Product Awareness, Industry Life Cycles, and Aggregate Profits*

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Do frictions in the expansion of consumer choice sets for new products explain firm growth, the industry life cycle, and aggregate profits? To explain industry and aggregate patterns, I introduce a mechanism by which consumers slowly become "aware" of differentiated products, expanding their choice sets. When aggregated, this information friction creates a wedge in an otherwise standard neoclassical growth model, which can help explain secular changes in factor shares in the calibrated model. New empirical evidence is shown to be consistent with this model: product creation and obsolescence rates are high, and markups tend to decrease as industries age.

Keywords: Macroeconomics, Firm Growth, Industry Life Cycle, Profit Share, Tobin's Q, Product Differentiation, Heterogeneous Markups, Customer Capital, Product Obsolescence, Information Frictions, Neoclassical Growth

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1 Introduction

> "In a typical year, 40 percent of household expenditures are on goods that were created in the last 4 years, and 20 percent of expenditures are in goods that disappear in the next 4 years." – Broda and Weinstein (2010)

As detailed scanner-level data has become available, the rapid changes in the products available to and purchased by consumers have become evident. For example, Broda and Weinstein (2010) documents a striking rate of churn in product availability compared to typical measures such as firm entry and exit. While these sorts of patterns have traditionally been analyzed through the lens of creative destruction and firm selection, this rapid turnover of products has the—possibly negative—side effect of distorting the product age distribution in consumption bundles. As the age distribution is determined by the rate of product obsolescence and product creation, a high rate of both leads to many immature products—and any permanent changes in these rates could lead to long-run changes in the distribution's shape.

Leaving aside any gains from creative destruction or growing productivity, a turbulent and skewed age distribution of products raises two related questions: From consumers' perspective, does the product age distribution have any impact on their welfare, purchasing power, and factor shares? From firms' perspective, do changes in product maturity throughout the life cycle of their industry lead to changes in profit margins and market power?

In most models of firm heterogeneity, the answer to both questions is clear: the age of the product is irrelevant. For example, under a monopolistically competitive benchmark, a rapid churn of products is important only insofar as it is an indicator of selection, or if older vintages have lower quality and/or productivity. Firms' profit margins and markups under monopolistic competition are constant and independent of the product's age. Even in fancier models with oligopolistic competition, the age distribution has no effect in and of itself.

This paper shows that the crucial assumption leading to the irrelevance of the product age distribution in a monopolistically competitive benchmark is that of full information sets for consumers and firms. To be concrete, the dynamics of demand are very different if every consumer can choose from every product in the economy, instead of consumers having incomplete and evolving choice sets of available products. In the long run, for a given product, most consumers interested in the product would be able to purchase it, but that assumption fails early in a product's life cycle. Moreover, if product obsolescence rates are as high as the empirical results suggest, then the age distribution is skewed towards relatively immature products, and, thus, a large number of products in the economy are early in their life cycle. In that case, the information frictions could lead to empirically relevant effects on welfare and factor shares.

If we consider demand as arising from the network of connections between firms and consumers, then product maturity is tightly connected to this network formation, and choice sets are determined by the firms to which a consumer is connected.[1] Early in an industry's life cycle, the network of connections is sparse. Later, due to advertising, word of mouth, direct sales, and other forces, the network becomes more dense, until an assumption of perfect connectivity may be reasonable. Therefore, setting aside changes in productivity or quality, the expanding set of relationships is the key to understanding the role of demand in an industry's life cycle.

To this perspective, I add a specification for the evolution of information sets to embody the network and use this to establish new micro-foundations for industry demand embedded in an otherwise standard neoclassical growth model with heterogeneous firms. Following the empirical literature on this topic—such as Goeree (2008) and De Los Santos, Hortaçsu, and Wildenbeest (2012)—I call the limited information sets within the network "awareness," with which I capture frictions, such as a consumer not having: knowledge of a product's existence; knowledge of the idiosyncratic match to her preferences; information on the particular location and method for purchasing; geographical proximity to a distributor of the product; etc. Over time, consumers are likely to expand their awareness of firms' existence and characteristics, but the process could be slow. In addition to incomplete information and heterogeneous choice sets, I assume that consumers have heterogeneous preferences for each good and that they consider the quality of each match when making their consumption decisions.

With these model features, the slow expansion of product awareness generates an industry life cycle through changes in both the number of customers and the demand per customer. As new firms enter, the time it takes for information to reach consumers provides a simple explanation for the slow growth of firms and industries as a time-varying limitation on the number of potential customers. Additionally, as consumers become aware of these new firms, they increase their demand when they find a new good that they prefer.

For a firm, the expansion of product awareness causes two countervailing effects on profits. First, increased awareness among consumers increases the level of competition and, hence, decreases market power and prices. Intuitively, if all of a firm's customers know only of that firm, it would have monopoly pricing power over them. But if some of those customers are aware of multiple firms, then a firm without the ability to price discriminate needs to lower its prices to compete. Second, increased awareness among consumers gives them more choice

[1] Empirical facts on the slow expansion of customer capital has spawned a recent and influential literature on demand. See Gourio and Rudanko (2014a), in particular, for motivation on the role of customer capital in forming intangible capital. My paper builds on top of these motivations by showing that it is not just accumulated customers that matter, but also the features of the interconnected network that aggregate to form the customer capital.

and allows for better matches, on average, between firm and consumer. Over time, consumers sort into their preferred products, which increases demand. These competing forces drive industry profits and valuations. At the aggregate level, the strength of these effects depends on the level of awareness maturity of different product categories and vintages, which may change due to industry composition and technological innovations.

The key to understanding the degree of market power and sorting in the economy is that the time-varying *effective number* of competitors for each consumer, rather than the *total number* of competitors for all consumers, determines market power and the implications of this demand friction at the industry or aggregate level. If information and choice sets are complete, then the total and effective number of competitors is the same. But early in a product's life cycle—and perhaps even after maturity—a fully connected network is a poor assumption.

Throughout this paper, the story to have in mind is the invention of a new product category, such as the personal computer (PC) industry. At the beginning, firms may have small absolute demand but can charge high markups since customers tend to have less information about the market and difficulty shopping around. Over time, competition intensifies and markups are driven towards the razor thin margins we see today in the PC industry, even though the market is heavy concentrated. To see this within the model, since it is the *effective number* of competitors that really matters, there may be a large number of competing firms at the beginning, and a small number of oligopolistic firms when the industry matures, and one can still have the pattern of decreasing profit margins if the effective number of competitors is increasing. Keep in mind that this is in contrast to a product entering into an existing and mature market. This has been the focus of most of the literature, in which an entering firm may have incentives to lower its prices *relative* to the incumbents'. My model is consistent with both results: lower prices as industries mature, and lower prices compared to incumbents when entering an industry

At the aggregate level, profits and demand are simply a composite of all industries, which are affected by age as a proxy for the degree of product maturity. Hence, any changes in the distribution of product and industry ages will change average markups, profits, and factor shares in the economy. For example, a temporary influx of new products will tend to distort the age distribution towards younger firms (with less accumulated awareness, but higher market power). When aggregated up, this could show as a lagged impulse of increased markups and profit margins, but as a lagged effect on output since it takes time for awareness to build.

When considering the role of secular changes, the dominating effect on aggregates comes from the skew of the age distribution, which, in turn, is driven primarily by the obsolescence rate. For example, if products are retired rapidly (e.g., consumers have fickle tastes), then for any product creation rate, the stationary age distribution is skewed towards immature products. As discussed, immature industries provide firms with more market power (and less absolute demand), which could have a large impact on welfare in a carefully calibrated model. Even with a technological innovation that better spreads awareness (e.g., the invention of the web or television), the effects on profit margins tend to be small compared to increases in the speed of product obsolescence.

Summary of Contributions Empirically, this paper first establishes new indicators for the creation and obsolescence of products using intellectual property (IP) data from the United States Patent and Trademark Office (USPTO). I argue that trademarks are the best proxy for the creation of a new product, and that the rates of abandonment and obsolescence give us a sense of the age distribution of products in the economy. In particular, note that the age distribution of products rather than that of firms is the object of interest, as one product per firm is a poor approximation when connecting the data to my model. I show strong evidence from trademarks (and patents) that the rate of obsolescence is high, as well as moderately strong evidence that the obsolescence rate has undergone a secular change since the 1980s. Finally, to connect to the aggregates of interest, I show that the profit share has been increasing, and that this increase is connected to markups. The conclusion of the aggregate evidence is that obsolescence rates are high; average product ages are, consequently, low; and there is some evidence of secular changes.

As discussed, when compared to monopolistic competition, a skewed age distribution of products may be irrelevant unless there are systematic effects of age on industry profits and markups. To show this, my next empirical contribution is an analysis of a panel of industries using WRDS/Compustat and US Census of Manufacturers data to uncover the role of age in the industry life cycle. I find new evidence that—even after controlling for industry concentration, the number of firms in the industry, and year fixed effects—markups decline over the industry life cycle, as the theory predicts.

