi_{mk} + \varepsilon_{nk}, \text{ for } k \in \{D, R\},$$

where λ_{sk} is a state-specific preference intercept that captures state-level heterogeneity in voter preferences. ξ_{mk} and ε_{nk} are unobserved random preference shocks at the county level and at the individual level, respectively. ξ_{mk} captures the unobserved factors that affect preferences at the county level, such as the benefits that the voters in county m receive from policies supported by candidate k . Then, the expression for the utility difference is as follows:

$$b_{nR} - b_{nD} = b(\mathbf{x}_n) + \lambda_s + \xi_m + \varepsilon_n,$$

where $b(\mathbf{x}_n) \equiv b_R(\mathbf{x}_n) - b_D(\mathbf{x}_n)$, $\lambda_s \equiv \lambda_{sR} - \lambda_{sD}$, $\xi_m \equiv \xi_{mR} - \xi_{mD}$, and $\varepsilon_n \equiv \varepsilon_{nR} - \varepsilon_{nD}$. We assume that ε_n follows the standard normal distribution.

We also let voting cost c_n be a function of voter n 's characteristics as

$$c_n = c_s(\mathbf{x}_n) + \eta_m,$$

¹⁸Note that expressions (1) and (2) take the familiar form of $pb - c$.

where $c_s(\mathbf{x}_n)$ is the cost function and η_m is a county-level shock on the cost of voting. The cost function, $c_s(\cdot)$, is allowed to depend on s . Given that previous studies (e.g., Smith, 2001) find that neither the presence nor the closeness of congressional elections affect turnout in presidential elections, we do not incorporate other elections as a cost shifter in our model. We assume that ξ_m and η_m are both independent of \mathbf{x}_n , but we allow ξ_m and η_m to be correlated with each other.

We let the voting efficacy term, p_n , depend on both the demographic characteristics of voter n and the state in which she votes as follows:

$$p_n = p_s(\mathbf{x}_n) = p_s \times \tilde{p}(\mathbf{x}_n),$$

where p_s is a state specific coefficient that we estimate. It is important to let p_n depend on the state in which the voter votes because of the winner-take-all nature of the electoral votes in each state.¹⁹ For example, in the 2004 presidential election, a vote in key states such as Ohio was predicted to matter considerably more than a vote elsewhere. Our specification also allows for the possibility that p_n depends on voters' characteristics, \mathbf{x}_n . Previous work has shown that voters' social and economic status affects her general sense of political efficacy (see, e.g., Karp and Banducci, 2008).

Note that the behavior of the voters under our model is the same as the equilibrium play of the voters under the pivotal voter model if we set p_s equal to the actual pivot probability in state s and set $\tilde{p}(\mathbf{x}_n)$ equal to 1. In this sense, our specification nests the pivotal voter model as a special case. However, instead of imposing the pivotal voter model (and, hence, placing equilibrium restrictions on p_n), we estimate p_s and $\tilde{p}(\cdot)$ directly from the data. This approach allows us to interpret p_n consistently with models of turnout that endogenize p_n in ways other than pivot probabilities under rational expectations.

Substituting the expressions for $b_{nR} - b_{nD}$, c_n , and p_n into equations (1) and (2),

¹⁹In U.S. presidential elections, the winner is determined by the Electoral College. Each U.S. state is allocated a number of electoral votes, roughly in proportion to the state's population. The electoral votes of each state are awarded on a winner-takes-all basis in all states, except for Maine and Nebraska. The Presidential candidate who wins the plurality of electoral votes becomes the winner of the election.

the utility from choosing each of the alternatives can be expressed as follows:

$$\begin{aligned} U_{nD}(\mathbf{x}_n) &= p_s(\mathbf{x}_n) [-b_s(\mathbf{x}_n) - \xi_m - \varepsilon_n] - c_s(\mathbf{x}_n) - \eta_m, \\ U_{nR}(\mathbf{x}_n) &= p_s(\mathbf{x}_n) [b_s(\mathbf{x}_n) + \xi_m + \varepsilon_n] - c_s(\mathbf{x}_n) - \eta_m, \\ U_{n0}(\mathbf{x}_n) &= 0, \end{aligned}$$

where $b_s(\mathbf{x}_n)$ denotes $b(\mathbf{x}_n) + \lambda_s$.

A Voter's Decision Voter n 's problem is to choose the alternative $k \in \{D, R, 0\}$ that provides her with the highest utility:

$$k = \arg \max_{\kappa \in \{D, R, 0\}} U_{n\kappa}(\mathbf{x}_n). \quad (3)$$

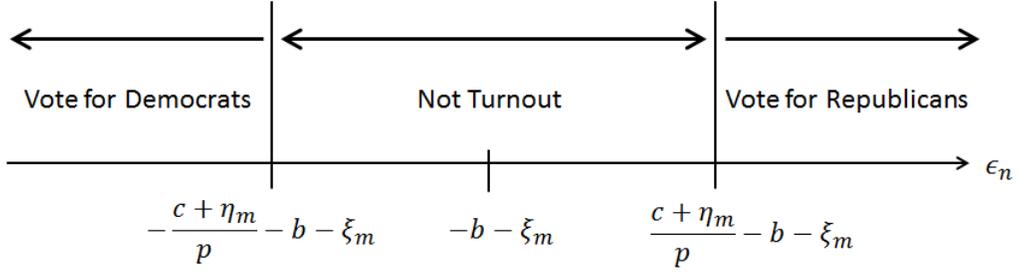
We can write the probability that voter n votes for candidate R as

$$\begin{aligned} \Pr(R = \arg \max_{\kappa \in \{D, R, 0\}} U_{n\kappa}) &= \Pr(U_{nR} > U_{nD} \text{ and } U_{nR} > 0) \\ &= \Pr\left(\varepsilon_n > -b_s(\mathbf{x}_n) - \xi_m \text{ and } \varepsilon_n > -b_s(\mathbf{x}_n) - \xi_m + \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)}\right) \\ &= 1 - \Phi\left(\max\left\{-b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m + \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)}\right\}\right), \end{aligned}$$

where Φ is the CDF of the standard normal. We can derive a similar expression for candidate D .

Figure 1 depicts the behavior of a voter as a function of ε_n . There are two cases to consider: one in which the cost of voting is positive (Case 1) and the other in which the cost of voting is negative (Case 2). In Case 1, a voter with a strong preference for one of the candidates (which corresponds to a large positive realization or a large negative realization of ε_n) votes for her preferred candidate, while a voter who is relatively indifferent between the two candidates does not turn out. That is, a voter with high preference intensity relative to cost turns out, while a voter with low preference intensity does not. In Case 2, a voter always votes, regardless of her preference intensity, as the cost of voting is negative (e.g., a voter obtains positive net-benefit from fulfilling her civic duty.)

Case 1: $c + \eta_m > 0$



Case 2: $c + \eta_m < 0$

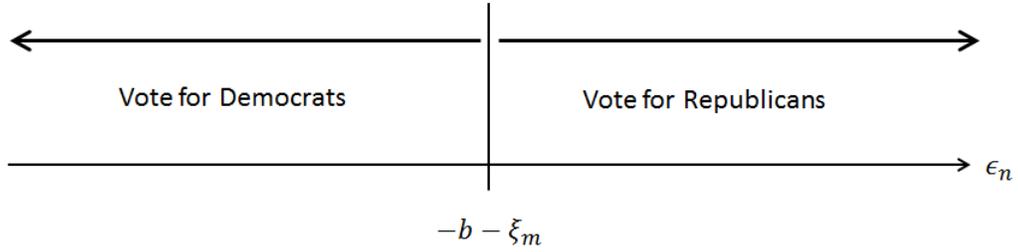


Figure 1: Voter's Decision as a Function of ϵ_n . The top panel corresponds to the case in which a voter has positive costs of voting. The bottom panel corresponds to the case in which a voter has negative costs of voting.

Vote Share and Voter Turnout We can express the vote share for candidate k in county m , $v_{k,m}$, and the fraction of voters who do not turn out, $v_{0,m}$, as follows:

$$v_{R,m} \equiv \int 1 - \Phi \left(\max \left\{ -b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m + \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)} \right\} \right) dF_{\mathbf{x},m}(\mathbf{x}_n), \quad (4)$$

$$v_{D,m} \equiv \int \Phi \left(\min \left\{ -b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m - \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)} \right\} \right) dF_{\mathbf{x},m}(\mathbf{x}_n), \quad (5)$$

$$v_{0,m} \equiv 1 - v_{D,m} - v_{R,m} \quad (6)$$

where $F_{\mathbf{x},m}$ denotes the distribution of \mathbf{x} in county m . Denoting the number of eligible voters in county m by N_m and the number of counties in state s as M_s , the vote share for candidate k in state s can be expressed as $\sum_{m=1}^{M_s} N_m v_{k,m} / \sum_{m=1}^{M_s} N_m$. The candidate with the highest vote share in state s is allocated all of the electors of that state.²⁰

²⁰Maine and Nebraska use a different allocation method. Hence, we drop these two states from our sample.

The candidate who wins the plurality of the electors becomes the overall winner of the presidential election.

Advertising and Campaign Visits An important feature of presidential elections not explicitly modeled thus far is the campaign activities of candidates. Candidates target key states with advertisements and campaign visits during the election. These campaign activities are endogenous and depend on the expected closeness of the race in each state (see, e.g., Strömberg, 2008; Gordon and Hartmann, 2013).

While we do not have a specific model of political campaigns, the model accounts for the effect of campaigns on voters through the state-specific preference intercept λ_s . Because we treat λ_s as parameters to be estimated, λ_s may be arbitrarily correlated with the characteristics of the state, the closeness of the race in the state, etc. Hence, our estimates of the primitives of the model are consistent even in the presence of campaign activities. We note, however, that the results of our counterfactual experiments take the level of campaigning as given.

Discussion on Voter's Information Another factor that we do not specifically model is voter's information. One way to explicitly model information is by endogenizing the voters' information acquisition, e.g., Matsusaka (1995) and Degan and Merlo (2011). In these models, the voters decide on the amount of information to acquire about the candidates by paying the cost of information acquisition. Our specification of voter preference and costs can be thought of as the indirect utility of these models to the extent that information acquisition costs are functions of voter demographics. In fact, in our estimation results, we find that income and education are associated with low voting costs, which suggests that the information acquisition cost may comprise an important part of the voting cost (as the opportunity cost for these voters tend to be higher).

Another way to model information is to consider a common value environment in which voters obtain signals about the quality of the candidates (see, e.g., Feddersen and Pesendorfer, 1996, 1997). In these models, voters' utility consists partly of the expected quality of the candidates, which is computed by conditioning on the event that the voter is pivotal. To the extent that the prior beliefs over candidate quality and the signal distribution depend on the voter's demographic characteristics, the common

value component is also a function of these characteristics. Hence, our specification of the utility can also be interpreted as a reduced-form of model with a common value component.

Discussion on p The modeling in our paper is purposely agnostic about how p is endogenized: We do not impose a particular model of p , such as rational expectations (Palfrey and Rosenthal, 1983, 1985), overconfidence (Duffy and Tavits, 2008), or cognitive dissonance (Minozzi, 2013). Similarly, our estimation approach avoids using restrictions specific to a particular way of modeling voter beliefs. The important point for our purpose is that there exists some p that generates the data regardless of the way in which p is endogenized. Our approach is to identify and estimate both the model primitives and the equilibrium p directly from the data with as little structure as possible. This empirical strategy is similar in spirit to that in the estimation of incomplete models, in which some primitives are estimated from the data without fully specifying a model. For example, Haile and Tamer (2003) recovers bidder values without fully specifying a model of the English auction, using only the restriction that the winning bid lies between the valuations of the losers and the winner. Given that their estimation procedure also avoids using restrictions specific to a particular model of the English auction, the estimates are consistent under a variety of models.

In Section 4, we show that the key primitives of the model are identified without fully specifying how voters form p . We show that the equilibrium p is also identified directly from the data.²¹ The strength of our approach is that we impose few restrictions on beliefs, and, thus, our estimates of preferences and costs are consistent under a variety of behavioral assumptions regarding how p is formed. On the other hand, this approach limits the types of counterfactual experiments that we can conduct since we do not specify a particular model regarding p .

3 Data

In this section, we describe our data and provide summary statistics. We combine county-level voting data and demographics data. The county-level voting data is obtained from David Leip’s Atlas of U.S. Presidential Elections. This dataset is a com-

²¹More precisely, $p_s/p_{s'}$ is identified for any states s and s' , and $\tilde{p}(\cdot)$ is identified up to a scalar normalization. See Section 4 for details.

pilation of election data from official sources such as state boards of elections. The demographics data is obtained from the U.S. Census Bureau. We construct the data on eligible voters for each county by combining the population estimates from the 2004 Annual Estimates of the Resident Population and age and citizenship information from the 2000 Census. We then adjust for the number of felons at the state level using the data from McDonald (2016). Hence, our data account for the difference between the voting age population and voting eligible population (see McDonald and Popkin, 2001).

We construct the joint distribution of voters' demographic characteristics and citizenship at the county level from the 2000 Census by combining the county-level marginal distribution of each demographic variable and the 5% Public Use Microdata Sample (see Appendix B for details). We augment the Census data with county-level information on religion using the Religious Congregations and Membership Study 2000. In particular, we define the variable Religious using adherence to either "Evangelical Denominations" or "Church of Jesus Christ of Latter-day Saints."

Our data consist of a total of 2,909 counties from forty states. Because we need a large number of counties within each state to identify the state-specific parameters, p_s and λ_s , we drop states that have fewer than 15 counties. These states are Alaska, Connecticut, District of Columbia, Delaware, Hawaii, Massachusetts, New Hampshire, Rhode Island, and Vermont. In addition, we drop Maine and Nebraska because these two states do not adopt the winner-takes-all rule to allocate electors. We also drop counties with a population below 1,000 because their vote shares and turnout rates can be extreme due to small population size.²² Table 1 presents the summary statistics of the county-level vote share, turnout, and demographic characteristics. Note that a Hispanic person may be of any race according to the definition used in the Census.

In order to illustrate the degree to which turnout and expected closeness are related, Figure 2 plots the relationship between the (ex-post) winning margin and voter turnout at the state level. The two variables are negatively correlated, although the fitted line is relatively flat. The slope of the fitted line implies that a decrease in the (ex-post) winning margin of ten percentage points is associated with an increase in turnout of only about 1.6 percentage points. While the negative correlation may be capturing some of the forces of the rational-expectations pivotal voter model, the flatness of the

²²In addition, we drop one county, Chattahoochee, GA, as the turnout rate is extremely low (18.8%) relative to all other counties. The turnout rate for the next lowest county is 33%.

Voting Data					
	Obs	Mean	Std. Dev.	Min	Max
Vote Share: Democrat	2,909	0.24	0.08	0.04	0.57
Vote Share: Republican	2,909	0.37	0.09	0.07	0.70
1 - Turnout Rate	2,909	0.40	0.09	0.12	0.67

County Demographics					
	Obs	Mean	Std. Dev.	Min	Max
% Hispanic	2,909	0.05	0.12	0.00	0.97
% Black/African American	2,909	0.09	0.15	0.00	0.87
% Neither Black nor White	2,909	0.05	0.08	0.00	0.95
Mean Age	2,909	46.75	2.64	35.94	56.07
Mean Income (USD 1,000)	2,909	42.71	9.23	23.33	93.40
Mean Years of Schooling	2,909	12.86	0.60	10.84	15.18
% Religious	2,909	0.26	0.17	0.00	1.00

Table 1: Summary Statistics of Voting Outcome and Demographic Characteristics of Eligible Voters. For Age, Income, and Years of Schooling, the table reports the mean, standard deviation, minimum, and maximum of the county mean. “% Religious” is the share of the population with adherence to either “Evangelical Denomination” or “Church of Jesus Christ of Latter-day Saints.”

slope suggests that turnout is unlikely to be fully accounted for by the pivotal voter model.

4 Identification

In this section, we discuss the identification of our model as the number of counties *within each state* becomes large ($M_s \rightarrow \infty$). Given that we have state-specific parameters for $p_s(\cdot)$ and $b_s(\cdot)$, we require the number of observations in each state to be large. Our discussion in this section builds on the idea initially proposed by Barkume (1976) in the context of property tax referenda for school districts.

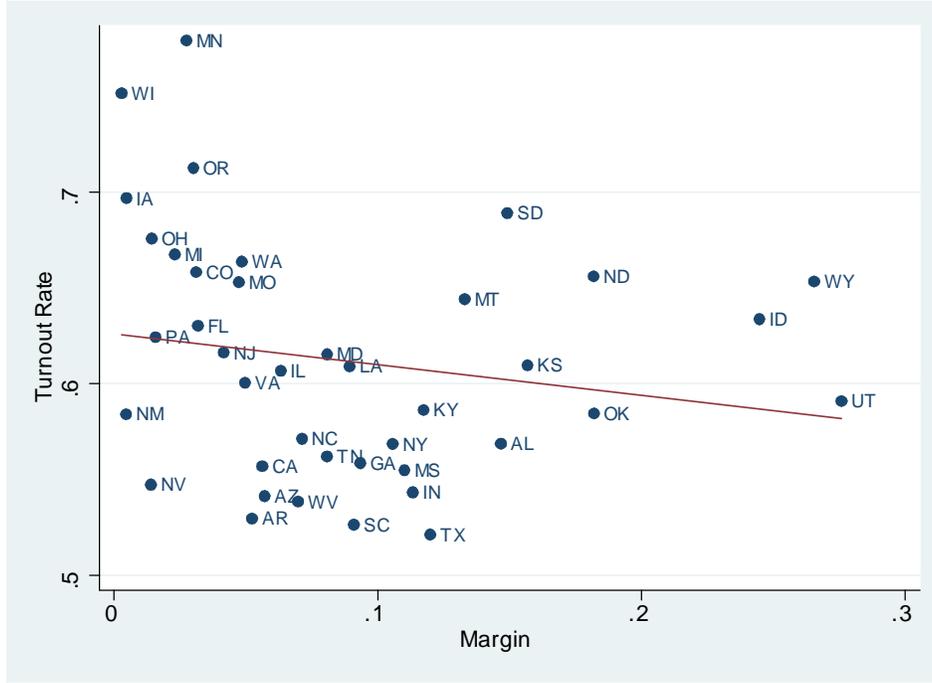


Figure 2: Relationship between the Ex-Post Winning Margin and Voter Turnout. The slope coefficient is -0.16 and not statistically significant.

Recall that the observed vote shares are expressed as:

$$\begin{aligned}
 v_{R,m} &\equiv \int 1 - \Phi \left(\max \left\{ -b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m + \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)} \right\} \right) dF_{\mathbf{x},m}(\mathbf{x}_n), \\
 v_{D,m} &\equiv \int \Phi \left(\min \left\{ -b_s(\mathbf{x}_n) - \xi_m, -b_s(\mathbf{x}_n) - \xi_m - \frac{c_s(\mathbf{x}_n) + \eta_m}{p_s(\mathbf{x}_n)} \right\} \right) dF_{\mathbf{x},m}(\mathbf{x}_n), \\
 v_{0,m} &\equiv 1 - v_{D,m} - v_{R,m}.
 \end{aligned}$$

For exposition, consider the simple case in which there is no heterogeneity in voters' observable characteristics, so that $\mathbf{x}_n = \bar{\mathbf{x}}_m$ for all n in county m .²³ In this case, the

²³Note that we are well aware of the issues of ecological fallacy. In what follows, we consider a simplified setup with $\mathbf{x}_n = \bar{\mathbf{x}}_m$ for all n in county m , just for expositional purposes. In our empirical exercise, we fully address the fact that each county has a distribution of \mathbf{x} by integrating the vote share for each \mathbf{x} with respect to $F_{\mathbf{x},m}(\cdot)$.

above expressions simplify as follows:

$$v_{R,m} \equiv 1 - \Phi \left(\max \left\{ -b_s(\bar{\mathbf{x}}_m) - \xi_m, -b_s(\bar{\mathbf{x}}_m) - \xi_m + \frac{c_s(\bar{\mathbf{x}}_m) + \eta_m}{p_s(\bar{\mathbf{x}}_m)} \right\} \right), \quad (7)$$

$$v_{D,m} \equiv \Phi \left(\min \left\{ -b_s(\bar{\mathbf{x}}_m) - \xi_m, -b_s(\bar{\mathbf{x}}_m) - \xi_m - \frac{c_s(\bar{\mathbf{x}}_m) + \eta_m}{p_s(\bar{\mathbf{x}}_m)} \right\} \right), \quad (8)$$

$$v_{0,m} \equiv 1 - v_{D,m} - v_{R,m}. \quad (9)$$

We now show that the primitives of the model are identified from expressions (7), (8), and (9).

Using the fact that Φ is a strictly increasing function, we can rewrite expressions (7) and (8) as follows:

$$\begin{aligned} \Phi^{-1}(1 - v_{R,m}) &= \max \left\{ -b_s(\bar{\mathbf{x}}_m) - \xi_m, -b_s(\bar{\mathbf{x}}_m) - \xi_m + \frac{c_s(\bar{\mathbf{x}}_m) + \eta_m}{p_s(\bar{\mathbf{x}}_m)} \right\}, \\ \Phi^{-1}(v_{D,m}) &= \min \left\{ -b_s(\bar{\mathbf{x}}_m) - \xi_m, -b_s(\bar{\mathbf{x}}_m) - \xi_m - \frac{c_s(\bar{\mathbf{x}}_m) + \eta_m}{p_s(\bar{\mathbf{x}}_m)} \right\}. \end{aligned}$$

Rearranging these two equations, we obtain the following expressions:

$$\frac{\Phi^{-1}(1 - v_{R,m}) + \Phi^{-1}(v_{D,m})}{-2} = b_s(\bar{\mathbf{x}}_m) + \xi_m, \text{ and} \quad (10)$$

$$\frac{\Phi^{-1}(1 - v_{R,m}) - \Phi^{-1}(v_{D,m})}{2} = \max \left\{ 0, \frac{c_s(\bar{\mathbf{x}}_m)}{p_s(\bar{\mathbf{x}}_m)} + \frac{\eta_m}{p_s(\bar{\mathbf{x}}_m)} \right\}. \quad (11)$$

Note that the left hand side of (10) closely reflects the difference in vote share, and the left hand side of (11) reflects the turnout rate. This is because, if we ignore the nonlinearity of $\Phi^{-1}(\cdot)$ and the denominator, the left hand side of (10) reduces to $1 - v_{R,m} + v_{D,m}$ and the left hand side of (11) to $1 - v_{R,m} - v_{D,m}$. The former is one minus the difference in vote share, and the latter is one minus voter turnout. The left hand side of expressions (10) and (11) can be directly computed using data on vote shares, $v_{D,m}$ and $v_{R,m}$.

We first consider the identification of $b_s(\cdot)$ and the distribution of ξ , $F_\xi(\cdot)$. Taking the expectation of (10) conditional on $\bar{\mathbf{x}}_m$, we have

$$\mathbf{E} \left[\frac{\Phi^{-1}(1 - v_{R,m}) + \Phi^{-1}(v_{D,m})}{-2} \middle| \bar{\mathbf{x}}_m \right] = b_s(\bar{\mathbf{x}}_m), \quad (12)$$

because $\mathbf{E}[\xi_m | \bar{\mathbf{x}}_m] = 0$. As the left hand side of the above expression is identified, $b_s(\cdot)$ is (nonparametrically) identified for each s (note that the asymptotics is with respect to the number of counties within each state). Given that $b_s(\cdot)$ is the utility difference between Republicans and Democrats, it is intuitive that $b_s(\cdot)$ is identified by the difference in vote share. In this model, changes in $b_s(\cdot)$ is reflected in the vote share margin which cannot be undone by changes in $c_s(\cdot)$.

Now, consider the identification of $F_\xi(\cdot)$. Given that $b_s(\cdot)$ is identified and the left hand side of (10) is observable, each realization of ξ_m can be recovered from (10). Hence, $F_\xi(\cdot)$ is also identified. Note that if $b_s(\cdot)$ is linear in $\bar{\mathbf{x}}_m$ (i.e., $b_s(\bar{\mathbf{x}}_m) = \beta \bar{\mathbf{x}}_m$), one can simply regress the left hand side of expression (10) on $\bar{\mathbf{x}}_m$ by OLS to obtain β as coefficients and ξ_m as residuals.

We now discuss the identification of $p_s(\cdot)$, $c_s(\cdot)$, and $F_\eta(\cdot)$. For simplicity, consider the case in which the second term inside the max operator of expression (11) is positive with probability 1;

$$\frac{\Phi^{-1}(1 - v_{R,m}) - \Phi^{-1}(v_{D,m})}{2} = \frac{c_s(\bar{\mathbf{x}}_m)}{p_s(\bar{\mathbf{x}}_m)} + \frac{\eta_m}{p_s(\bar{\mathbf{x}}_m)}. \quad (13)$$

This corresponds to the case in which the turnout rate is always less than 100%. We show in Appendix C that $p_s(\cdot)$, $c_s(\cdot)$, and $F_\eta(\cdot)$ are identified without this assumption.

Taking the conditional moments of (13), we have

$$\mathbf{E} \left[\frac{\Phi^{-1}(1 - v_{R,m}) - \Phi^{-1}(v_{D,m})}{2} \middle| \bar{\mathbf{x}}_m \right] = \frac{c_s(\bar{\mathbf{x}}_m)}{p_s(\bar{\mathbf{x}}_m)}, \text{ and} \quad (14)$$

$$\mathbf{Var} \left[\frac{\Phi^{-1}(1 - v_{R,m}) - \Phi^{-1}(v_{D,m})}{2} \middle| \bar{\mathbf{x}}_m \right] = \frac{\sigma_\eta^2}{(p_s(\bar{\mathbf{x}}_m))^2}, \quad (15)$$

where σ_η^2 is the variance of η_m . Using (15), $p_s(\cdot)$ is identified up to a scalar constant (i.e., up to σ_η^2) because the left hand side of (15) is identified. This implies that $c_s(\bar{\mathbf{x}}_m)$ is also identified up to σ_η^2 using (14). Given that $p_s(\cdot)$ and $c_s(\cdot)$ are identified, we can recover the realization of η_m from (13), implying that $F_\eta(\cdot)$ is also identified up to σ_η^2 . Intuitively, the left hand side of (14) and (15) closely reflect the mean and variance of the rate of abstention. Hence, the average cost of voting normalized by p_s (i.e., $c_s(\cdot)/p_s(\cdot)$) is identified from (14).

Importantly, while $p_s(\cdot)$ is identified only up to a scalar constant, the ratio $p_{s'}/p_{s''}$ for any states s' and s'' is identified given our specification of $p_s(\cdot)$ as $p_s(\cdot) = p_s \times \tilde{p}(\cdot)$.

To see this, note that the ratio of (15) for two counties with the same demographics in states s' and s'' directly identify $p_{s'}/p_{s''}$.

The discussion has, thus far, been based on the simplified case in which all voters in county m have the same demographic characteristics—i.e., $\mathbf{x}_n = \bar{\mathbf{x}}_m$ for all n in m . As long as there is sufficient variation in $F_{\mathbf{x},m}(\mathbf{x})$, we can recover the vote shares conditional on each \mathbf{x} and apply the identification discussion above.

Correlation between Unobserved Cost and Preference Shocks Our identification makes no assumptions regarding the correlation between the unobservables ξ_m and η_m . As ξ_m and η_m enter separately in (10) and (11), $\xi_m \perp \bar{\mathbf{x}}_m$ and $\eta_m \perp \bar{\mathbf{x}}_m$ are sufficient to identify the unknown primitives on the right hand side in each equation. Hence, we do not require any restrictions on the joint distribution of ξ_m and η_m . In fact, we can nonparametrically identify the joint distribution of ξ_m and η_m from the joint distribution of the residuals in each equation. In our estimation, we specify the joint distribution of ξ_m and η_m as a bivariate Normal with correlation coefficient ρ .

Exogenous Cost Shifters Lastly, we discuss identification when there exist instruments (e.g., rainfall) that shift the cost of voting but not the preferences of the voters. The point we wish to make is that the existence of exogenous cost shifters are neither necessary nor sufficient for identification.