The primary theoretical contribution of this paper is a novel theory of demand, isolated from quality and productivity changes and kept stylized to aid in aggregation. The approach is: introduce a single friction to consumers' choice sets (and, hence, frictions in the underlying network of relationships between consumers and firms), but keep the rest of the model as close to a neoclassical growth model under monopolistic competition as possible. I then aggregate up from the network of consumer-to-firm relationships to derive firm decisions, aggregate up firms to derive an industry life cycle, and aggregate up industries to form the economy as a whole. The relatively simple change in the standard framework leads to a variety of rich effects, but ultimately aggregates to a familiar neoclassical growth model with an awareness wedge.

Next, using a version of the theoretical model calibrated with the empirical results, I show that the effects of these information frictions are significant for factor shares and welfare. Conducting a comparative static of the changes in obsolescence rates uncovered in the empirical analysis, I show that this could be a contributor to the secular changes in profit shares. In contrast, I find with these calibrated values that an increase in the speed of awareness diffusion (e.g., invention of new advertising technology, such as the web) generates a large change in the number of products, but has a modest effect on profit margins compared to changes in obsolescence rate. Finally, I show that significant variation at the business cycle frequency are unlikely to be generated by these frictions since the calibrated growth of awareness is slow enough to smooth any significant changes.

Paper Organization The paper begins with a literature review in Section 1.1, and presents direct evidence from micro-studies that "awareness" (i.e., limited choice sets) is empirically important.

The empirical results are in Section 2. Section 2.1 first analyzes general patterns of obsolescence and product creation using the USPTO data, and then provides evidence of secular changes to obsolescence and the profit share. Next, Section 2.2 uses industry panel data to establish facts about the role of age in markups and firm profits. Finally, Section 2.3 summarizes existing micro-studies on firm dynamics and demand, which are relevant for the age-dependent forces in this model.

Section 3 discusses a stylized model to explain this evidence and implement the product awareness mechanism. The novel modeling contribution of awareness as limited choice sets is described in Section 3.2. Using a general structure for awareness processes, Sections 3.3 and 3.4 embed limited choice sets in an industry equilibrium. Finally, the industries are aggregated in Section 4 to form a variation of the neoclassical growth model, and to nest the familiar neoclassical growth model with monopolistic competition.

The calibrated simulations and analysis in Section 5 show some of the novel predictions of the model and provide a sense of the magnitude of the effects. Section 5.1 simulates a single industry to show the general patterns in the industry life cycle. Aggregating to the macro-economy in Section 5.2, the information friction becomes a wedge in productivity and factor shares, compared to an otherwise standard neoclassical growth model, and I conduct comparative statics to show the important role of product obsolescence and other key parameters to explain secular changes in factor shares. Finally, Section 5.3 presents a version of an impulse response of a large cohort of new product entry. The effects are modest and slow—cementing the intuition that new products have long lags before they can affect aggregates.

While the environment in much of Section 3 is kept simple to highlight the unique predictions of the model, it is amenable to extensions. The first is in Section 6, in which I connect different types of asymmetry in product entry and quality to the empirical results of Section 2.3. Second, while the paper focuses more on the implications of the expansion of product awareness than on its endogeneity, Section 7 provides a simple model of investment in expanding consumer choice sets—leading to the conclusion that the earlier comparative statics do not unravel with control over the awareness process.

1.1 Related Literature

This paper fits primarily into the macro and international literature on consumer capital and demand, such as Arkolakis (2010, 2015), Drozd and Nosal (2012), and Gourio and Rudanko (2014a,b).[2] By concentrating on information frictions and heterogeneous choice sets, I am able to provide a novel perspective on demand to complement those models.[3] Additionally, while I emphasize information frictions on profitability, the role of within- vs. between-industry heterogeneity and the creation/obsolescence of new products/innovations complement papers such as Atkeson and Burstein (2008, 2015). Finally, the paper fits into the literature on the role of information in industry equilibrium, such as Dinlersoz and Yorukoglu (2012).

The main competing theory—industry selection driven through stochastic productivity or imperfect signals on productivity/demand—is the core mechanism of papers such as Jovanovic (1982) and Hopenhayn (1992).[4] To isolate my contribution, I purposely turn off the selection and productivity process, but the standard approaches nest well with my model. Further technological explanations for the industry life cycle and the eventual decline of firms go back to Gort and Klepper (1982) and continue with papers such as Dinlersoz and MacDonald (2009). For the most part, these explanations are based on technological cycles of product introduction and refinement, rather than on theories of demand.

Motivations for the Product Awareness Friction Recall that the information friction, which I label "awareness," is any friction that prevents consumers' choice sets from includ-

[2]To compare to my paper: (1) Arkolakis (2010, 2015) investigates the market access margin in trade and firm growth models, and relates it to advertising expenditures; (2) Drozd and Nosal (2012) has a notion of investment in market capital, interpreted very similarly to to the accumulated awareness here, and relates it to frictions in international price elasticities; (3) Gourio and Rudanko (2014b) models customer capital as a two-sided search and matching friction between consumers and salespeople, and it connects the friction to general sales and marketing expenditures; and, (4) Gourio and Rudanko (2014a) discusses the aggregate implications of customer capital wedges. Other papers on this topic include Molinari and Turino (2009), which emphasizes the role of advertising in business cycles, with a DSGE-style equilibrium.

[3]The cost of these micro-foundations and increased heterogeneity is that business cycles, in the style of Drozd and Nosal (2012) and Gourio and Rudanko (2014b), are more difficult to analyze, and control over advertising, in the style of Arkolakis (2010), is kept much simpler.

[4]Models with learning in this literature, such as Jovanovic (1982) and Arkolakis, Papageorgiou, and Timoshenko (2014), are an exception to the irrelevance of age in industry models. There, age becomes important as a proxy for the precision of the prior on productivity or demand.

ing every product in the economy. Hence, product awareness captures several potentially independent effects that have been examined in the literature.

Imperfect Information and Advertising: Explaining demand and market imperfections is a key goal of models with informative advertising in customer markets, as discussed in Bagwell (2007). A classic example is Butters (1977), which examines the steady-state price distribution when firms advertise the location and price of their product. Within the empirical IO literature, the closest counterpart to this paper is Goeree (2008), which estimates a static discrete-choice model of the personal computer industry, in which advertising affects the probability of a consumer being aware of a product. Dinlersoz and Yorukoglu (2012) takes a Butters (1977) style model dynamic, and provides a different perspective on the role of accumulated information frictions in customer acquisition. The authors emphasize the role of limited "memory" in the dynamics, and the interaction with stochastic productivity.[5]

Customer Markets and Customer Capital: Phelps and Winter (1970), Hall (2008), and Luttmer (2006, 2011) interpret the process of firm growth through customer capital acquisition.[6] Within applied theory, many papers, such as Bergemann and Välimäki (2006), have modeled a dynamic demand choice for consumers.[7] The related role of customer markets and market power is discussed in Bils (1989) and Rotemberg and Woodford (1991).

Limited Attention and Bounded Rationality: Bordalo, Gennaioli, and Shleifer (2015) has firms competing for attention of product attributes. If the attention leads to sparsity of information sets, it could manifest in a way similar to my model—even if consumers have access to all products. Similarly, the complementary literature on inattention and sparsity—such as Hellwig, Kohls, and Veldkamp (2012) and Gabaix (2014)—shows how limited information sets may arise. While my model provides no theory of how consumers might affect their information sets, it shows the implications of incomplete information for industry and aggregate profits.

[5] Some specifications of the awareness evolution process—in particular, with $\mu > 0$ in Example 1 —could provide a rudimentary approximation of limited consumer memory.

[6] Customer capital is often explained by switching costs, as described in Klemperer (1995), or through habits in consumer preferences. For example, Paciello, Pozzi, and Trachter (2014) shows the role of switching costs in a dynamic game between customers and firms, leading to a habits-like dynamics. Ravn, Schmitt-Grohe, and Uribe (2006) explores the creation of habits on a good-by-good basis. In contrast to these approaches, my model assumes that no intrinsic habits exist in the preferences, as consumers can costlessly switch between goods that they are aware of. Instead, customer turnover decreases over time due to customer sorting into their preferred firm, thus decreasing switching probabilities.

[7] In particular, the decreasing prices over time in this model are similar to those in Bergemann and Välimäki (2006), albeit due to different forces. The expansion of informed consumers over time in that paper is analogous to the expansion of consumer awareness, and the "niche" vs. "mass market" dichotomy is captured in the σ parameter in my model (as well as possible mixed-strategy equilibria discussed in Proposition 3).

Are Limited Choice Sets Empirically Relevant? While the literature described above suggests a variety of theoretical mechanisms for limited choice sets, that does not mean that the sets are small enough to be important. However, empirical studies able to connect individuals to choice sets consistently show that the effective choice sets are very small.