To illustrate this point, consider the following discrete choice setup with instruments \mathbf{z}_n ,

$$\begin{aligned} V_A &= u_A(\mathbf{x}_n) - c_{\mathbf{x}}(\mathbf{x}_n) - c_{\mathbf{z}}(\mathbf{z}_n), \\ V_B &= u_B(\mathbf{x}_n) - c_{\mathbf{x}}(\mathbf{x}_n) - c_{\mathbf{z}}(\mathbf{z}_n), \\ V_0 &= 0, \end{aligned}$$

where V_k denotes the mean utility of choosing $k \in \{A, B, 0\}$. Here, $u_A(\mathbf{x}_n)$ is not necessarily equal to $-u_B(\mathbf{x}_n)$, and the cost function is separated into two components, $c_{\mathbf{x}}(\mathbf{x}_n)$ and $c_{\mathbf{z}}(\mathbf{z}_n)$, where \mathbf{z}_n is a vector of cost shifters excluded from $u_k(\mathbf{x}_n)$. For any arbitrary function $g(\mathbf{x}_n)$, consider an alternative model with $\tilde{u}_k(\mathbf{x}_n) = u_k(\mathbf{x}_n) + g(\mathbf{x}_n)$ ($k \in \{A, B\}$) and $\tilde{c}_{\mathbf{x}}(\mathbf{x}_n) = c_{\mathbf{x}}(\mathbf{x}_n) + g(\mathbf{x}_n)$, as follows:

$$\begin{aligned}
V_A &= \tilde{u}_A(\mathbf{x}_n) - \tilde{c}_x(\mathbf{x}_n) - c_z(\mathbf{z}_n), \\
V_B &= \tilde{u}_B(\mathbf{x}_n) - \tilde{c}_x(\mathbf{x}_n) - c_z(\mathbf{z}_n), \\
V_0 &= 0.
\end{aligned}$$

Because $\tilde{u}_k(\mathbf{x}_n) - \tilde{c}_x(\mathbf{x}_n) = u_k(\mathbf{x}_n) - c_x(\mathbf{x}_n)$, the two models are observationally equivalent, and thus, $u_k(\cdot)$ and $c_x(\cdot)$ are not separately identified. In this model, one cannot differentiate between the case in which voters have intense preferences and large voting costs and one in which voters have weak preferences and small costs. However, in the model we take to the data, intense preferences for one of the parties are reflected in the vote share margin, which cannot be offset by increasing the voting cost. It is *not* the availability of instruments, but rather the observation that we can express $u_A(\mathbf{x}_n) = -u_B(\mathbf{x}_n)$ that identifies the primitives of the model.

5 Specification and Estimation

5.1 Specification

We now specify $b_s(\cdot)$, $c_s(\cdot)$, $p_s(\cdot)$ and the joint distribution of ξ_m and η_m for our estimation. We specify $b_s(\cdot)$, which is the utility difference from having candidates R and D in office, as a function of a state-specific intercept, λ_s , and demographic characteristics, \mathbf{x}_n , consisting of age, race, income, religion, and years of schooling:

$$b_s(\mathbf{x}_n) = \lambda_s + \beta'_b \mathbf{x}_n.$$

The intercept, λ_s , is a parameter that we estimate for each state. It captures the state-level net preference shock for the Republicans that demographic characteristics do not account for. Note that the linear specification for the utility difference can be derived from a spatial voting model in which a voter has quadratic loss and her bliss point is linear in \mathbf{x}_n .²⁴

²⁴To illustrate this point, consider a unidimensional spatial voting model in which candidate D 's ideological position is 0; candidate R 's position is 1; and a voter's bliss point is $\alpha_n = \beta^{bliss} \mathbf{x}_n$. Under the quadratic loss function, the utility from electing candidates D and R are $-\alpha_n^2$ and $-(1 - \alpha_n)^2$, respectively, and the utility difference, $b_s(\mathbf{x}_n)$, is written as $-(1 - \alpha_n)^2 + \alpha_n^2 = 2\beta^{bliss} \mathbf{x}_n - 1$. Thus,

Voting cost, $c(\cdot)$, is also specified as a linear function of \mathbf{x}_n as follows:

$$c(\mathbf{x}_n) = \beta_c[1, \mathbf{x}_n]'$$

We do not specifically model the presence of other elections, such as gubernatorial and congressional elections, because previous studies (e.g., Smith, 2001) find that neither the presence nor the closeness of other elections affects turnout in presidential elections. Ballot initiatives are found not to increase turnout in presidential elections either (Childers and Binder, 2012).

We also do not include weather-related variables in $c(\cdot)$ because there was insufficient variation in precipitation and temperature on the day of the 2004 presidential election to have affected turnout significantly.²⁵ Although $c(\cdot)$ is allowed to be state-specific in the identification, we opt for a relatively simple specification to keep the number of parameters manageable.²⁶

We specify the voter's perception of efficacy as $p_s \times \tilde{p}(\mathbf{x}_n)$, where $\tilde{p}(\cdot)$ is a function of her age, income, and years of schooling, as follows:²⁷

$$\tilde{p}(\mathbf{x}_n) = \exp(\beta'_p \mathbf{x}_n).$$

We normalize $p_s = 1$ for Alabama and normalize $\tilde{p}(\cdot)$ such that $\tilde{p}(\bar{\mathbf{x}}) = 1$, where $\bar{\mathbf{x}}$ is the national average of \mathbf{x}_n .²⁸

We specify the joint distribution of county-level preference shock ξ and cost shock

$b_s(\mathbf{x}_n)$ is linear in \mathbf{x}_n in such a model.

²⁵We included weather variables in the simple model that assumes $\mathbf{x}_n = \bar{\mathbf{x}}_m$ (i.e., the demographic characteristics of voters in each county are assumed to be the same within county) and found the coefficients on the weather variables to be small and insignificant.

²⁶If we include a state-specific cost intercept, this increases the number of parameters by the number of states. Given that some states have only a moderate number of counties, we found it hard to include a state-specific cost term in addition to λ_s and p_s .

²⁷The set of variables included in $p_s(\mathbf{x}_n)$ is a subset of \mathbf{x}_n that takes continuous values. Here, we do not include dummy variables such as race, and religion. The reason is as follows. The variation in $c(\cdot)$ changes the utility level additively, while the variation in $p_s(\cdot)$ changes it multiplicatively as $p_s \times b_s$. As dummy variables take only 0 and 1, it is difficult, in practice, to distinguish whether the effects of those variables are additive or multiplicative. Thus, estimating the model with dummy variables in both cost and efficacy is difficult, and we include only continuous variables in $p_s(\cdot)$.

²⁸Note that we need two normalizations. Because we express $p_s(\mathbf{x}_n)$ as $p_s \times \tilde{p}(\mathbf{x}_n)$, we need a scalar normalization on either p_s or $\tilde{p}(\mathbf{x}_n)$. We normalize $p_s = 1$ for Alabama. We also need an additional normalization because $p_s(\cdot)$ is identified only up to the variance of η —i.e., the level of p_s is not identified in our model. Assuming that $\tilde{p}(\bar{\mathbf{x}}) = 1$ eliminates this degree of freedom.

η as a bivariate normal, $N(0, \Sigma)$, where Σ is the variance-covariance matrix with diagonal elements equal to $\sigma_\xi^2, \sigma_\eta^2$ and off-diagonal elements $\rho\sigma_\xi\sigma_\eta$.

5.2 Estimation

We use the method of moments to estimate the model parameters.²⁹ Recall that the vote shares (as a fraction of eligible voters) and turnout in county m are given by expressions (4), (5) and (6), where $F_{\mathbf{x},m}$ is the distribution of \mathbf{x}_n in county m . For a fixed vector of the model parameters, $\boldsymbol{\theta} = (\boldsymbol{\beta}_b, \{\lambda_s\}, \boldsymbol{\beta}_c, \{p_s\}, \boldsymbol{\beta}_p, \sigma_\xi, \sigma_\eta, \rho)$, we can compute the moments of expressions (4), (5) and (6) by integrating over ξ and η . Our estimation is based on matching the moments generated by the model with the corresponding sample moments.

Specifically, we define the first and second order moments implied by the model as follows:

$$\begin{aligned}\hat{v}_{k,m}(\boldsymbol{\theta}) &= \mathbf{E}_{\xi,\eta}[v_{k,m}(\xi, \eta; \boldsymbol{\theta})], \forall k \in \{D, R\}, \\ \hat{v}_{k,m}^{squared}(\boldsymbol{\theta}) &= \mathbf{E}_{\xi,\eta}[v_{k,m}(\xi, \eta; \boldsymbol{\theta})^2], \forall k \in \{D, R\}, \\ \hat{v}_m^{cross}(\boldsymbol{\theta}) &= \mathbf{E}_{\xi,\eta}[v_{D,m}(\xi, \eta; \boldsymbol{\theta})v_{R,m}(\xi, \eta; \boldsymbol{\theta})],\end{aligned}$$

where $v_{k,m}(\xi, \eta; \boldsymbol{\theta})$ is the vote share of candidate k given a realization of (ξ, η) and parameter $\boldsymbol{\theta}$.^{30,31} Denoting the observed vote share of candidate k in county m as $\mathbf{v}_{k,m}$,

²⁹In contrast to maximum likelihood estimation, which requires us to solve for (ξ, η) that rationalizes the observed vote share for each parameter value, the method of moments only requires integration with respect to ξ and η by simulation. The latter is substantially less costly in terms of computation.

³⁰Computing $\hat{v}_{k,m}(\boldsymbol{\theta})$, $\hat{v}_{k,m}^{squared}(\boldsymbol{\theta})$, and $\hat{v}_m^{cross}(\boldsymbol{\theta})$ requires integration over (ξ, η) . For integration, we use a quadrature with $[5 \times 5]$ nodes and pruning (see Jäckel, 2005) with a total of 21 nodes.

³¹Note that we do not need to know the value of ρ for computing $v_{k,m}(\xi, \eta; \boldsymbol{\theta})$. Thus, for each $\boldsymbol{\theta} \setminus \{\rho\}$ (i.e., the parameters except for ρ), we can compute $v_{k,m}(\xi, \eta; \boldsymbol{\theta})$ and solve for the implied realizations (ξ_m, η_m) that give the observed vote shares in county m . Berry (1994) guarantees that there exists a unique pair of (ξ_m, η_m) that equates $v_{k,m}(\xi, \eta; \boldsymbol{\theta})$ to the observed shares. By computing the correlation between the implied realizations of ξ_m and η_m , we can obtain the value of ρ that is consistent with the observed data. We then impose this value of ρ to compute $\hat{v}_{k,m}(\boldsymbol{\theta})$, $\hat{v}_{k,m}^{squared}(\boldsymbol{\theta})$, and $\hat{v}_m^{cross}(\boldsymbol{\theta})$. Our estimation procedure can be thought of as minimizing the objective function with respect to $\boldsymbol{\theta} \setminus \{\rho\}$, and the estimate of ρ is obtained by computing the correlation between the implied realizations of ξ_m and η_m given the estimate of $\boldsymbol{\theta} \setminus \{\rho\}$.

our objective function, $J(\theta)$, is given by

$$J(\theta) = \sum_{k=\{D,R\}} \left(\frac{J_{1,k}(\theta)}{\widehat{\mathbf{Var}}(\mathbf{v}_{k,m})} + \frac{J_{2,k}(\theta)}{\widehat{\mathbf{Var}}(\mathbf{v}_{k,m}^2)} \right) + \frac{J_3(\theta)}{\widehat{\mathbf{Var}}(\mathbf{v}_{D,m}\mathbf{v}_{R,m})},$$

where

$$\begin{aligned} J_{1,k}(\theta) &= \frac{1}{M} \sum_{s=1}^S \sum_{m=1}^{M_s} (\hat{v}_{k,m}(\theta) - \mathbf{v}_{k,m})^2, \forall k \in \{D, R\}, \\ J_{2,k}(\theta) &= \frac{1}{M} \sum_{s=1}^S \sum_{m=1}^{M_s} (\hat{v}_{k,m}^{squared}(\theta) - \mathbf{v}_{k,m}^2)^2, \forall k \in \{D, R\}, \\ J_3(\theta) &= \frac{1}{M} \sum_{s=1}^S \sum_{m=1}^{M_s} (\hat{v}_m^{cross}(\theta) - \mathbf{v}_{D,m}\mathbf{v}_{R,m})^2. \end{aligned}$$

$J_{1,k}$ is the sum of the squared differences between the expectation of the predicted vote share ($\hat{v}_{k,m}(\theta)$) and the actual vote share ($\mathbf{v}_{k,m}$). $J_{2,k}$ is the sum of the squared differences between $\hat{v}_{k,m}^{squared}(\theta)$ and the squared vote share, $\mathbf{v}_{k,m}^2$. J_3 is the sum of the squared differences between the predicted and actual cross terms. M is the total number of counties across all states, $\sum_{s=1}^S M_s$, and $\widehat{\mathbf{Var}}(z)$ denotes the sample variance of z .

The construction of our objective function follows our identification argument closely. The first moment, $J_{1,k}$, matches the conditional expectation of the vote shares from the model with that from the data. Intuitively, $J_{1,k}$ corresponds to (12) and (14), and pins down β_b , $\{\lambda_s\}$, β_c/p_s , β_c/β_p and $p_s/p_{s'}$. The second and third moments, $J_{2,k}$ and J_3 , correspond to (15). These moments pin down $\beta_p, \sigma_\xi, \sigma_\eta$ and ρ .

6 Results

The set of parameters that we estimate include those that are common across all states ($\beta_b, \sigma_\xi, \beta_c, \sigma_\eta, \beta_p, \rho$) and those that are specific to each state ($\{\lambda_s\}, \{p_s\}$). Table 2 reports the estimates of the former set, while Figures 4 and 5 plot the parameter estimates of the latter set.

	Preference		Cost		Efficacy	
	Estimate	SE	Estimate	SE	Estimate	SE
Age	0.0185	(0.0036)	-0.0086	(0.0091)	0.0648	(0.0157)
Years of Schooling	-0.0788	(0.0160)	-0.2470	(0.0655)	-0.1694	(0.0623)
log(income)	0.3747	(0.0496)	-0.1996	(0.0825)	-0.0534	(0.1728)
Hispanic	-0.2474	(0.0843)	0.0750	(0.0726)		
African American	-1.3392	(0.0592)	0.2799	(0.0807)		
Other Races	-0.7380	(0.0745)	0.2502	(0.0859)		
Religious	0.6616	(0.0577)	0.1442	(0.0440)		
Constant	0.1034	(0.0374)	0.4778	(0.0341)		
Sigma	-0.1978	(0.0086)	0.0097	(0.9045)		
Rho	-0.0840 (0.0521)					

Table 2: Parameter Estimates. The table reports the parameter estimates of voters’ preferences, costs, and perception of voting efficacy. The estimate of the constant terms in the first and second columns corresponds to the preference and costs of the voter who has \mathbf{x}_n equal to the national average and has λ_s equal to that of Alabama. The variable log(income) is the log of income measured in units of 1,000 USD. Excluded categories are non-Hispanic, White, and non-Religious. Standard errors are computed by analytically deriving the asymptotic variance-covariance matrix. The standard errors are reported in parentheses.