The first example is a micro-study of online browsing data in De Los Santos, Hortaçsu, and Wildenbeest (2012). Using online browsing data logging every website visited by a large number of consumers, Figure 1 ("Consumer Bookstore Awareness") shows that close to 35% of consumers *visited* a *single* online bookstore in an 18-month period. The proportion purchasing from multiple online bookstores in that period is even smaller, despite the dozens of online bookstores, the ease of searching on the internet, and the relative homogeneity of online books to make price comparisons easy.[8] Perhaps the consumers were "aware" of stores visited prior to the 18 months of data, but the fact that they do very few price comparisons is clear evidence of an information friction (or improbably large search costs).

Another important example is Goeree (2008), which uses a similar (albeit largely static) concept of awareness as limited information sets. The author estimates, exploiting variation in advertising exposure, that median markups in the PC industry are 15% due to limited choice sets, as opposed to 5% under full information.

Finally, the literature examining marketing data, such as Bronnenberg, Dhar, and Dube (2009), show slow growth of product availability and product stickiness consistent with small choice sets and disparate regional product accessibility.[9]

2 Empirics

This section analyzes empirics at several levels of aggregation. For economy-wide empirics, Section 2.1 establishes new indicators of product creation and obsolescence, and explains why the product life cycle could have an important impact on welfare through level effects and factor shares. At the industry level, Section 2.2 shows new evidence from industry panels that is consistent with the mechanism in this paper, potentially causing some of the effects seen at the aggregate level. The panel regressions also confirm that monopolistic competition, as well as any models without age effects, can be easily rejected. Finally, Section 2.3 summarizes existing micro-studies on firm dynamics in the context of this research.

[8]More generally, any model with empirically relevant search frictions would exhibit limited choice sets. For example, looking at direct networks of buyers and sellers using Columbian export data, Eaton, Eslava, Krizan, Kugler, and Tybout (2014) finds very small networks, where "the average exporter sold to around 1.5 buyers while the average buyer had around 4 sellers."

[9]Earlier indirect evidence from Bils and Klenow (2001) shows accelerating variety growth—skewing the age distribution towards younger products.

2.1 Aggregates and Obsolescence

For aggregated models with monopolistic competition, the most straightforward assumption is typically one product per firm. While the assumption is often innocuous in modeling, it may be a terrible approximation for taking models to the data. For example, some papers use the one product per firm assumption with firm creation and destruction rates provide a proxy for product creation and destruction. However, as data on consumer panels—such as Nielsen scanner data—become more available, using firm creation/destruction as a proxy for creative destruction at the product level falls apart empirically. For example, as discussed in Section 1, Broda and Weinstein (2010) shows rapid churn in the availability of consumer products—far outstripping either churn in the labor markets or creation/destruction rates of firms. While a time-series of detailed consumer and firm purchases of both services and products would be ideal, the data does not yet exist.

IP Proxies for Product Creation Until product-level panels are sufficiently rich, another approach to getting a sense of the creation rate of products is to find a proxy for the invention of the product itself. A common approach in the growth literature has been to look at patents. While the data is useful as a test, it is indirect and problematic.[10] A more direct approach using intellectual property indicators is the underused data on trademarks. While a patent is supposed to provide temporary monopoly protection for publishing a non-obvious and novel invention to the world, a trademark is essentially a registration of the unique identifying information for a product (i.e., product name, logos, etc.).[11] Since it is low-cost and available in perpetuity, it has wide coverage across all sorts of industries in which patents are not financially viable. Moreover, if we assume that the number of names and logos per product is constant over time, we do not run into the knife-edge case of constant returns to scale on innovation required for connecting patents to products and GDP growth.

Figure 1 and Table 1 summarize patents, trademarks, and aggregate growth rates.[12] The statistics in Table 1 show that the average growth rate of TFP and GDP is significantly

[10] When looking at time-series of patents as an indicator of product creation rates in the economy, consider that: (1) significant patenting is used by only a small fraction of firms, and a subset of industries, even though innovation and product creation is much more widespread; (2) there is no reason to assume that the number of patents required per product has been stable over time. In fact, having constant returns to scale (CRS) R&D production functions in patents is a knife-edge case in endogenous growth models; (3) patent applications may or may not be accepted, and the product using it may never be commercialized; and (4) the type of inventions that can be patented, the financial incentives and secondary markets, and the regulatory environment have gone through profound changes in the last 30 years. See Technical Appendix F.2 for more on the life cycle of a patent, and changes in the patent environment.

[11] See Technical Appendix F.1 for details on trademarks and their life cycle. Trademark litigation is essentially showing that a firm is using some of your product's identifying information to confuse consumers. Hence, the legal environment is aligned to ensure that trademarks are a good proxy of products/varieties.

[12] The data on trademarks and patents come from the USPTO and include approximately 5.4 million and

	mean	sd	min	max
% Change in Trademarks Filed	6.16	11.90	-27.00	29.60
% Change in Patent Applications	5.15	5.75	-8.10	22.56
% Change in GDP	2.72	1.99	-2.82	7.01
% Change in TFP	0.93	0.99	-1.95	2.58

Table 1: Summary Statistics of IP Indicators from 1981 to 2012

less than that of trademark or patent applications. Thinking of the average growth rate of trademark or patent applications as an entry rate into a stock of products, either: (1) the stock of products is exploding compared to the growth rate of GDP; or (2) there is a large amount of product obsolescence to ensure that the stock of products relative to the size of the economy is kept relatively stationary. The evidence from Broda and Weinstein (2010) and Table 2 suggests that obsolescence is a more likely answer than an economy with an exploding products-to-GDP ratio. The business cycle patterns of Figure 1 are less obvious due to the volatility of new product creation and the lag in implementation, but trademarks are roughly pro-cyclical.[13]

IP Proxies for Product Obsolescence It is reasonable to assume that if a product becomes obsolete or is abandoned, then any IP associated with it would be abandoned as well. However, the faster growth rate of trademark filing and patent applications than GDP documented in Table 1 provides only indirect evidence of this type of product obsolescence. Luckily, both patents and trademarks have the advantage of requiring a stream of maintenance payments prior to becoming in-force, and then further scheduled payments afterwards to maintain their validity. The detailed USPTO data on every patent and trademark can be used to track its life cycle and to construct statistics for survival and abandonment.[14] Table 2 provides the summary of this survival analysis of approximately 5.4 million trademarks and 6.8 million patents. Over 30% of trademark applications are abandoned before

6.8 million observations, respectively. See Appendix F for more details.

[13] At the business cycle frequency, the correlation between % change of TFP growth and four-year lags of the % change of trademark applications is $-0.18, -0.06, 0.10$, and 0.25. This means that for the first two years, an increase in the number of trademark applications is coincident with negative or stagnant TFP growth, which then becomes positive in later lags. From the perspective of productivity wedges, I interpret this as there being a lag between creation of a product category (i.e., slow spread of awareness) and, potentially, a slow increase in the quality of a product after creation (i.e., quality growth due to the sorting effect). See the decomposition of aggregate TFP in (29).

[14] For details of the patent and trademark life cycle and terminology, see Technical Appendices F.1 and F.2. More details on the USPTO data are available in Graham, Hancock, Marco, and Myers (2013), Marco, Carley, Jackson, and Myers (2015), and Graham, Marco, and Miller (2016).

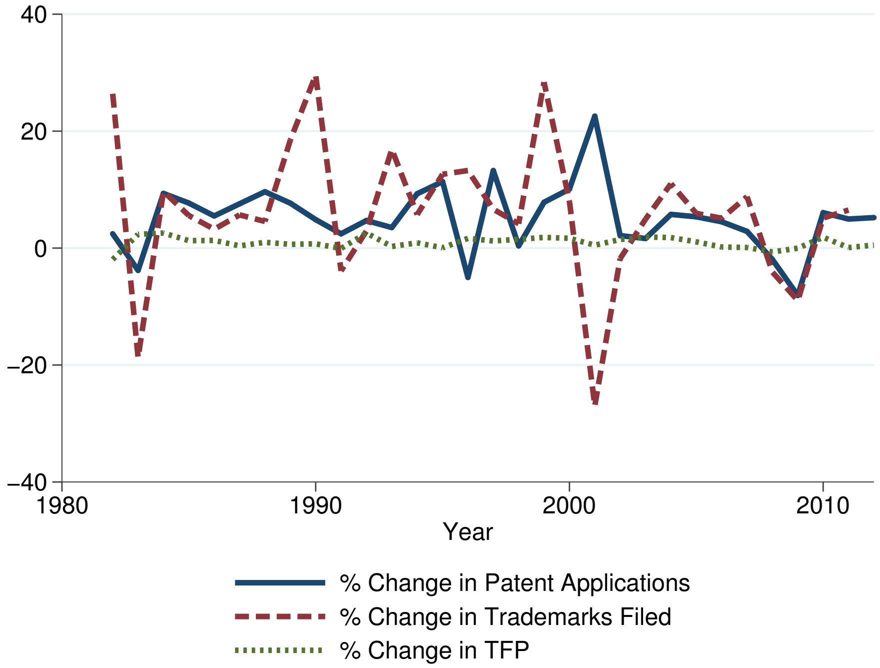

Figure 1: Intellectual Property and Growth Rates

	mean	sd	min	max
% of Trademarks Abandoned Before Registration	31.47	9.63	20.63	44.02
% of Trademarks Applications Surviving 10 years	15.99	2.48	12.59	20.82
% Patent Applications Abandoned Before Issue	14.20	2.81	7.24	18.01
% Issued Patents Expiring for Non-Payment	18.14	12.19	2.42	44.17

Table 2: Summary of Trademark and Patent Abandonment (1981-2009)

the registration process is complete (typically a two to three year process), and only 16% of trademark applications survive 10 years. Patents have a much higher upfront cost and greater option value if issued—and, consequently, have lower abandonment rates. Even then, close to 14% of patents are abandoned prior to being issued, and even 18% of issued patents are abandoned before the 17- to 20-year term ends.

Expanding from the unconditional averages, the time-series of Figure 2 shows the possibility of secular changes in obsolescence. Trademark abandonment rates increased from 20-30% in the 1980s to 40-55% in the 1990s. Even starting in the 1990s, the rates go from less than 40%, peaking at 55% and, finally, to around 45% by the mid 2000s. The increase in patent abandonment is even more extreme: Figure 2 shows an increase from approximately 10% to 30% in the abandonment rates of patents prior to being issued (a process which usually takes three to seven years). Even conditional on a patent being issued, the proportion of patents expiring due to non-payment increased from 10% to almost 40% in the sample.[15]

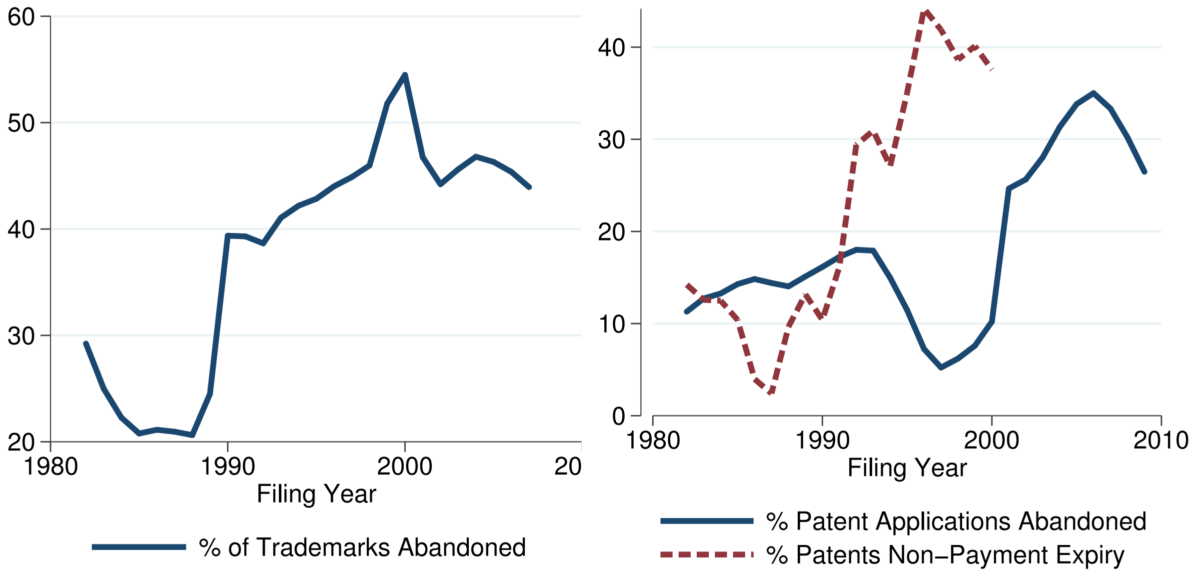

Figure 2: Abandoned and Expired Trademarks and Patents

Implications and Secular Changes To summarize the evidence on product creation and obsolescence: (1) IP proxies show that new products are created far faster than the growth rate of the economy; (2) this can be reconciled with direct evidence that abandonment and obsolescence rates are high; and (3) the obsolescence rates (and, consequently, the product creation rates in any stationary balanced growth path) seem to have increased since the 1980s.

In a standard model with monopolistic competition, this skewing of the age distribution of products would be irrelevant beyond a higher depreciation rate of innovation expenditures (to borrow the terminology of Atkeson and Burstein (2015)). To demonstrate that the age distribution matters empirically, I will show in Section 2.2 that there are significant age-dependent effects on measures of profitability—such as markups and Tobin's Q. At the aggregate level, factor shares are driven by the profit margins of the underlying industries, firms, and products. Consequently, if there were secular or business cycle changes in the age distribution of products, they would manifest as changes in the factor shares (and aggregate measures of profits). To get a sense of whether there have been significant changes in aggregate profits or factor shares coincident with changes in obsolescence rates, Figure 3 shows key indicators for the US economy.

[15]See Technical Appendix D.1 for more analysis of abandonment, and tests to remove the effects of any administrative changes. The data on patent expiry for non-payment is cut off around 2000 to ensure that patents have gone through the appropriate renewal events prior to the end of the sample period.

The secular decline of the labor share from the BEA data shown in Figure 3 is consistent with research such as Elsby, Hobijn, and Şahin (2013) and Karabarbounis and Neiman (2014). However, in most of that literature, the explanations for the changes in the labor share have tended not to focus on the role of the profit share, which seems to be increasing in Figure 3. To get a sense of the magnitudes, when HP-filtered, the corporate profit share increases from about 4.5% to over 6% over the sample. Hence, as the labor share is the total output minus the capital share and profit share, this could be a significant contributor to secular changes in the labor share. To show robustness to the idea that measures of corporate profits have increased, the stock market capitalization-to-GDP ratio, and the aggregate Tobin's Q are both shown—providing more evidence on secular changes in profitability.