Estimates of $\beta_b, \sigma_\xi, \beta_c, \sigma_\eta, \beta_p,$ and ρ The first column of Table 2 reports the estimates of the preference parameters. We find that Age and Income enter the utility difference, $b_R - b_D$, positively, implying that old and high-income voters are more likely to prefer Republicans. Years of Schooling enters the utility difference negatively, thus more-educated voters are more likely to prefer Democrats. We also find that Hispanics, African Americans, and Other Races prefer Democrats. The Religion variable carries a positive coefficient, implying that religious voters prefer Republicans. The estimate of the constant term corresponds to the preference of the voter who has \mathbf{x}_n equal to the national average and has λ_s equal to that of Alabama.

In the second column of Table 2, we report the estimates of the cost parameters. The estimated costs are inclusive of any benefits from fulfilling civic duty. Moreover, our cost estimates include not only physical and opportunity costs but also psychological costs, such as information acquisition costs. We find that Age, Years of Schooling, and Income enter voting cost negatively. This implies that older, more-educated, and

higher-income voters have a lower cost of voting.³² Hispanics, African Americans, and Other Races have a higher cost of voting relative to non-Hispanics and Whites. Religious voters also have a higher cost of voting compared to non-religious voters. The estimate of the constant term in the second column corresponds to the cost of a voter whose characteristics are set to the national mean.

The third column of Table 2 reports the estimates of the efficacy parameters. We find that Age enters the perception of efficacy positively, while Years of Schooling and Income enter negatively. This implies that older, less-educated, and lower-income citizens tend to have a higher perception of efficacy. Given that older voters have lower voting costs as well, they are more likely to be overrepresented than young voters. Regarding Years of Schooling and Income, the overall effect on participation depends on the relative magnitudes of the cost and efficacy coefficients. We discuss the net effect in the next subsection.

The last row of Table 2 reports the estimate of ρ , which is the correlation between unobservable shocks ξ and η . The estimate is negative (-0.084), implying that the correlation in the unobservable shocks tends to suppress turnout among voters who prefer Democrats.

Representation and Preference Aggregation To better understand what our estimates imply about representation across demographic groups and their preferences, Figure 3 plots the estimated share of voters who prefer Democrats over Republicans (right axis) and an estimated measure of voter representation (left axis) by demographic groups. The share of voters who prefer Democrats over Republicans is simply the two-party vote share of the Democrats unconditional on turnout, computed using the preference estimates. The representation measure that we use is based on Wolfinger and Rosenstone (1980), and it is defined as follows:

$$WR(\mathbf{x}) = \frac{\text{share of group } \mathbf{x} \text{ among those who turn out}}{\text{share of group } \mathbf{x} \text{ among the overall electorate}},$$

where \mathbf{x} is a demographic group. Overrepresented demographic groups have representation measures larger than one, while underrepresented groups have measures less

³²Given that high-income and more-educated voters tend to have high opportunity cost, the negative coefficients on income and education might suggest that information acquisition cost can be an important part of the voting cost.

than one.

Figure 3 shows that there are significant differences in the representation and preference measures among demographic groups. For example, panel (a) shows that the representation measure of 75-year-old voters is 1.65, while that of 25-year-old voters is 0.37. The fraction of 75-year-old voters who prefer Democrats is 36.4%, while that of 25-year-old voters is 59.9%.

Figure 3 also illustrates how demographic characteristics are related to preference aggregation. In particular, if the two curves in each panel have the same (opposite) slope, the voters who prefer Republicans (Democrats) are underrepresented. For example, in panels (a), (c), and (d), the overrepresented groups tend to prefer Republicans, while the underrepresented groups tend to prefer Democrats.

Panel (a) of Figure 3 shows that old voters are overrepresented *and* prefer Republicans, while young voters are underrepresented *and* tend to prefer Democrats. Similarly, low-income voters and those who are Hispanic, African American, and of Other Races are underrepresented and tend to prefer Democrats. On the other hand, panel (f) shows that religious voters are underrepresented and prefer Republicans. These results show that there is a systematic selection in the preferences of those who turn out.

Estimates of State-Specific Effects, λ_s and p_s Figure 4 plots our estimates of the state specific intercepts in the voter's utility relative to that of Alabama, which is normalized to 0 (Table 7 in the Online Appendix reports the point estimates and the standard errors). Larger values of λ_s imply that the voters in the corresponding state prefer Republicans, *net of the effect of demographic characteristics*. These state fixed effects may include the inherent preferences of voters and/or the effect of campaign activities by candidates. The figure show that states such as Wyoming, Kansas, and Louisiana have the strongest preference for Republicans, while states such as Arkansas, Tennessee, and Wisconsin have the strongest preference for Democrats. Overall, Democratic strongholds such as New York and California tend to have low estimated values of λ_s , while Republican strongholds such as Georgia and Texas tend to have high estimates.

Figure 5 plots the estimates of the state-specific component of the perception of voting efficacy, p_s , with normalization $p_{Alabama} = 1$ (Table 8 in the Online Appendix reports the point estimates and the standard errors). High values of p_s correspond to

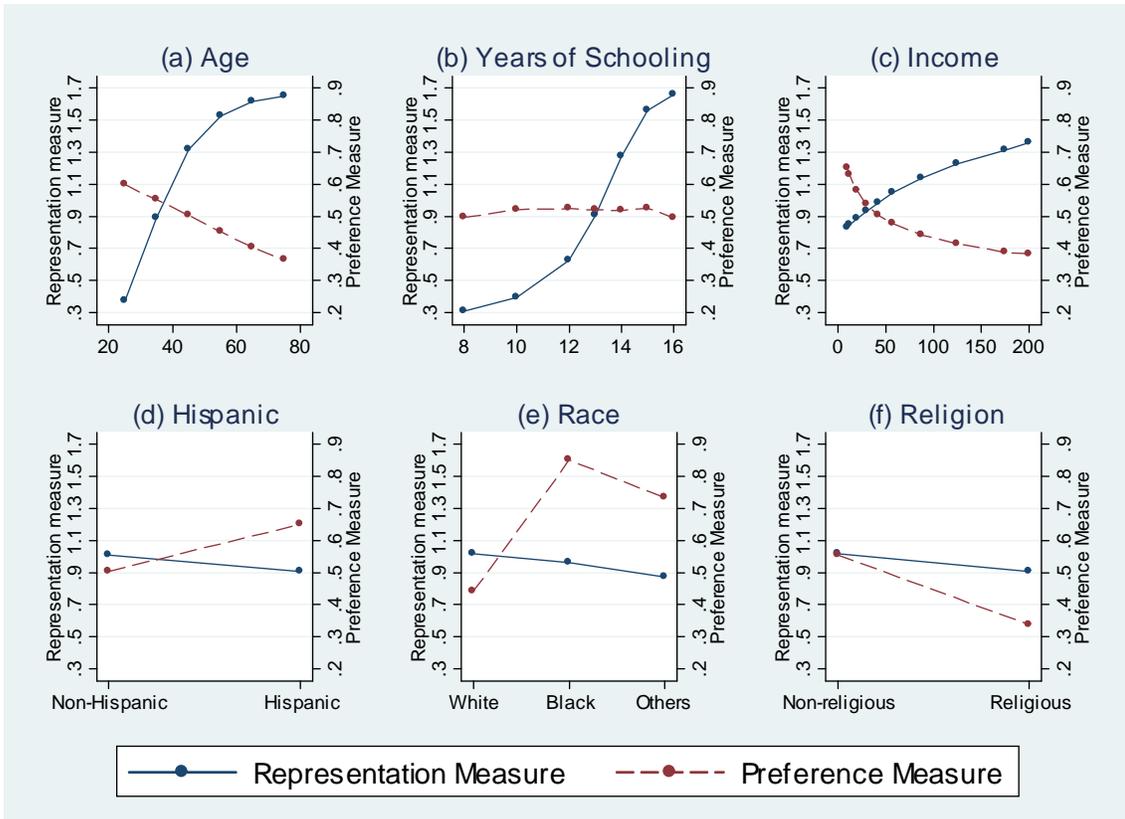


Figure 3: Representation and Preference by Demographic Characteristics. The horizontal axis corresponds to the level/category of the demographic variable. The left vertical axis corresponds to the representation measure, and the right vertical axis corresponds to preference of the group in terms of the two-party vote share. The horizontal axis in Panel (c) is income in units of 1,000 USD.

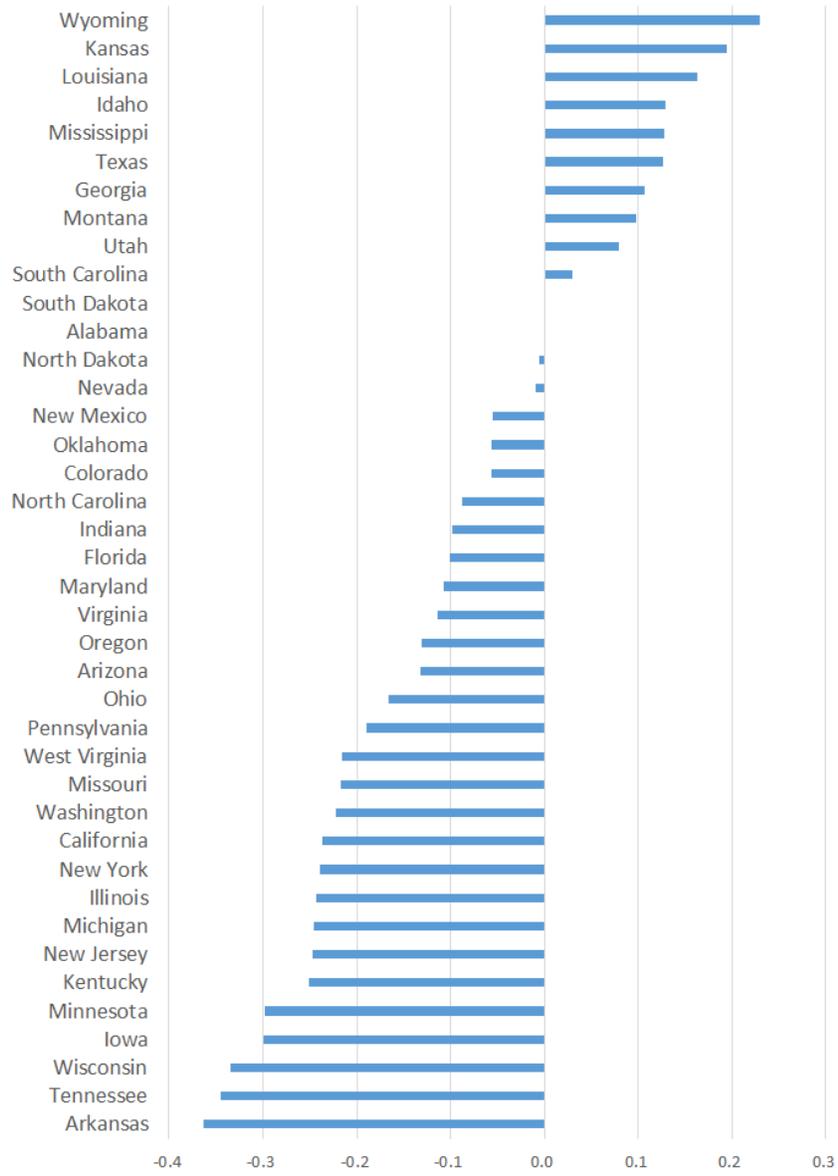


Figure 4: Estimates of the State-level Preference Intercept, λ_s . $\lambda_{Alabama}$ is normalized to zero.

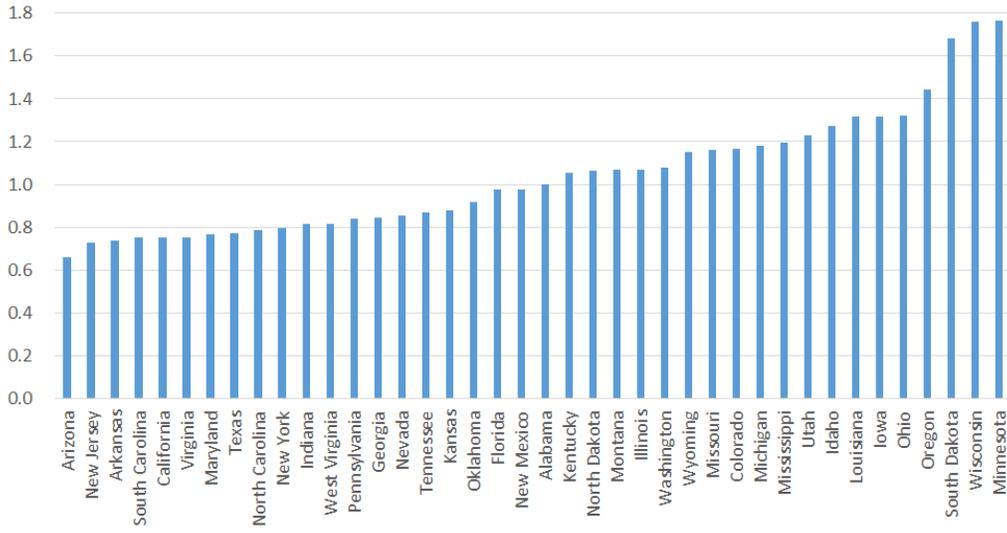


Figure 5: Estimates of the State-level Efficacy Coefficient, p_s . Our estimates of p_s are relative to $p_{Alabama}$ where $p_{Alabama}$ is normalized to 1.

high perception of voting efficacy, after controlling for demographics. The perception of voting efficacy varies across states, which may partly reflect the fact that the electors are determined at the state level. We find that battleground states such as Minnesota, Wisconsin, Ohio and Iowa have some of the highest estimated values of voting efficacy. We also find that states considered party strongholds, such as California and Texas, have low estimated values. These results suggest a positive relationship between perception of voting efficacy and pivot probability.