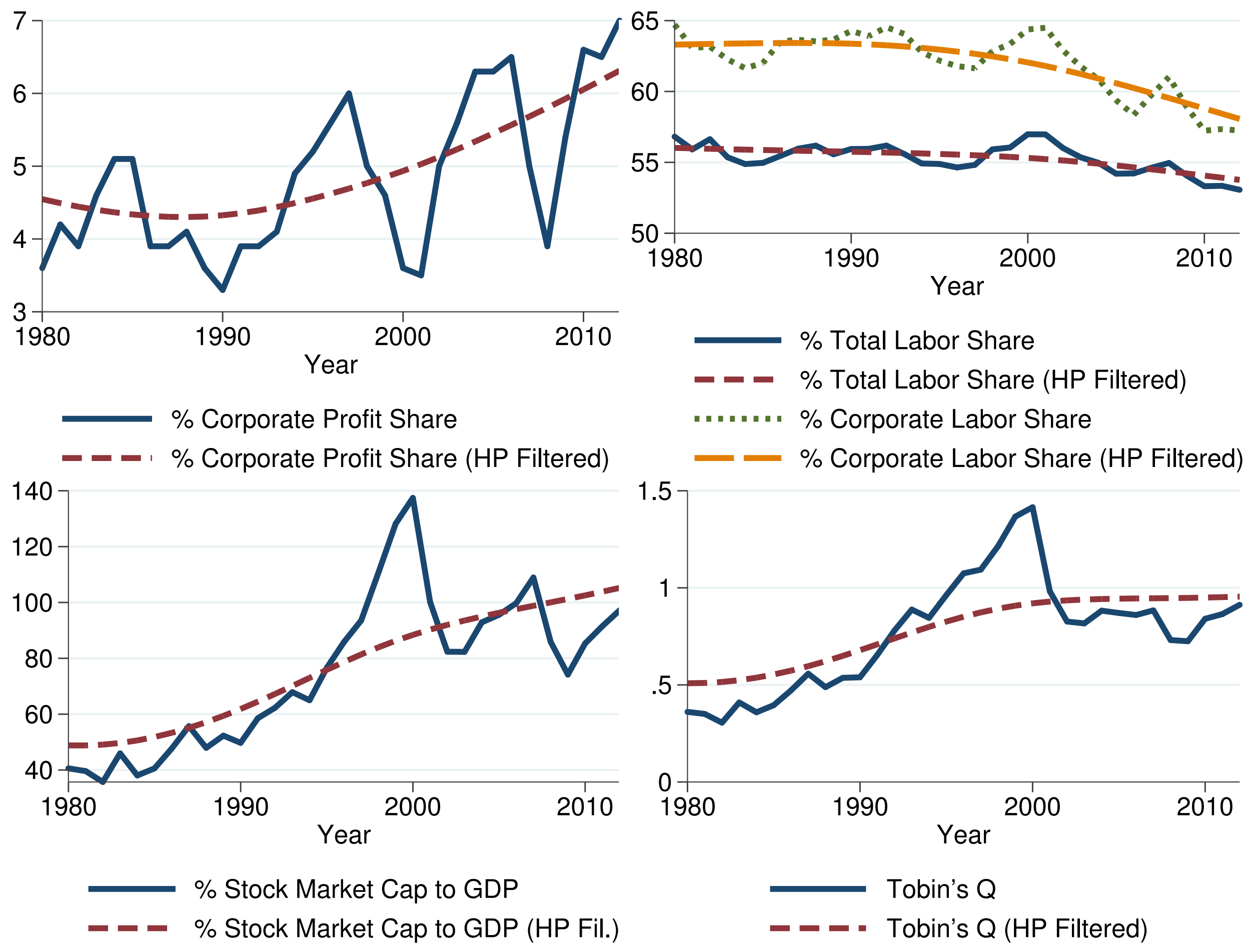

Figure 3: Aggregate Profits and Factor Shares

For asset pricing implications, a key reason to understand firm and industry growth is to explain the discrepancy between firms' stock market valuations and book values, as captured in Tobin's Q and discussed in Hall (2001). The general consensus is that sources of intangible capital are necessary to explain valuation growth rates and systematic deviations of Tobin's

Q from unity after controlling for stock market volatility and investment irreversibility. In my model, the accumulated spread of awareness is a key determinant of intangible capital of industries and the aggregate economy, as the value of intangible assets are sensitive to market power.

For evidence that the changes in profit share over time are related to a secular change in markups, and not simply an industry composition effect, see the panel data results in Appendix D.3.

2.2 Industry Evolution

While Section 2.1 shows that measures of product creation/destruction are high—generating an age distribution skewed towards younger products—it does not show why this would affect the measures of profitability. Moreover, to connect to any secular evidence at the aggregate level, I need to demonstrate that product age has a sufficiently strong effect on profits and market power, so that changes in the age distribution could explain part of the secular changes in Figure 3 and Figure 22 of Appendix D.3.

While no panel on products themselves exists, we can use industry panels to get a sense of the role of age. From NBER-CES Manufacturing Industry Database (MID), the Census Concentration Ratios, and Compustat, I analyze a panel of 189 six-digit NAICS manufacturing industries from 1961 to 2012.[16]

In order to understand the role of age, I need to define birth. As NAICS codes are generated infrequently, and after the industry/product category already exists, I cannot use the first appearance of a particular NAICS code as the birth year. Instead, I use a threshold where birth is defined as the point at which employment reaches 5% of the industry's maximum level in the data. Robustness checks using alternative birth definitions are done in Technical Appendix E.2. Another concern is that different types of industries may have different life cycle lengths. To account for this, I can rescale the age by defining the industry life cycle length as the age at which it hits the maximum employment.[17]

Figure 4 shows a fixed-effects regression on the industry panel to look at the effects of these relative age decile bins on various indicators.[18] The regressions show a semi-elasticity

[16]Data prior to 1997 is converted from SIC4 to NAICS6 through concordance tables. For details on the data sources, see Appendix F. In the robustness check without controls (Technical Appendix E.1) 502 manufacturing and non-manufacturing industries are used instead since industry concentration data is not required.

[17]From this, I can define decile age bins where the first bin is age 0-10% of age at max employment, age 10-20% of age at max employment, etc. See Technical Appendix F for details on this normalization process, and Figure 17 of Appendix D.1 for a histogram of the peak and birth years. The approach of stretching the life cycle based on the year of birth and peak follows the business cycle literature.

[18]For a robustness check using the age directly and no rescaling based on life cycle length, see Figure 20 and Appendix D.2. The general patterns remain the same.

Figure 4: Effects of Age Relative to Peak Employment (Controls for # Firms & Concentration & Year Fixed Effects)

for the marginal effect of being in an age bin—or a direct age in the robustness check of Figure 20 in Appendix D.2. For example, compared to being in the first age bin (i.e., at birth to 10% of the industry peak age), operating profit margins are roughly 20% less when the industry reaches the 10th age bin (i.e., at 100% of the industry peak age). In all cases, Figure 4 controls for the number of firms in the industry and a measure of the concentration of the industry (e.g., the proportion of industry revenue in the top eight firms), and it has a year fixed effect to control for any aggregate changes. The thin lines show one standard error confidence intervals around each estimate.

The first measure of markups—average operating profit margins as calculated from Compustat—shows a significant change in profitability, dropping by nearly 20% from birth. The second measure is the price-cost-margin as calculated from the NBER-Census MID, dropping nearly 10% after birth. Robustness to other markup measures is given in Figure 19 of Appendix D.1. Tobin's Q, calculated from Compustat as a measure of the future profits relative to installed capital, also drops nearly 25%. Finally, to get a sense of the role of productivity, five-factor TFP from the Census MID is shown to have few statistically significant changes.[19]

For the measures of profitability, such as markups, it is important to control for measures of market concentration, such as the number of firms and the concentration index from the Census. This helps rule out that the decline in profitability is simply due to more firm entry or a less-concentrated industry structure.[20]

To summarize the results of this empirical contribution: robust to different measures of profitability and definitions of age, there is a statistically significant drop in profit margins as industries age. Consequently, given the evidence of high product creation and obsolescence rates in Section 2.1, the age distribution of products could have a large impact on aggregated profits and factor shares.

2.3 Existing Studies with Firm Data

While Section 2.2 provides new evidence on the evolution of industry aggregates, a number of studies have analyzed industries with firm-level panel data. This evidence is important for the extensions in Section 6.

[19] If anything, the five-factor TFP is decreasing with age here. The simplest explanation is due to the year fixed effect, which has generally increasing TFP across the economy. If there were some sort of a vintage technology effect upon creation, then industry TFP growth that does not keep up with aggregate TFP growth would end up as a negative age effect on industry TFP in this regression. Regardless, this regression emphasizes that TFP changes with age are not statistically significant compared to TFP changes directly coming from spillovers in the economy.

[20] On the other hand, to allay worries about endogeneity, see the robustness check without any controls in Technical Appendix Figure 4, which shows that the general pattern holds.

The existing literature has emphasized the role of productivity. Studies in which physical productivity can be measured directly, such as Foster, Haltiwanger, and Syverson (2008, 2016), find a number of empirical regularities. For example, Foster, Haltiwanger, and Syverson (2016) finds that TFP is highly persistent, with an average auto-correlation of idiosyncratic TFP of approximately 0.80. Comparing entrants and incumbents: (1) it often takes over 15 years for a new entrant to reach 73% of an incumbent's size; (2) while there is a small TFP advantage of entrants, the difference disappears after five years; and (3) entrants' prices are significantly lower, but converge to the incumbent's. This casts some doubt on the role of productivity in industry dynamics. Entrants are small and grow very slowly in spite of having lower prices and slightly higher productivity. The authors suggest that frictions in accumulating demand offer an explanation for the puzzling low growth of entrants.

The striking result of these studies is that productivity is extremely important for determining profits and selection, but it does not explain firm size or growth rates especially well. Small firms are frequently productive, have low prices, and yet grow slowly, while large firms are often unproductive and only slowly removed through selection. In summarizing these studies, Syverson (2011) cites understanding demand as an important approach for future research in order to explain how these productivity disparities can be sustained over time.

At first, the evidence that new entrants have lower prices seems to contradict the evidence in Section 2.2 that younger firms tend to have higher markups. However, this paper makes clear the important distinction between a firm being young because it has just entered an existing industry and a firm being young because it entered a new industry (and, hence, all producers within the industry are young).

When looking at the distribution of TFP by industry, most studies find large, sustained differences in productivity—even in the most homogeneous of markets. For example, Hsieh and Klenow (2009) finds that the ratio of the 75th and 25th percentile physical TFP is 5.0 in India and 3.2 in the US. While traditional stories for this dispersion have focused on misallocation due to market inefficiencies, I am emphasizing the role of customer capital and information frictions as the primary reason for slow transition dynamics and selection. Perhaps many firms are small because of frictions to accumulate customers, and policies to address standard distortions like financial frictions or rent-seeking would have little effect.

A related result to TFP dispersion is that the skewness of profits and market shares within an industry can be large, and the profit dispersion even higher than that of the market shares. For example, Apple had 17% of the smartphone market share, but 91% of the profit share in 2015.[21]

[21] See `http://www.forbes.com/sites/chuckjones/2016/02/21/apples-iphone-market-share-vs-profits`

3 Model

This section first summarizes the model and introduces key notation in Section 3.1; describes general processes for awareness evolution in Section 3.2; and then solves for the key decisions of the network of consumers and firms in Sections 3.3 and 3.4. These decisions are further tied together in Section 4, which aggregates to a stationary equilibrium.