While some of the forces of the pivotal voter model seem to be at play, the estimated values of p_s suggest that the rational expectations pivotal voter model is unlikely to explain the overall voting pattern very well. Models of voting based on rational expectations would require p_s in battleground states to be orders of magnitude greater than those in party strongholds (see, e.g., Shachar and Nalebuff, 1999). However, our estimates of p_s fall within a narrow range: the ratio of the estimated state-level efficacy parameters, $p_s/p_{s'}$, is, at most, three. Our results highlight the importance of relaxing the assumption of rational expectations on the pivot probability.

Fit To assess the fit of our model, Figure 6 plots the county-level vote share, voter turnout, and vote share margin predicted from the model against the data. The predicted vote share, turnout, and vote share margin are computed by evaluating ex-

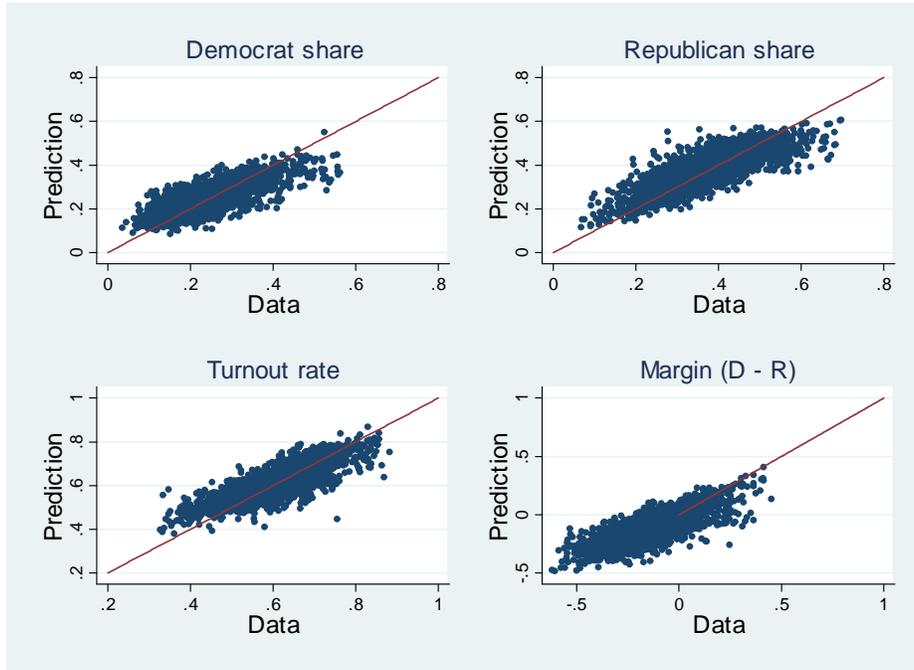


Figure 6: Fit. The figure plots the predicted vote share, turnout, and vote share margin against the data for each county.

pressions (4), (5), and (6) at the estimated parameter values and integrating out ξ and η . The plots line up around the 45-degree line, which suggests that the model fits the data well. In Online Appendix B, we provide further discussion of fit.

In previous work, Coate et al. (2008) discusses the difficulty of fitting the winning margin using the rational expectations pivotal voter model. In our paper, we do not impose the rational expectations assumption, and the model fits the winning margin in the data well.

Turnout and Preference Intensity We now discuss how preference intensity is related to turnout and how this affects preference aggregation. Our discussion is closely related to the theoretical work of Campbell (1999) that shows that minorities with intense preferences can win elections with costly voting (see, also, Casella, 2005; Lalley and Weyl, 2015). Note that our discussion of intensity in this subsection depends on the distributional assumption of idiosyncratic preference error ε_n .

Figure 7 plots the histogram of $b(\mathbf{x}_n) \equiv b_s(\mathbf{x}_n) + \varepsilon_n$ for all eligible voters in the forty states included in our sample (top panel), and the proportion of those who turn

out for given levels of $b(\mathbf{x}_n)$ (bottom panel). The top panel shows that the distribution of the utility difference is roughly centered around zero, and has a slightly fatter tail on the Democrats’ side. The bottom panel shows that turnout is higher among Republican supporters than Democratic supporters at the same level of preference intensity. For example, voters with preferences in the bin $[-2.5, -2)$ turns out with 70.3%, while voters with preference in the bin $[2, 2.5)$ turns out with 87.6%. Overall, we estimate that turnout is significantly lower among the electorate who prefer Democrats over Republicans, at 55.7%, than turnout among those who prefer Republicans over Democrats, at 64.5%.

The panel also shows that there is high turnout among voters with high preference intensity for either party. For example, voters with preferences in the bin $[-2.5, -2)$ are almost twice as likely to turn out as those with preferences in the bin $[-0.5, 0)$ (70.3% compared to 36.5%). This implies that voters with intense preferences effectively have “more votes” than those who are indifferent, as pointed out by Campbell (1999).

7 Counterfactual Experiments

We conduct two counterfactual experiments to quantify the degree to which the correlation among preferences, voting cost, and efficacy affects preference aggregation.

7.1 Preference Aggregation under Compulsory Voting

In our first counterfactual experiment, we consider what the election outcome would be if the preferences of all eligible voters were aggregated. The counterfactual result can be thought of as the election outcome under compulsory voting. The election outcome under this counterfactual can be computed by setting voting cost to zero. That is, individuals vote for Democrats or Republicans depending on the sign of $\widehat{b}_s(\mathbf{x}_n) + \widehat{\xi}_m + \varepsilon_n$, where $\widehat{b}_s(\cdot)$ and $\widehat{\xi}_m$ are the estimates of the net utility difference and county-level preference shock.³³ We can express the counterfactual county-level vote shares, $\tilde{v}_{D,m}$

³³Note that there is a unique value of $(\widehat{\xi}_m, \widehat{\eta}_m)$ that rationalizes the actual vote outcome given our estimates of preference, cost and perception of efficacy, as discussed in footnote 31. We use these values of $(\widehat{\xi}_m, \widehat{\eta}_m)$ to compute our counterfactual outcome.

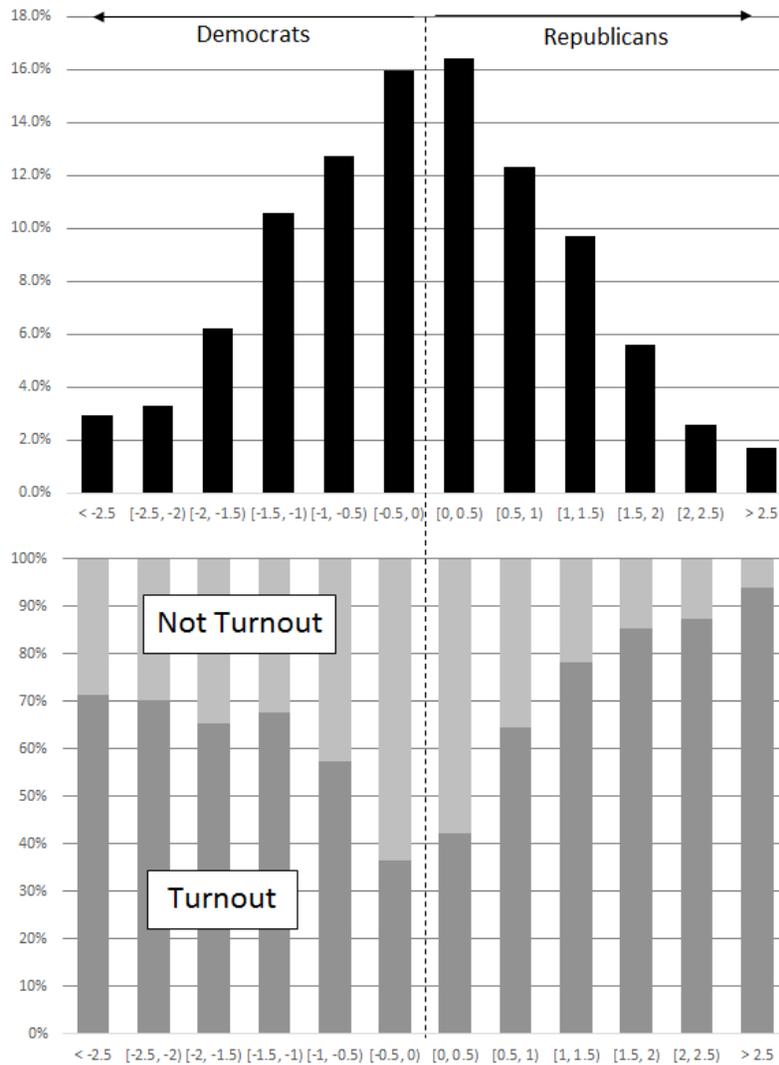


Figure 7: Histogram of Preference Intensity and Turnout Rate by Preference Intensity. The top panel plots the histogram of the utility difference, $b(x_n)$. The bottom panel plots the proportion of those who turn out for given levels of preference intensity.

	Two-Party Vote Share		Turnout Rate	# of Electors	
	Democrats	Republicans		Democrats	Republicans
Actual	48.2%	51.8%	60.1%	208	278
Counterfactual	51.9%	48.1%	100.0%	310	176
	(1.1%)	(1.1%)	n.a.	(30.5)	(30.5)

Table 3: Counterfactual Outcome Under Full Turnout. The table compares the actual outcome with the counterfactual outcome in which all voters turn out. The reported outcomes do not include the results for the eleven states that we drop from the sample. Standard errors are reported in parentheses.

and $\tilde{v}_{R,m}$, as follows:

$$\begin{aligned}\tilde{v}_{D,m} &= \int \Phi\left(-\hat{b}_s(\mathbf{x}_n) - \hat{\xi}_m\right) dF_{\mathbf{x},m}(\mathbf{x}_n), \\ \tilde{v}_{R,m} &= 1 - \tilde{v}_{D,m}.\end{aligned}$$

Note that our counterfactual results are robust to equilibrium adjustments to voters' perception of efficacy because a voter's decision depends only on the sign of the utility difference irrespective of their perception of efficacy.

Table 3 compares the actual outcomes with the counterfactual outcomes for the forty states in our sample. The first row of Table 3 reports the actual vote share, turnout rate, and the number of electors for the two parties. We report our counterfactual results in the second row. We find that the Democratic two-party vote share increases from 48.2% to 51.9% in the counterfactual, reflecting our earlier finding that the preference for Democrats and voting costs are positively correlated.

In terms of electoral votes, we find that the Democrats increase the number of electoral votes by 102, from 208 to 310. Although ten states and the District of Columbia (D.C.) are not included in our sample, 310 electoral votes is larger than the threshold needed to win the election (270) even if the 10 excluded states and D.C. all vote for the Republican electors.³⁴ Hence, our estimates suggest that the Democrats would likely have won the 2004 presidential election if the preferences of all voters had been aggregated. The standard errors in our parameter estimates translate to an 87.7% confidence level that the number of electors for the Democrats exceeds 270. Note that this number

³⁴There are a total of 538 electoral votes, including the states that are excluded from our sample. A candidate needs 270 electoral votes to win.

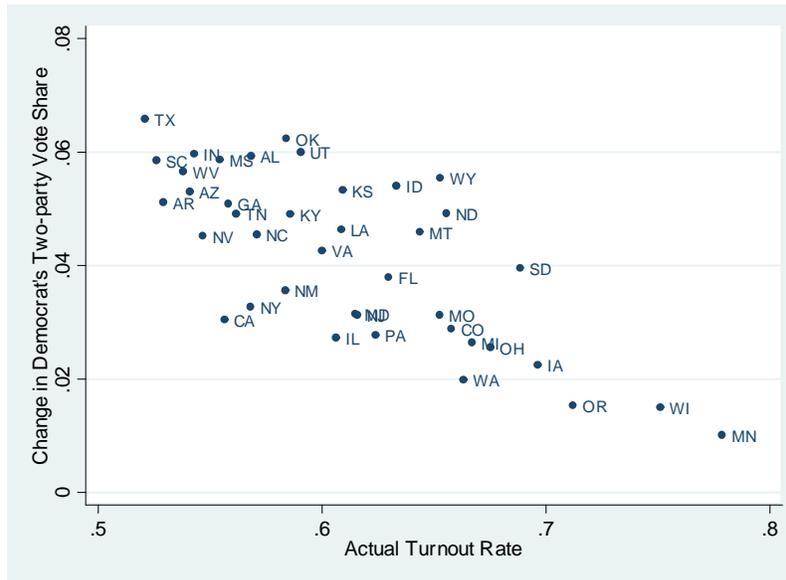


Figure 8: Changes in the Democrat’s Vote Share by State. The figure plots the changes in the Democrat’s two-party vote share between the actual and the counterfactual against the level of actual turnout rate.

is a lower bound on the confidence level that the Democrats win the election because it assumes that all states excluded from our sample vote for the Republicans. If we assume, instead, that all states excluded from our sample vote in the same way as they did in the actual election, the confidence level that the Democrats win is 96.0%.

Table 4 presents the state-level breakdown of the counterfactual results for the forty states in our sample. We find that the two-party vote share of the Democrats increases in the counterfactual in all states, and that the results are overturned in nine states (shaded in the table) in the counterfactual. The table also shows that there is considerable heterogeneity in the magnitude of the change across states. For example, in Texas, we find that the change in the two-party vote share for the Democrats is more than 5 percentage points (from 38.5% to 45.1%), while, in Minnesota, the change is only 1.0 percentage point. An important variable that explains the heterogeneity is the actual turnout. Figure 8 plots the state-level change in the two-party vote share against turnout, and shows that the change tends to be small in states with high voter turnout, while it tends to be large in states with low voter turnout.

	Two-Party Vote Share of Democrats			Turnout Rate	# of Electors for Democrats			Prob(Democrats win in counterfactual)
	Actual	Counterfactual	(1.2%)	Actual	Actual	Counterfactual	(0.0)	
Alabama	37.1%	43.0%	(1.2%)	56.9%	0	0	(0.0)	0.0%
Arizona	44.7%	50.0%	(1.4%)	54.1%	0	10	(5.0)	48.8%
Arkansas	45.1%	50.2%	(1.4%)	53.0%	0	6	(3.0)	52.9%
California	55.0%	58.1%	(1.4%)	55.7%	55	55	(0.0)	100.0%
Colorado	47.6%	50.5%	(0.9%)	65.8%	0	9	(4.3)	65.3%
Florida	47.5%	51.2%	(1.1%)	63.0%	0	27	(8.6)	88.6%
Georgia	41.6%	46.7%	(1.2%)	55.9%	0	0	(0.9)	0.3%
Idaho	30.7%	36.1%	(1.2%)	63.4%	0	0	(0.0)	0.0%
Illinois	55.2%	57.9%	(1.2%)	60.7%	21	21	(0.0)	100.0%
Indiana	39.6%	45.5%	(1.3%)	54.3%	0	0	(0.0)	0.0%
Iowa	49.7%	51.9%	(0.9%)	69.7%	0	7	(0.4)	99.7%
Kansas	37.1%	42.5%	(1.2%)	61.0%	0	0	(0.0)	0.0%
Kentucky	40.0%	44.9%	(1.2%)	58.6%	0	0	(0.0)	0.0%
Louisiana	42.7%	47.3%	(1.0%)	60.9%	0	0	(0.9)	1.0%
Maryland	56.6%	59.7%	(1.1%)	61.5%	10	10	(0.0)	100.0%
Michigan	51.7%	54.4%	(1.0%)	66.7%	17	17	(0.0)	100.0%
Minnesota	51.8%	52.8%	(0.7%)	77.9%	10	10	(0.0)	100.0%
Mississippi	40.1%	45.9%	(1.2%)	55.5%	0	0	(0.0)	0.0%
Missouri	46.4%	49.5%	(0.9%)	65.3%	0	0	(4.7)	24.2%
Montana	39.7%	44.2%	(1.0%)	64.4%	0	0	(0.0)	0.0%
Nevada	48.7%	53.2%	(1.3%)	54.7%	0	5	(0.3)	99.7%
New Jersey	53.4%	56.5%	(1.2%)	61.6%	15	15	(0.0)	100.0%
New Mexico	49.6%	53.2%	(1.2%)	58.4%	0	5	(0.3)	99.7%
New York	59.3%	62.5%	(1.3%)	56.9%	31	31	(0.0)	100.0%
North Carolina	43.8%	48.3%	(1.2%)	57.1%	0	0	(3.8)	6.7%
North Dakota	36.1%	41.0%	(1.0%)	65.6%	0	0	(0.0)	0.0%
Ohio	48.9%	51.5%	(0.9%)	67.6%	0	20	(3.4)	97.0%
Oklahoma	34.4%	40.6%	(1.3%)	58.4%	0	0	(0.0)	0.0%
Oregon	52.1%	53.6%	(0.8%)	71.3%	7	7	(0.0)	100.0%
Pennsylvania	51.3%	54.0%	(1.1%)	62.4%	21	21	(0.0)	100.0%
South Carolina	41.4%	47.2%	(1.3%)	52.6%	0	0	(0.8)	1.0%
South Dakota	39.2%	43.1%	(0.8%)	68.9%	0	0	(0.0)	0.0%
Tennessee	42.8%	47.7%	(1.2%)	56.2%	0	0	(1.8)	2.7%
Texas	38.5%	45.1%	(1.5%)	52.1%	0	0	(0.0)	0.0%
Utah	26.7%	32.6%	(1.5%)	59.1%	0	0	(0.0)	0.0%
Virginia	45.9%	50.1%	(1.1%)	60.1%	0	13	(6.5)	48.5%
Washington	53.6%	55.6%	(1.0%)	66.4%	11	11	(0.0)	100.0%
West Virginia	43.5%	49.2%	(1.4%)	53.8%	0	0	(2.2)	25.3%
Wisconsin	50.2%	51.7%	(0.7%)	75.2%	10	10	(0.0)	100.0%
Wyoming	29.7%	35.2%	(1.1%)	65.3%	0	0	(0.0)	0.0%

Table 4: State-level Simulation Results under Full Turnout. The shaded rows correspond to the states in which the winning party in the counterfactual differs from that in the actual data. The total number of electors is 538 and the number of electors for the states included in our data is 486. 270 electors are needed to win the election. Standard errors are reported in parenthesis.

7.2 Efficiency Gap and Endogenous Turnout

In our second counterfactual experiment, we study the implications of endogenous turnout for using the efficiency gap as a measure of gerrymandering. Our counterfactual is motivated by a recent U.S. Supreme Court case involving districting for Wisconsin’s state legislature (*Gill v. Whitford*) in which the plaintiffs introduce a metric called the “efficiency gap” to measure the extent of gerrymandering.

The efficiency gap, proposed by Stephanopoulos and McGhee (2015) is a measure of how well vote shares map into seat shares, and it is defined as the difference in the wasted votes between the parties divided by the total votes. The top half of Table 5 is a numerical example from Stephanopoulos and McGhee (2015) that illustrates how the efficiency gap is computed. The wasted vote for party k in district d is either $v_{kd} - (1/2)(v_{kd} + v_{-kd})$ or v_{kd} , depending on whether v_{kd} is greater than v_{-kd} , where v_{kd} is the votes obtained by party k in district d and v_{-kd} is the votes obtained by the other party. The efficiency gap is simply the sum of the difference in the wasted votes across districts, divided by the total votes. Stephanopoulos and McGhee (2015) argue that an efficiency gap exceeding 8% should be deemed presumptively unlawful. Table 5 is an example of a districting plan that favors party A (party A wins districts 1 through 8 with a state-wide vote share of 55%), and indeed, this is reflected in the high efficiency gap of 20% for party A.

While the efficiency gap captures the imbalance in the way votes are translated to seat shares, the validity of the efficiency gap as a measure of gerrymandering requires that turnout is exogenous. To see this, consider again the example in Table 5, but suppose now that half of the voters are low cost types, who always vote, and the other half are high cost types, who vote only when the partisanship of the district is relatively even. The bottom part of Table 5 illustrates a redistricting plan in which the shares of A and B supporters are kept the same as in the original Stephanopoulos and McGhee (2015)’s example, but voters now have heterogeneous costs. Suppose that the high cost voters turn out only in districts 4 to 8, which are the districts with a relatively even partisanship. This would lower the efficiency gap from 20% to 4.87% while keeping the identity of the winning party in each district unchanged (i.e., party A wins districts 1 through 8).

This example illustrates the potential issue with using the efficiency gap as a measure of gerrymandering. The original example of Stephanopoulos and McGhee (2015)

District	1	2	3	4	5	6	7	8	9	10	Total	
Stephanopolous and McGhee (2015)'s example												
Total Population	100	100	100	100	100	100	100	100	100	100	1000	
A Supporter	70	70	70	54	54	54	54	54	35	35	550	
B Supporter	30	30	30	46	46	46	46	46	65	65	450	
A Wasted Vote	20	20	20	4	4	4	4	4	35	35	150	Efficiency Gap
B Wasted Vote	30	30	30	46	46	46	46	46	15	15	350	20.00%
Endogenous Turnout												
Low Cost A Supporter	70	70	70	0	0	0	0	0	33	32	275	
High Cost A Supporter	0	0	0	54	54	54	54	54	2	3	275	
Low Cost B Supproter	0	0	0	46	43	23	23	23	34	33	225	
High Cost B Supporter	30	30	30	0	3	23	23	23	31	32	225	
A Vote	70	70	70	54	54	54	54	54	33	32	545	
B Vote	0	0	0	46	46	46	46	46	34	33	297	
Turnout	70	70	70	100	100	100	100	100	67	65	842	
A Wasted Vote	35	35	35	4	4	4	4	4	33	32	190	Efficiency Gap
B Wasted Vote	0	0	0	46	46	46	46	46	0.5	0.5	231	4.87%

Table 5: Calculation of the Efficiency Gap. This example is taken from Figure 1 of Stephanopolous and McGhee (2015). The top half is the original example. The bottom half is our modification with two cost types and endogenous turnout.

and the modified example are just as biased for party A in terms of mapping the overall preferences of the voters to seat shares, and yet they result in very different efficiency gap measures. Moreover, districting planners can take advantage of the heterogeneity in voting costs to draw plans that give one party a disproportionate advantage while keeping the efficiency gap low. Whether the efficiency gap is a good measure of gerrymandering depends on the extent to which turnout levels can be manipulated through districting plans.

In order to empirically evaluate the robustness of the efficiency gap to endogenous turnout, we compute the efficiency gap for the 2004 U.S. Presidential election when we equalize the state-specific component of efficacy across states (set $p_s = p_{s'}$). To the extent that variation in p_s across states reflects how state boundaries affect turnout, equalizing p_s across states can be interpreted as eliminating endogeneity in turnout that is state specific. Although the intended use of the efficiency gap measure is mainly for Congressional and state legislative elections, it is possible to compute the efficiency gap for presidential elections as well. If the efficiency gap is sensitive to turnout with respect to state boundaries which are not subject to gerrymandering, it may be more sensitive in settings in which boundaries are subject to manipulation.

	Turnout rate	Efficiency Gap
Actual	60.1%	-0.47%
Counterfactual with various turnout rate	40%	-9.35%
	50%	-7.21%
	60%	-2.61%
	60.1%	-2.64%
	70%	-3.38%
	80%	6.89%

Table 6: Election Outcomes and Efficiency Gap when p_s is Equalized across States. Efficiency gap is defined for Democrat candidate. Positive (negative) efficiency gap means that the districting plan gives Democrats (Republicans) an advantage.

Table 6 reports the efficiency gap computed using the actual data (Row 1) and that computed when p is equalized across states (Rows 2 through 7). We consider six different levels of p to target aggregate turnout levels between 40% and 80% in 10% increments and the actual aggregate turnout level (60.1%). We compute the efficiency gap by counting the wasted votes in each state for the two parties and then summing the difference across all of the states. Comparing the first row to the 5th row (60.1%), we find that the efficiency gap changes by 2.2%. This implies that equalizing p_s across states in a way that does not change the overall turnout affects the efficiency gap by 2.2%. The table also shows that the efficiency gap changes by about 14% when turnout is exogenously increased from 50% to 80%. These changes in the efficiency measure seem significant especially in light of the fact that Stephanopoulos and McGhee (2015) argue that districting plans exceeding the threshold value of 8% should be deemed presumptively unlawful.

The sensitivity of the efficiency gap to endogeneity of turnout points to a more general problem with comparing actual votes and seat shares as a measure of gerrymandering. While our discussion has so far focused on the efficiency gap, any measure based on a comparison between actual votes and seat shares is subject to the same concerns. This includes the concept of partisan symmetry, which a majority of the Supreme court justices expressed support for in the case *League of United Latin American Citizens v Perry*. Much of the extant arguments take turnout as fixed and exogenous. Our results illustrate the importance of considering the implications of endogenous turnout when thinking about how to measure gerrymandering.

One natural alternative is to compare the difference between the actual underlying voter preferences and seat shares. If we think about elections as a way to aggregate preferences into outcomes, evaluating the electoral system in terms of its ability to aggregate preferences seems most coherent. The methods developed in this paper can be used for that purpose.

References

- Akerlof, George A and William T Dickens**, “The Economic Consequences of Cognitive Dissonance,” *American Economic Review*, 1982, pp. 307–319.
- Atkeson, Lonna Rae**, ““Sure, I Voted for the Winner!” Overreport of the Primary Vote for the Party Nominee in the National Election Studies,” *Political Behavior*, 1999, 21 (3), 197–215.
- Barkume, Anthony J**, “Identification of Preference for Election Outcomes from Aggregate Voting Data,” *Public Choice*, 1976, 27 (1), 41–58.
- Bendor, Jonathan, Daniel Diermeier, and Michael Ting**, “A Behavioral Model of Turnout,” *American Political Science Review*, 2003, 97 (02), 261–280.
- Berry, Steven T**, “Estimating Discrete-Choice Models of Product Differentiation,” *RAND Journal of Economics*, 1994, 25 (2), 242–262.
- Best, Robin E., Shawn J. Donahue, Jonathan Krasno, Daniel B. Magleby, and Michael D. McDonald**, “Considering the prospects for establishing a packing gerrymandering standard,” *Election Law Journal*, 2018, 17 (1), 1–20.
- Brunell, Thomas L and John DiNardo**, “A Propensity Score Reweighting Approach to Estimating the Partisan Effects of Full Turnout in American Presidential Elections,” *Political Analysis*, 2004, 12 (1), 28–45.
- Brunnermeier, Markus K and Jonathan A Parker**, “Optimal Expectations,” *American Economic Review*, 2005, 95 (4).
- Campbell, Colin M**, “Large Electorates and Decisive Minorities,” *Journal of Political Economy*, 1999, 107 (6), 1199–1217.
- Cascio, Elizabeth U and Ebonya Washington**, “Valuing the Vote: The Redistribution of Voting Rights and State Funds Following the Voting Rights Act of 1965,” *Quarterly Journal of Economics*, 2013.
- Casella, Alessandra**, “Storable Votes,” *Games and Economic Behavior*, 2005, 51 (2), 391–419.

- Childers, Matt and Mike Binder**, “Engaged by the initiative? How the use of citizen initiatives increases voter turnout,” *Political Research Quarterly*, 2012, 65 (1), 93–103.
- Citrin, Jack, Eric Schickler, and John Sides**, “What if Everyone Voted? Simulating the Impact of Increased Turnout in Senate Elections,” *American Journal of Political Science*, 2003, 47 (1), 75–90.
- Coate, Stephen and Brian Knight**, “Socially optimal districting: a theoretical and empirical exploration,” *The Quarterly Journal of Economics*, 2007, 122 (4), 1409–1471.
- and **Michael Conlin**, “A Group Rule-Utilitarian Approach to Voter Turnout: Theory and Evidence,” *American Economic Review*, 2004, pp. 1476–1504.
- , —, and **Andrea Moro**, “The Performance of Pivotal-Voter Models in Small-Scale Elections: Evidence from Texas Liquor Referenda,” *Journal of Public Economics*, 2008, 92 (3), 582–596.
- Degan, Arianna and Antonio Merlo**, “A Structural Model of Turnout and Voting in Multiple Elections,” *Journal of the European Economic Association*, 2011, 9 (2), 209–245.
- DellaVigna, Stefano, John A List, Ulrike Malmendier, and Gautam Rao**, “Voting to Tell Others,” *Working Paper*, 2015.
- DeNardo, James**, “Turnout and the Vote: The Joke’s on the Democrats,” *American Political Science Review*, 1980, 74 (02), 406–420.
- Dhillon, Amrita and Susana Peralta**, “Economic Theories of Voter Turnout,” *Economic Journal*, 2002, 112 (480), 332–352.
- Downs, Anthony**, *An Economic Theory of Democracy*, New York, 1957.
- Duffy, John and Margit Tavits**, “Beliefs and Voting Decisions: A Test of the Pivotal Voter Model,” *American Journal of Political Science*, 2008, 52 (3), 603–618.
- Esponda, Ignacio and Demian Pouzo**, “Conditional Retrospective Voting in Large Elections,” *American Economic Journal: Microeconomics*, *Forthcoming*, 2016.