3.1 Summary and Notation

Time, t, is continuous. There are two types of agents in the economy: a continuum of infinitely-lived consumers labeled by $j \in [0, 1]$ and firms organized by product categories. The continuum of product categories is indexed by $m \in [0, M(t)]$, where each m can be interpreted as a product category from which a consumer derives utility, and the mass of product categories available at time t is $M(t)$. Within each product category, the finite set of firms producing each variety is arbitrarily indexed by i, such that (i, m) uniquely denotes a firm and its product. The indices of firms producing in product category m is denoted $\mathcal{I}_m$.[22] While I will describe the related set of products and the preferences for those products as a "product category," I will use the term "industry" to discuss the concrete firms producing within that product category. While you can usually think of them as synonymous, the subtle distinction is more important when discussing the endogenous generation of new product categories in Section 4.3.[23]

Consumers have permanent heterogeneous preferences for each product and are aware of an evolving subset of the firms in the economy (i.e., can purchase only from some firms due to frictions in access and information). Symmetrically, firms are heterogeneous over the set of consumers who are aware of them (i.e., are in only a subset of consumer choice sets), and they make their decisions based on the expected evolution of that distribution. When aggregating, consumers rent labor and capital to firms and invest in new product categories to license to operating firms—thereby creating a new industry. The physical and R&D investment to create more capital and product categories is kept as standard as possible to

[22]For interpreting the model and taking to the data, I *do not* consider each (i, m) as representing *a single firm* in the data. Many firms produce a wide range of varieties, often in different product categories and with different vintages. This disconnects the average age of a firm in the economy (which has been increasing in the United States) from the average age of a product category or variety (which may not have been increasing). This baseline model makes the strong assumption that there are no complementarities in producing for different industries and no product cannibalization within an industry—ensuring that firm decisions per product category would be independent and that the organization of products across physical firms would be indeterminate.

[23]Of course, having each firm belong to a single industry and produce a single product is unrealistic. Similarly, monopolistically competitive models assume one product per firm—and show that without complementaries in production, it is an innocuous assumption.

enable aggregation to a standard neoclassical growth model and to directly nest a neoclassical growth model with monopolistically competitive firms.

Consumers have constant elasticity of substitution (CES) preferences across product categories, but varieties are perfectly substitutable within a product category after controlling for prices and match-specific preferences. Firms compete by simultaneously choosing a price (i.e., repeated Bertrand pricing). Given prices, consumers choose the quantity demanded for each product that they are aware of. Over time, consumers become aware of new products through a stochastic process. I first analyze the evolution of awareness as an exogenous process, and then endogenize it through firm investment in Section 7. As I abstract from physical TFP changes, the distribution of product awareness among consumers is the primary time and industry-age varying state in the economy.

Assume, for simplicity, that all of the products have identical intrinsic quality, common to all consumers.[24] There are two sources of heterogeneity in the model: a permanent quality of the idiosyncratic match between each product and each consumer, $\xi_{imj} > 0$; and a consumer-specific subset of firms in the awareness set, varying as the product category ages, $A_{mj}(a) \subseteq \mathcal{I}_m$.

For clarity when isolating a particular product category, drop the m subscript where possible, so that an i uniquely identifies a firm (or variety) within a particular industry producing the product category. Use a as the age of a particular product category, and drop t except when product categories are aggregated.

3.2 Awareness (i.e., Incomplete Choice Sets)

Limited consumer choice sets (which I have labeled "awareness") are the model's only deviation from a traditional model such as monopolistic competition in a neoclassical growth model. Therefore, this section introduces the model of awareness and explains its evolution, while Sections 3.3 and 3.4 integrate the choice set heterogeneity into as standard a model as possible. Fix a particular product category m for notational clarity.

Distinguishing the Total and the Effective Number of Competitors Consumer j of product category age a is aware of a subset of the operating firms in each product category, $A_j(a) \subseteq \mathcal{I}$. The intuition for awareness is given in the Venn Diagram in Figure 5. In this duopoly example, the intersection of the two sets is the mass of customers who are aware of both firms, and those in the uncolored regions are aware of none. Over time, the mass of consumers aware of each firm increases, along with the proportionate size of the intersection. It is the growth in the average number of firms in the awareness set—conditional on there

[24]This assumption is relaxed in Appendix A and Section 6, where the model is derived using differentiated products or firms. The symmetric case is presented in the main body of this paper for clarity with aggregation.

being at least one—that drives the interesting dynamics of this model.

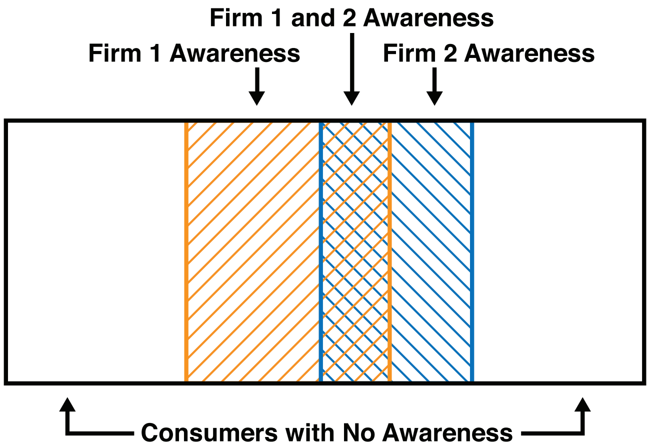

Figure 5: Example Venn Diagram of Awareness in a Duopoly

In order to isolate the effects of expanding awareness from the properties of a particular awareness process, this model will derive most of its results for a general law of motion.[25] For now, assume that the evolution of awareness is an exogenously given stochastic process. Later, in Section 7, I solve a model in which the stochastic process for the evolution of awareness is an optimal choice of firms competing within a product category.

For simplicity, assume that N firms enter at the birth of the product category at age $a = 0$. There are no fixed costs generating exit, and—assuming natural conditions on the stochastic process such that the returns to entry are decreasing in industry age—the number of firms is fixed at N forever. The case of firms entering at different times is discussed in Section 6. As I assume a large N when aggregating, the particular value or commonality between product categories is not essential. Instead, what is critical for demand, supply, and pricing decisions is the distribution of awareness sets across consumers (i.e., the *effective number* and type of competitors for each consumer, rather than the *total number* of competitors). There is a tight connection between this concept and the effective number of competitors in papers such as Fajgelbaum, Grossman, and Helpman (2011) and Gabaix, Laibson, Li, Li, Resnick, and de Vries (2016).

Stochastic Process for Awareness Evolution The evolution of awareness for firms entering at different times, and with different quality, is generalized in Technical Appendix C. Fortunately, if firms have some symmetry in the time of entry or in the intrinsic quality (or productivity), a simpler state-space than tracking all A_j is sufficient for firms to calculate

[25] The model's novel contribution is seen in the *implications* from evolving awareness, rather than in the specification of a particular process. Advertising, consumer search, and other motivations for the awareness process are discussed in Section 1.1. The baseline version in Example 1 specifies word-of-mouth contagion to better match moments of the data, but simpler processes are explored in Technical Appendix B.

profits and make optimal pricing decisions. In particular, if all firms enter at $a = 0$ and have identical intrinsic quality (or productivity), then the distribution of the count of firms in the consumer's awareness set, with cardinality $N + 1$, is a sufficient statistic to calculate firm profits and prices.[26]

Define the proportion of consumers aware of $n \in \{0, \dots N\}$ firms as the probability mass function (pmf) $f_n(a)$, with $\sum_{n=0}^{N} f_n(a) = 1$. Stacking as a vector,

$$f(a) \equiv \begin{bmatrix} f_0(a) & f_1(a) & \dots & f_N(a) \end{bmatrix} \in \mathbb{R}^{N+1}$$

Technical Appendix C.3 gives an example of a mapping between a stochastic process for the choice sets $A_j(\cdot)$ and n. Except in examples such as those in Section 6.1, assume that consumers start with a newly invented product category of age 0 with no awareness: $A_j(0) = \emptyset$ for all j—i.e., $f_0(0) = 1$.