- Feddersen, Timothy**, “Rational Choice Theory and the Paradox of Not Voting,” *Journal of Economic Perspectives*, 2004, pp. 99–112.
- **and Alvaro Sandroni**, “A Theory of Participation in Elections,” *American Economic Review*, 2006, pp. 1271–1282.
- **and Wolfgang Pesendorfer**, “Voting behavior and information aggregation in elections with private information,” *Econometrica*, 1997, pp. 1029–1058.
- Feddersen, Timothy J and Wolfgang Pesendorfer**, “The swing voter’s curse,” *American economic review*, 1996, pp. 408–424.
- Friedman, John N and Richard T Holden**, “Optimal gerrymandering: sometimes pack, but never crack,” *American Economic Review*, 2008, 98 (1), 113–44.
- Fujiwara, Thomas**, “Voting Technology, Political Responsiveness, and Infant Health: Evidence from Brazil,” *Econometrica*, 2015, 83 (2), 423–464.
- Gordon, Brett R and Wesley R Hartmann**, “Advertising Effects in Presidential Elections,” *Marketing Science*, 2013, 32 (1), 19–35.
- Grofman, Bernard and Gary King**, “The future of partisan symmetry as a judicial test for partisan gerrymandering after LULAC v. Perry,” *Election Law Journal*, 2007, 6 (1), 2–35.
- Haile, Philip A and Elie Tamer**, “Inference with an Incomplete Model of English Auctions,” *Journal of Political Economy*, 2003, 111 (1), 1–51.
- Hansford, Thomas G and Brad T Gomez**, “Estimating the Electoral Effects of Voter Turnout,” *American Political Science Review*, 2010, 104 (02), 268–288.
- Harsanyi, John C**, “Rule Utilitarianism, Rights, Obligations and the Theory of Rational Behavior,” *Theory and Decision*, 1980, 12 (2), 115–133.
- Hotz, V Joseph and Robert A Miller**, “Conditional Choice Probabilities and the Estimation of Dynamic Models,” *Review of Economic Studies*, 1993, 60 (3), 497–529.

- Husted, Thomas A and Lawrence W Kenny**, “The Effect of the Expansion of the Voting Franchise on the Size of Government,” *Journal of Political Economy*, 1997, pp. 54–82.
- Jäckel, Peter**, “A Note on Multivariate Gauss-Hermite Quadrature,” Technical Report 2005. Sourced from: www.pjaeckel.webspace.virginia-media.com/ANoteOnMultivariateGaussHermiteQuadrature.pdf.
- Kanazawa, Satoshi**, “A Possible Solution to the Paradox of Voter Turnout,” *Journal of Politics*, 1998, 60 (04), 974–995.
- Karp, Jeffrey A and Susan A Banducci**, “Political Efficacy and Participation in Twenty-Seven Democracies: How Electoral Systems Shape Political Behaviour,” *British Journal of Political Science*, 2008, 38 (02), 311–334.
- Kroth, Verena, Valentino Larcinese, and Joachim Wehner**, “A Better Life For All? Democratization and Electrification in Post-Apartheid South Africa,” Technical Report 2013.
- Lalley, P Steven and E Glen Weyl**, “Quadratic Voting,” *Working Paper*, 2015.
- Ledyard, John O**, “The Pure Theory of Large Two-Candidate Elections,” *Public Choice*, 1984, 44 (1), 7–41.
- Leighley, Jan E and Jonathan Nagler**, *Who Votes Now?: Demographics, Issues, Inequality, and Turnout in the United States*, Princeton University Press, 2013.
- Levine, David K and Thomas R Palfrey**, “The Paradox of Voter Participation? A Laboratory Study,” *American Political Science Review*, 2007, 101 (01), 143–158.
- Lijphart, Arend**, “Unequal Participation: Democracy’s Unresolved Dilemma Presidential Address,” *American Political Science Review*, 1997, 91 (01), 1–14.
- Martinez, Michael D and Jeff Gill**, “The Effects of Turnout on Partisan Outcomes in US Presidential Elections 1960–2000,” *Journal of Politics*, 2005, 67 (4), 1248–1274.
- Matsusaka, John G**, “Explaining voter turnout patterns: An information theory,” *Public choice*, 1995, 84 (1), 91–117.

- McDonald, Michael P.**, “1980-2014 November General Election,” 2016. accessed May 26th, 2016.
- McDonald, Michael P and Samuel L Popkin**, “The Myth of the Vanishing Voter,” *American Political Science Review*, 2001, 95 (4), 963–974.
- Merlo, A and Aureo de Paula**, “Identification and Estimation of Preference Distributions When Voters Are Ideological,” *Working Paper*, 2016.
- Merlo, Antonio**, “Whither Political Economy? Theories, Facts and Issues,” in Whitney Newey Richard Blundell and Torsten Persson, eds., *Advances in Economics and Econometrics, Theory and Applications: Ninth World Congress of the Econometric Society*, Vol. I, Cambridge: Cambridge University Press, 2006, pp. 381–421.
- Miller, Grant**, “Women’s Suffrage, Political Responsiveness, and Child Survival in American History,” *The Quarterly Journal of Economics*, 2008, 123 (3), 1287.
- Minozzi, William**, “Endogenous Beliefs in Models of Politics,” *American Journal of Political Science*, 2013, 57 (3), 566–581.
- Myatt, David**, “A Rational Choice Theory of Voter Turnout,” Technical Report 2012.
- Myerson, Roger B and Robert J Weber**, “A Theory of Voting Equilibria.,” *American Political Science Review*, 1993, 87 (01), 102–114.
- Nagle, John F**, “Measures of partisan bias for legislating fair elections,” *Election Law Journal*, 2015, 14 (4), 346–360.
- Palfrey, Thomas R and Howard Rosenthal**, “A Strategic Calculus of Voting,” *Public Choice*, 1983, 41 (1), 7–53.
- **and** —, “Voter Participation and Strategic Uncertainty,” *American Political Science Review*, 1985, 79 (01), 62–78.
- Riker, William H and Peter C Ordeshook**, “A Theory of the Calculus of Voting,” *American Political Science Review*, 1968, 62 (01), 25–42.
- Shachar, Ron and Barry Nalebuff**, “Follow the Leader: Theory and Evidence on Political Participation,” *American Economic Review*, 1999, pp. 525–547.

Smith, Mark A, “The Contingent Effects of Ballot Initiatives and Candidate Races on Turnout,” *American Journal of Political Science*, 2001, pp. 700–706.

Stephanopoulos, Nicholas O and Eric M McGhee, “Partisan gerrymandering and the efficiency gap,” *The University of Chicago Law Review*, 2015, pp. 831–900.

Strömberg, David, “How the Electoral College Influences Campaigns and Policy: the Probability of Being Florida,” *American Economic Review*, 2008, 98 (3), 769–807.

Tucker, Harvey J and James DeNardo, “Does Heavy Turnout Help Democrats in Presidential Elections?,” *American Political Science Review*, 1986, 80 (04), 1291–1304.

Wolfinger, Raymond E and Steven J Rosenstone, *Who votes?*, Vol. 22, Yale University Press, 1980.

8 Appendix

8.1 Appendix A: Derivation of the Calculus of Voting Model

In this Appendix, we follow Riker and Ordeshook (1968) and present a derivation of expressions (1) and (2). We classify the situation of a voter into the following five mutually exclusive events:

E_0 : votes for D and R are tied without her vote;

E_{D1} : D has exactly one more vote than R without her vote;

E_{R1} : R has exactly one more vote than D without her vote;

E_{D2} : D has two or more votes than R without her vote;

E_{R2} : R has two or more votes than D without her vote.

Let q_l denote the probability of E_l for $l \in \{0, D1, R1, D2, R2\}$. Let π be the probability that D wins the election in case of a tie. Then, the utility of the voter for voting for candidates D and R , as well as not voting, are written as

$$U_D = q_0 b_D + q_{D1} b_D + q_{R1} (\pi b_D + (1 - \pi) b_R) + q_{D2} b_D + q_{R2} b_R - c,$$

$$U_R = q_0 b_R + q_{D1} (\pi b_D + (1 - \pi) b_R) + q_{R1} b_R + q_{D2} b_D + q_{R2} b_R - c,$$

$$U_0 = q_0 (\pi b_D + (1 - \pi) b_R) + (q_{D1} + q_{D2}) b_D + (q_{R1} + q_{R2}) b_R.$$

By taking the difference between voting for D and not voting, we have

$$\begin{aligned} U_D - U_0 &= q_0 b_D + q_{D1} b_D + q_{R1} (\pi b_D + (1 - \pi) b_R) + q_{D2} b_D + q_{R2} b_R - c \\ &\quad - q_0 (\pi b_D + (1 - \pi) b_R) - (q_{D1} + q_{D2}) b_D - (q_{R1} + q_{R2}) b_R \\ &= (q_0 (1 - \pi) + q_{R1} \pi) b_D - (q_0 (1 - \pi) + q_{R1} \pi) b_R - c \\ &= (q_0 (1 - \pi) + q_{R1} \pi) (b_D - b_R) - c. \end{aligned}$$

Similarly, we have

$$\begin{aligned} U_R - U_0 &= q_0 b_R + q_{R1} b_R + q_{D1} (\pi b_D + (1 - \pi) b_R) + q_{R2} b_R + q_{D2} b_D - c \\ &\quad - q_0 (\pi b_D + (1 - \pi) b_R) - (q_{R1} + q_{R2}) b_R - (q_{D1} + q_{D2}) b_D \\ &= (q_{D1} (1 - \pi) + q_0 \pi) b_R + (-q_{D1} (1 - \pi) - q_0 \pi) b_D \\ &= (q_{D1} (1 - \pi) + q_0 \pi) (b_R - b_D) - c. \end{aligned}$$

Assuming that $q_0 = q_{D1} = q_{R1} \equiv p$ in a large election (see page 103 of Myerson and Weber, 1993, for a justification), we can rewrite U_D , U_R , and U_0 as

$$\begin{aligned} U_D &= p(b_D - b_R) - c \\ U_R &= p(b_R - b_D) - c \\ U_0 &= 0. \end{aligned}$$

8.2 Appendix B: Data Construction

In this Appendix, we explain how we construct the joint distribution of demographic characteristics and citizenship status at the county level. We first use the 5% Public Use Microdata Sample of the 2000 U.S. Census (hereafter PUMS), which is an individual-level dataset, to estimate the covariance matrix between the demographic variables and citizenship information within each public use microdata area (PUMA). In particular, we estimate the joint distribution of the discrete demographic characteristics (Race, Hispanic, Citizenship) by counting the frequency of occurrence. We then estimate a covariance matrix for the continuous demographic variables (Age, Income, Years of Schooling) for each bin. Because the PUMA and counties do not necessarily coincide, we estimate covariance matrices for each PUMA and then use the correspondence chart provided in the PUMS website to obtain estimates at the county level.

In the second step, we construct the joint distribution of demographic characteristics by combining the covariance matrix estimated in the first step and the marginal distributions of each of the demographic variables at the county level obtained from Census Summary File 1 through File 3. We discretize continuous variables into coarse bins. We discretize age into three bins: (1) 18-34 years old; (2) 35-59 years old; and (3) above 60 years old; income into 6 bins: (1) \$0-\$25,000; (2) \$25,000-\$50,000; (3) \$50,000-\$75,000; (4) \$75,000-\$100,000; (5) \$100,000-\$150,000; and (6) above \$150,000; and years of schooling into 5 bins: (1) Less than 9th grade; (2) 9th-12th grade with no diploma; (3) high school graduate; (4) some college with no degree or associate degree; and (5) bachelor's degree or higher. Thus, there are 540 bins in total. The joint distribution of demographic characteristics that we create gives us a probability mass over each of the 540 bins for each county.

Finally, we augment the census data with religion data obtained from Religious Congregations and Membership Study 2000. These data contain information on the

share of the population with adherence to either “Evangelical Denominations” or “Church of Jesus Christ of Latter-day Saints” at the county level. Because the Census does not collect information on religion, we do not know the correlation between the religion variable and the demographic characteristics in the Census. Thus, we assume independence of the religion variable and other demographic variables. As a result, there are 1,080 bins in our demographics distribution.

8.3 Appendix C: Identification of $c(\cdot)$, $p(\cdot)$, and F_η in the general case

In this Appendix, we show that $c(\cdot)$, $p(\cdot)$, and F_η are identified even when the max operator in equation (11) binds with positive probability. Note that our argument in the main text considered only the case in which the max operator never binds. Recall that

$$\underbrace{\frac{\Phi^{-1}(1 - v_{R,m}) - \Phi^{-1}(v_{D,m})}{2}}_{\equiv Y_m} = \max \left\{ 0, \frac{c(\bar{\mathbf{x}}_m)}{p(\bar{\mathbf{x}}_m)} + \frac{\eta_m}{p(\bar{\mathbf{x}}_m)} \right\}, \eta_m \perp \bar{\mathbf{x}}_m. \quad (16)$$

In this Appendix, we work with the normalization that the value of $p(\cdot)$ at some $\bar{\mathbf{x}}_m = \mathbf{x}_0$ as $p(\mathbf{x}_0) = 1$. This amounts to a particular normalization of variance of η . Note that the distribution of Y_m (the left hand side of equation (16)) conditional on $\bar{\mathbf{x}}_m = \mathbf{x}_0$ is a truncated distribution with mass at zero. Figure 9 illustrates this when the mass at zero is less than 50%, and F_η is symmetric and single-peaked at zero.

First, we present our identification discussion for the case that F_η is symmetric and single-peaked at zero. As Figure 9 illustrates, the median of Y_m conditional on $\bar{\mathbf{x}}_m = \mathbf{x}_0$ directly identifies $c(\mathbf{x}_0)$ under these assumptions. Also, the density of η , f_η , is identified above the point of truncation. Formally, $f_\eta(F_\eta^{-1}(t))$ is identified for any $t > t(\mathbf{x}_0)$, where

$$t(\mathbf{x}_0) = \Pr(Y_m = 0 | \mathbf{x}_0).$$

Hence, $f_\eta(0)$ is identified from the height of the density of Y_m at the median.

Now, consider $\mathbf{x}_1 \neq \mathbf{x}_0$. Assume, again, that $t(\mathbf{x}_1) < 0.5$. Then, $c(\mathbf{x}_1)/p(\mathbf{x}_1)$ is identified from the conditional median of Y_m , and $p(\mathbf{x}_1)f_\eta(0)$ is identified by the height of the conditional density of Y_m at the median. Given that $f_\eta(0)$ is identified, $c(\mathbf{x}_1)$ and $p(\mathbf{x}_1)$ are both identified. Moreover, F_η is identified over its full support if

there exists sufficient variation in \mathbf{x} , i.e., $\inf_{\mathbf{x}} t(\mathbf{x}) = 0$.

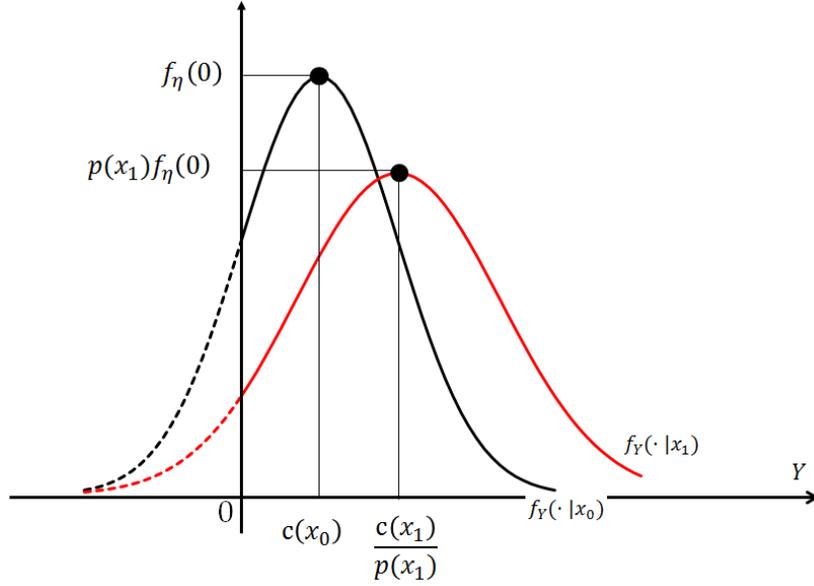


Figure 9: The distribution of Y_m conditional on $\mathbf{x} = \mathbf{x}_0$ and $\mathbf{x} = \mathbf{x}_1$ when the distribution of η is symmetric and single-peaked, and $t(\mathbf{x}_0), t(\mathbf{x}_1) < 0.5$, where $t(\mathbf{x})$ is the probability that Y_m is equal to zero conditional on \mathbf{x} . Note that the distribution of Y_m is truncated at zero. The conditional median of Y_m identifies $c(\mathbf{x}_0)$ and $c(\mathbf{x}_1)/p(\mathbf{x}_1)$, and the height of the density at the conditional median identifies $f_\eta(0)$ and $p(\mathbf{x}_1)f_\eta(0)$.

We now consider the case in which F_η is not restricted to being symmetric and single-peaked and $t(\mathbf{x}_0)$ may be less than 0.5. The distribution of Y_m is identified above $t(\mathbf{x}_0)$, as before. Now, consider $\mathbf{x}_1 \neq \mathbf{x}_0$. Similar to before, we identify $p(\mathbf{x}_1)f_\eta(F_\eta^{-1}(\tau))$ for τ above $t(\mathbf{x}_1)$.³⁵ If we let τ be any number larger than $\max\{t(\mathbf{x}_0), t(\mathbf{x}_1)\}$, both $f_\eta(F_\eta^{-1}(\tau))$ and $p(\mathbf{x}_1)f_\eta(F_\eta^{-1}(\tau))$ are identified. Hence, $p(\mathbf{x}_1)$ is identified. Similarly, $p(\cdot)$ is identified for all \mathbf{x} .

We now consider identification of $c(\cdot)$. We present two alternative assumptions on F_η and show that $c(\cdot)$ can be identified under either assumption. First, assume that the median of η is zero, $Med(\eta) = 0$, and that there exists $\mathbf{x} = \mathbf{x}_2$ such that $t(\mathbf{x}_2) < 1/2$. The latter assumption means that more than half of the counties have turnout less than 100% when $\mathbf{x} = \mathbf{x}_2$. Then, the median of Y_m conditional on \mathbf{x}_2 identifies $c(\mathbf{x}_2)/p(\mathbf{x}_2)$.

³⁵Note that we identify $f_{\eta/p(\mathbf{x}_1)}\left(F_{\eta/p(\mathbf{x}_1)}^{-1}(t)\right)$, where $f_{\eta/p(\mathbf{x}_1)}(\cdot)$ and $F_{\eta/p(\mathbf{x}_1)}^{-1}(\cdot)$ are the density of $\eta/p(\mathbf{x}_1)$ and the inverse distribution of $\eta/p(\mathbf{x}_1)$, respectively. Note that $f_{\eta/p(\mathbf{x}_1)}\left(F_{\eta/p(\mathbf{x}_1)}^{-1}(t)\right) = p(\mathbf{x}_1)f_\eta(F_\eta^{-1}(t))$.

Now, consider any $\mathbf{x}_1 \neq \mathbf{x}_2$ and let τ be any number larger than $\max\{t(\mathbf{x}_2), t(\mathbf{x}_1)\}$. Let z_1 and z_2 be the τ quantile of Y_m conditional on \mathbf{x}_1 and \mathbf{x}_2 , respectively. z_1 and z_2 are clearly identified. Then,

$$\begin{aligned}
p(\mathbf{x}_1) \left[F_{\eta/p(\mathbf{x}_1)}^{-1}(\tau) - F_{\eta/p(\mathbf{x}_1)}^{-1}(1/2) \right] &= p(\mathbf{x}_2) \left[F_{\eta/p(\mathbf{x}_2)}^{-1}(\tau) - F_{\eta/p(\mathbf{x}_2)}^{-1}(1/2) \right] \\
\Leftrightarrow p(\mathbf{x}_1) \left[z_1 - \frac{c(\mathbf{x}_1)}{p(\mathbf{x}_1)} \right] &= p(\mathbf{x}_2) \left[z_2 - \frac{c(\mathbf{x}_2)}{p(\mathbf{x}_2)} \right] \\
\Leftrightarrow \frac{c(\mathbf{x}_1)}{p(\mathbf{x}_1)} &= z_1 - \frac{p(\mathbf{x}_1)}{p(\mathbf{x}_2)} \left(z_2 - \frac{c(\mathbf{x}_2)}{p(\mathbf{x}_2)} \right). \tag{17}
\end{aligned}$$

Given that all of the terms on the right hand side of (17) are identified, $c(\mathbf{x}_1)/p(\mathbf{x}_1)$ is identified.

Alternatively, assume that $\mathbf{E}(\eta) = 0$ and $\inf_{\mathbf{x}} t(\mathbf{x}) = 0$. We now show that $c(\cdot)$ is identified under these alternative assumptions. Intuitively, this latter assumption means that there exist values of \mathbf{x} for which the max operator is never binding. In this case, we can fully recover the distribution of $F_{\eta}(\cdot)$. Then, we can identify the distribution of $c(\mathbf{x})/p(\mathbf{x}) + \eta_m/p(\mathbf{x})$ for any \mathbf{x} . Hence, we identify $c(\cdot)/p(\cdot)$.

9 Online Appendix

9.1 Online Appendix A: Additional Tables

We report the estimates of state-specific effects on preference and efficacy in Tables 7 and 8, which we use to plot Figures 4 and 5.

	Estimate	SE		Estimate	SE
Alabama	0 (Normalized)		Nevada	-0.009	(0.059)
Arizona	-0.132	(0.050)	New Jersey	-0.247	(0.041)
Arkansas	-0.363	(0.034)	New Mexico	-0.055	(0.069)
California	-0.236	(0.060)	New York	-0.239	(0.036)
Colorado	-0.057	(0.055)	North Carolina	-0.088	(0.033)
Florida	-0.101	(0.039)	North Dakota	-0.006	(0.040)
Georgia	0.107	(0.028)	Ohio	-0.166	(0.035)
Idaho	0.129	(0.044)	Oklahoma	-0.057	(0.041)
Illinois	-0.243	(0.030)	Oregon	-0.130	(0.054)
Indiana	-0.099	(0.031)	Pennsylvania	-0.190	(0.037)
Iowa	-0.300	(0.032)	South Carolina	0.030	(0.031)
Kansas	0.194	(0.038)	South Dakota	0.000	(0.039)
Kentucky	-0.251	(0.032)	Tennessee	-0.345	(0.034)
Louisiana	0.163	(0.030)	Texas	0.127	(0.034)
Maryland	-0.107	(0.053)	Utah	0.079	(0.062)
Michigan	-0.246	(0.031)	Virginia	-0.114	(0.035)
Minnesota	-0.297	(0.032)	Washington	-0.223	(0.055)
Mississippi	0.128	(0.033)	West Virginia	-0.216	(0.037)
Missouri	-0.217	(0.029)	Wisconsin	-0.335	(0.033)
Montana	0.098	(0.044)	Wyoming	0.230	(0.057)

Table 7: Estimates of State Preference Fixed Effects Relative to $\lambda_{Alabama}$. Standard errors are reported in parentheses. Higher values imply a stronger preference for Democrats.

9.2 Online Appendix B: Fit

In this Appendix, we report further on the fit of the model. Figure 10 plots the distribution of Democratic and Republican vote shares in the data and in the model prediction.

	Estimate	SE		Estimate	SE
Alabama	1 (Normalized)		Nevada	0.854	(0.075)
Arizona	0.660	(0.109)	New Jersey	0.725	(0.054)
Arkansas	0.739	(0.043)	New Mexico	0.976	(0.083)
California	0.751	(0.053)	New York	0.795	(0.052)
Colorado	1.165	(0.073)	North Carolina	0.787	(0.040)
Florida	0.975	(0.054)	North Dakota	1.062	(0.056)
Georgia	0.842	(0.039)	Ohio	1.323	(0.057)
Idaho	1.274	(0.081)	Oklahoma	0.919	(0.041)
Illinois	1.067	(0.041)	Oregon	1.441	(0.086)
Indiana	0.814	(0.042)	Pennsylvania	0.838	(0.046)
Iowa	1.319	(0.055)	South Carolina	0.751	(0.045)
Kansas	0.877	(0.043)	South Dakota	1.680	(0.105)
Kentucky	1.052	(0.037)	Tennessee	0.867	(0.041)
Louisiana	1.316	(0.079)	Texas	0.769	(0.041)
Maryland	0.767	(0.050)	Utah	1.229	(0.087)
Michigan	1.180	(0.048)	Virginia	0.754	(0.043)
Minnesota	1.765	(0.103)	Washington	1.080	(0.051)
Mississippi	1.193	(0.067)	West Virginia	0.814	(0.043)
Missouri	1.159	(0.040)	Wisconsin	1.762	(0.108)
Montana	1.066	(0.054)	Wyoming	1.151	(0.060)

Table 8: Estimates of State-level Fixed Effects of Voting Efficacy. Standard errors are reported in parentheses. Alabama is set to 1 for normalization.

The figure shows that the model fits the data well for all ranges of the Democratic and Republican vote shares. Lastly, we compute the χ^2 statistic for the goodness-of-fit test. The χ^2 test statistics for Democratic and Republican vote shares are 14.07 and 45.29, respectively. The former do not reject the null that these two distributions are the same at 5% level, while the latter rejects it.

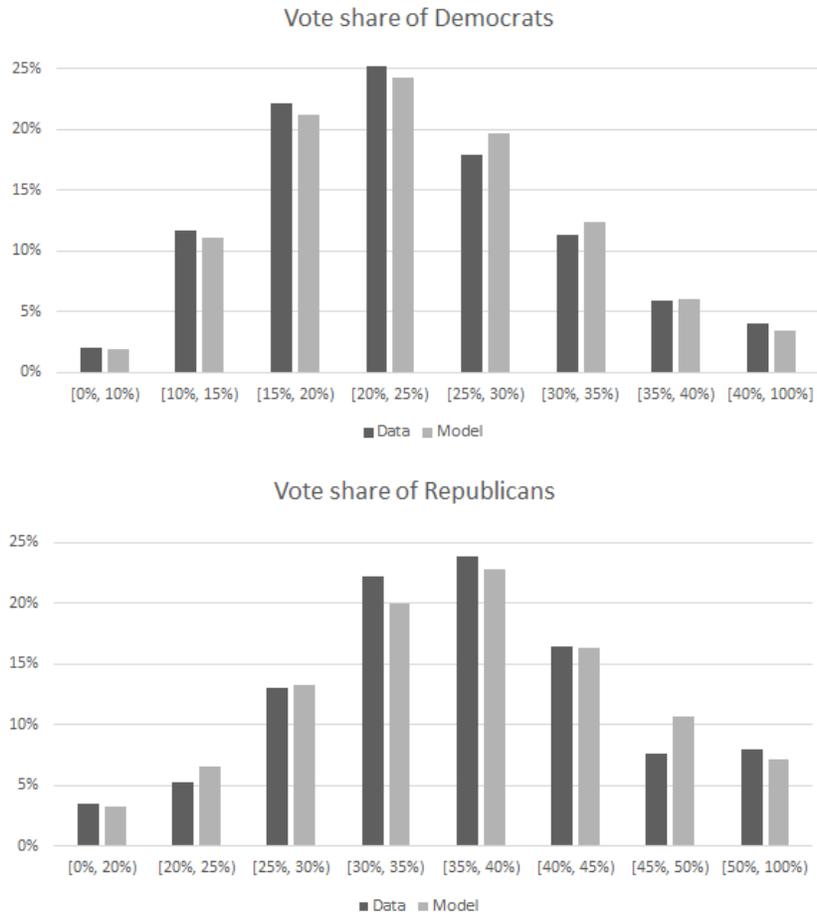


Figure 10: Model Fit. The top panel plots the distributions of Democratic vote share in the data and in the model prediction. The bottom panel plots the distributions for the Republican vote share.