Any stochastic process for the choice sets $A_j(a)$ has an accompanying count process, so to concentrate on symmetric equilibria, I will directly specify a process for the firm count in awareness sets. As this is a continuous-time process, and there are a discrete number of states, the evolution of the count is a continuous-time Markov chain. Denote the intensity matrix (or infinitesimal generator) of the process as $\mathbb{Q}$. With this Markov chain (and denoting the partial derivative with respect to a as the operator $\boldsymbol{\partial}_a$), a Kolmogorov forward equation (KFE) provides a system of $N + 1$ ordinary differential equations for the evolution of the distribution

$$\boldsymbol{\partial}_a f(a) = f(a) \cdot \mathbb{Q}(a), \quad \text{given initial condition } f(0) \in \mathbb{R}^{N+1} \tag{1}$$

The solution to (1) in terms of matrix exponentials is,

$$f(a) = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \cdot e^{\int_0^a \mathbb{Q}(s) \mathrm{d}s} = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \cdot e^{a\mathbb{Q}}, \text{ for an age invariant } \mathbb{Q} \tag{2}$$

Given a particular intensity matrix, the evolution of awareness is isomorphic to models from queuing theory (and Poisson counting processes).[27] $\mathbb{Q}$ will be defined as a model

[26] The count of awareness is sufficient to solve the problem only under symmetry in entry and intrinsic quality, but the fully differentiated version is discussed in Technical Appendix C and is simulated in the extensions of Section 6. As the awareness distribution evolves after entry, firms with the same intrinsic quality are truly symmetric only if they enter at the same date. For example, if there are two firm qualities or two cohorts of entry, then a distribution of counts over these two degrees of firm heterogeneity would be necessary, with cardinality $N^2 + 1$. See Technical Appendix C.1 for more details, and Section 6.1 for an example of this asymmetry.

[27] This connection provides a variety of useful formulas and theory from the operations research literature. For example, the stationary mean number of firms in consumer awareness sets is given by the Little Formula for the particular $\mathbb{Q}$. While simpler formulas often exist for counting processes in continuous-time, there is

intrinsic in this section, or parameterized as an endogenous choice of a controlled Markov process in Section 7. Due to the continuum of consumers, in the absence of any aggregate shocks to the stochastic process for awareness, the evolution of awareness is deterministic for each product category.[28]

Zero-Removed Count Distribution As discussed, the interesting dynamics of the model come from the overlapping choice sets (conditional on there being at least one product). Hence, the notation can be simplified considerably by calculating defining moments of the distribution of the choice set sizes, n, conditional on $n > 0$. To remove the $n = 0$ point, take the pmf $f_n(a)$ and left-truncate it at $n = 1$. This gives a new pmf of $\frac{f_n(a)}{1-f_0(a)}$ for $n \geq 1$. Define this as a random variable $\hat{n}$ (i.e., $n \,|\, n > 0$). For any function, $g(n) : \mathbb{N}_+ \to \mathbb{R}$, the expectation of the (0 truncated) awareness state among consumers is defined as

$$\mathbb{E}_a \left[g(\hat{n})\right] \equiv \sum_{n=1}^{N} \frac{f_n(a)}{1 - f_0(a)} g(n) \tag{3}$$

To understand the role of these moments, Figure 6 shows an alternative interpretation as a simple network with a duopoly and a continuum of consumers (sorted on the vertical axis by their connections, for graphical clarity).[29] As the product category matures, fewer consumers have an awareness set with $n = 0$, which leads to mechanical growth. However, the mean awareness set size conditional on $n > 0$—i.e., $\mathbb{E}_a \left[\hat{n}\right] \equiv \mathbb{E}_a \left[n | n > 0\right]$—also increases, leading to more intense competition and sorting.

Baseline Awareness Process To fix a particular awareness process $\mathbb{Q}$ for the calibration and comparative statics, assume that each consumer has an intensity $\theta > 0$ of becoming aware of a firm in a product category, and an equal probability of becoming aware of a particular firm (including the potential of repeating a meeting with an existing firm in her information set, which does not add to the count). For generality, assume that customers can forget an

nothing in this model or mechanism that prevents using a discrete-time Markov chain.

[28]The continuum of consumers here is convenient for aggregation, but the mechanism is present in models with connections between a discrete number of consumers and producers. The discreteness is especially important in international trade, where the "consumers" are typically importers and distributors of a product, as in Eaton, Eslava, Krizan, Kugler, and Tybout (2014).

[29]If the network interpretation were extended to allows links between consumers in a social network, then the expansion of awareness could also be modeled as information diffusion over a network. A simple version of this is given in the word-of-mouth contagion process of Example 1. This is a different mechanism than in Rob and Fishman (2005), where word-of-mouth (from consumers living one period) spreads information about product quality, spurring further quality investment. Here, quality growth comes "passively" from sorting.

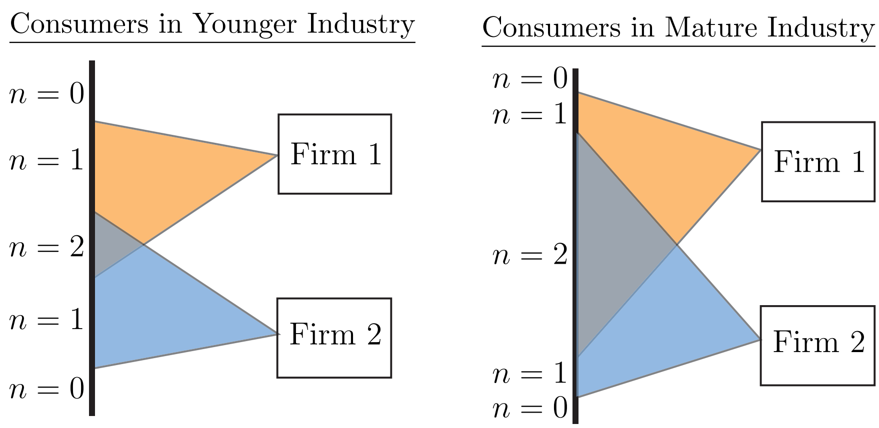

Figure 6: Awareness Sets as an Expanding Network

existing firm at rate $\mu \geq 0$.[30] To capture a typical S-curve of new product adoption—as in Mahajan, Muller, and Bass (1990)—also add word-of-mouth diffusion between existing customers. An arrival rate, $\theta_d \geq 0$, parameterizes this spreading of awareness, which is adjusted by the proportion of consumers already aware. To keep this process simple, I assume that the word-of-mouth spreads about adoption of the product category itself and not about customers of a particular firm (i.e., the additional θ_d arrivals are only to consumers with $n = 0$).

Example 1 (Baseline Awareness Process)**.** *Given μ, θ, and θ_d, awareness evolves according to the following age-dependent intensity matrix.*

$$\mathbb{Q}(a) = \begin{bmatrix} -(\theta+\theta_d(1-f_0(a))) & \theta+\theta_d(1-f_0(a)) & 0 & \dots & & \dots & 0 \\ \mu & -\mu-\frac{N-1}{N}\theta & \frac{N-1}{N}\theta & 0 & \dots & \dots & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & 0 & 0 \dots & \mu & -\mu-\frac{1}{N}\theta & \frac{1}{N}\theta \\ 0 & 0 & 0 & 0 \dots & 0 & \mu & -\mu \end{bmatrix} \in \mathbb{R}^{(N+1)\times(N+1)} \tag{4}$$

When $\mu = 0$, the KFE (2) with generator (4) separates into an ODE in the number of customers aware of no products in the category, $\boldsymbol{\partial}_a f_0(a) = -f_0(a)\left(\theta + \theta_d(1 - f_0(a))\right)$. With

[30] In queuing theory, this is called an M/M/1/K with customer balking (alternatively, "discouraged arrivals"). See Kleinrock (1975) Section 3.3). If $\mu > 0$, the "forgetting rate" is analogous to the "service time" in queuing theory. If $\mu = 0$, this has a simple degenerate stationary distribution of $f(\infty) = \begin{bmatrix} 0 & \dots & 1 \end{bmatrix}$. Some find a rapid depreciation of customer capital; for example, Eaton, Eslava, Krizan, Kugler, and Tybout (2014) calibrate a 27% yearly separation rate for distributors in a related search model. The decreasing hazard of separation found in these studies could be crudely approximated by having μ depend on age and n in $\mathbb{Q}$. With Example 1 and $\mu = 0$, as $f_0(a)$ is truncated when calculating moments of $\hat{n}$, calculations of $\mathbb{E}_a\left[g(\hat{n})\right]$ are identical to those in *Technical Appendix* Examples 1 and 2, with $\mu = 0$.

the initial condition $f_0(0) = 1$*, the solution to this ODE is*

$$f_0(a) = \frac{\theta_d + \theta}{\theta_d + \theta \exp\left((\theta_d + \theta)a\right)} \tag{5}$$

3.3 Consumers

Beyond the embedding of awareness in the consumer preferences, I keep the model and aggregation as close to the benchmark model (i.e., the neoclassical growth with CES preferences and monopolistic competition) as possible.

Consumers have constant elasticity of substitution (CES) preferences between product categories with an elasticity of $\kappa \equiv \frac{1}{1-\varsigma} > 1$. Define $\bar{y}_{mj}(a)$ as the quality-adjusted quantity of products purchased in product category $m \in [0, M]$. Nested within the standard Dixit-Stiglitz aggregation of product categories, quality- and price-adjusted goods within a product category are perfectly substitutable.[31]

Recall that ξ_{imj} is a time-invariant, idiosyncratic preference for each product in the economy. For simplicity, I will assume that these are distributed as draws from an independent Gumbel distribution upon firm entry.[32] The vector of time-invariant ξ_{imj} and the time-varying A_{mj} are the idiosyncratic state of a consumer.

While I will leave the evolution of awareness, parameterized by $\mathbb{Q}$, general when not simulating results, I will maintain that the distribution of A evolves independently from ξ—as derived more formally in Appendix A.1 and Assumption 2.

Preferences and Budget Constraints Assume that consumers discount the future at rate $\rho > 0$ and have a constant relative risk aversion (CRRA) of $\gamma > 1$. Furthermore, all consumers have a real income $\Omega(t)$ and a bundle price $P(t)$—both derived in general equilibrium in Section 4.

[31] While perfectly substitutable goods are atypical for a model aggregating to a representative agent, consumers here are differentiated more at the level of an IO discrete-choice model—which typically has each consumer purchasing a single good from each category to match the data (i.e., a consumer with full awareness of the ketchup product category would be unlikely to purchase both Heinz and Hunt's Ketchup during any period without relative price changes). In those models, the elasticity between goods occurs only *after* aggregation—which I derive as market shares for my model in Technical Appendix C.4. While each m is a narrow product category, these will integrate to a representative agent in Section 4, with variations in aggregated CES preferences, such as in (A.36).

[32] This formulation is equivalent to assuming that ξ change but are IID over time, as is often assumed in discrete-choice models. The Gumbel distribution is also known as the Extreme Value Type 1 Distribution, and has a cdf of $G(\xi) = e^{-e^{-\xi}}$. A key property that enables analytic solutions for the demand system is that the difference between two Gumbel random variables is distributed as a Logistic. Furthermore, the max-stability of the Gumbel distribution leads to convenient order-statistics for the aggregation in Section 4, where an individual consumer has a distribution of awareness set sizes across product categories. Without the Gumbel or max-stable distributions for ξ, numerical integrals might be required for calculating demand.

Given awareness sets $A_{mj}(t)$, idiosyncratic preferences ξ_{mj}, and nominal prices $\hat{p}_m(t)$ for all product categories and products, the consumer chooses demand per good $c_{imj}(t)$ to maximize the welfare,

$$\int_0^\infty e^{-\rho t} \frac{1}{1-\gamma} \left[\overbrace{\left(\int_0^{M(t)} \left(\sum_{i \in A_{mj}(t)} \bar{\Gamma} e^{\sigma \xi_{imj}} c_{imj}(t) \right)^{\varsigma} \mathrm{d}m \right)^{1/\varsigma}}^{\equiv C_j(t), \text{ a "Composite Good"}} \right]^{1-\gamma} \mathrm{d}t \tag{6}$$

The income of consumers includes: (1) rental of their inelastic supply of one unit of labor, at real wage $w(t)$; (2) rental of their capital stock $k(t)$ at real rate $r(t)$; and (3) real profits from ownership of the firms, $\Pi(t)$. The budget constraint for all purchases with intensive demand y_{imj} is,

$$\int_0^{M(t)} \left[\sum_{i \in A_{mj}(t)} \hat{p}_{im}(t) y_{imj}(t) \right] \mathrm{d}m \leq P(t)\Omega(t) \equiv P(t) \left(w(t) + r(t)k(t) + \Pi(t) \right) \tag{7}$$

where $\bar{\Gamma} \equiv \Gamma(1 - \sigma(\kappa - 1))^{1/(1-\kappa)}$ is a normalizing constant to adjust for $\mathbb{E}\left[e^{\sigma(\kappa-1)\xi}\right] \neq 1$, with $\Gamma(\cdot)$ the Gamma function.[33]

To compare (6) to CES and discrete-choice utility specifications: (1) Unlike many discrete-choice preferences, the idiosyncratic ξ_{ij} term is multiplicative rather than additive, and there is an intensive choice of quantity. Economically, this represents a consumer's demand increasing as she meets firms that better match her needs (i.e., an implication of customer sorting) and enables homothetic aggregate preferences; (2) the parameter $\sigma > 0$ determines the degree of differentiation within a product category and has a direct analogy to the variance of the random utility shocks in discrete-choice theory; and, (3) the consumer can purchase only from firms in the set $A_j(a)$. This constraint breaks the standard aggregation to a representative consumer with nested CES preferences since the set of goods that each consumer can substitute between is idiosyncratic and evolving. However, as derived in Section 4, max-stable (e.g., Gumbel) ξ distributions will enable simplifications in the aggregation process.

[33] Within the trade and macro literature, the preference specification here is similar to preferences in Atkeson and Burstein (2007) and Fajgelbaum, Grossman, and Helpman (2011). The closest specification of discrete-choice preferences for a single industry with heterogeneous choice sets is that of Goeree (2008)—although I have added multiplicative idiosyncratic preferences, an intensive margin, and choice set dynamics. As discussed earlier, the perfect quality- and price-adjusted substitutability between goods within the category is consistent with each m being a narrowly defined product category.

Dynamic Decisions To nest the neoclassical growth model and determine marginal costs to compute factor shares, allow both capital savings and an entrepreneurship decision to endogenize the number of product categories available. This setup is nearly standard, and I use the simplifying assumption that capital goods are created using the same technology and with the same elasticities as consumption goods. Hence, some of y_{imj} purchased through (7) is used for direct consumption c_{imj} in (6), while others will produce homogeneous capital goods or create new product categories. Following models of monopolistic competition with capital, these types of innocuous assumptions will save on notation, but are qualitatively unimportant since a competitive final goods aggregator is isomorphic to CES aggregation within the preferences. See Section 4.3 for more details and Appendix B.2 for the aggregation proof.

Once a product category is invented, it is licensed to operating firms to create a particular industry (i.e., m). I will solve the planning problem in which the consumer directly chooses the allocations for consumption $C(t)$, capital investment to expand $k(t)$, and R&D to expand $M(t)$.[34] With the standard assumption that the consumer can distribute aggregated production into consumption, $C(t)$, investment in capital, $i_k(t)$, and investment in new product categories, $i_M(t)$, the resource constraint is,

$$Y(t) = C(t) + i_k(t) + i_M(t) \tag{8}$$

The consumer rents the capital and inelastic labor to firms, and owns the licensing rights which for each product category. When a new product category is created, the household licenses the blueprints to the N firms starting up production in the industry. As the consumer owns a perfectly diversified portfolio of both the entrepreneurial licensing and the operating firms, the particular split of the surplus between the licensor and licensee does not matter for aggregates.

Capital depreciates at rate $\delta_k > 0$; product categories effectively depreciate at rate $\delta_M > 0$; and the relative productivity of inventing new product categories is $z_M(t)$—all described

[34] An alternative approach would be to assume free entry of entrepreneurs, markets for the production of capital and new industries, and decentralization through the competitive rental rates, such as in (31). However, from standard growth theory and the lack of any externalities (unlike the examples in Atkeson and Burstein (2015)), this decentralized equilibrium would have identical allocations. As the aggregated production function is homothetic, another interpretation (if investment is weakly positive) is that there are parallel technologies for the creation of homogeneous capital and homogeneous new product categories from differentiated intermediates.

in more detail in Section 4.2 and (34):

$$\boldsymbol{\partial}_t k(t) = -\delta_K k(t) + i_k(t) \tag{9}$$
$$\boldsymbol{\partial}_t M(t) = -\delta_M M(t) + z_M(t) i_M(t) \tag{10}$$

The dynamic decisions will be easier to analyze after the consumption decisions are complete and the model is aggregated to a representative agent in Section 4.

Static Decisions As consumers' choices do not affect their future awareness (e.g., they have no habits directly in preferences), demand can be solved as a static optimization problem. For notational clarity, drop the t index and use the age a of a given product category. Define the real price as $p_i(a) \equiv \hat{p}_i(a)/P$, and solve for the demand functions:

Proposition 1 (Intensive Demand)**.** *Given real prices $p(a)$ and real income Ω, for each product category m:*

1. *Of those with non-empty awareness sets, almost every consumer purchases from a single firm per product category. A consumer purchases product i and no others if and only if*

$$\log\left(p_{i'}(a)\right) - \log\left(p_i(a)\right) > \sigma\left(\xi_{i'j} - \xi_{ij}\right), \quad \forall\, i' \in A_j(a) \setminus \{i\} \tag{11}$$

2. *The intensive demand for product i is*

$$y_{ij}(a, \xi_{ij}) = \bar{\Gamma}^{\kappa-1} e^{\sigma(\kappa-1)\xi_{ij}} p_i(a)^{-\kappa} \Omega \tag{12}$$

3. *The price index is a function of the preferences, ξ_{mj}, and nominal prices, $\hat{p}_m$,*

$$P_j(a) \equiv \bar{\Gamma}^{-1} \left(\int_{|\mathcal{I}_{jm}(a)|>0} e^{\sigma(\kappa-1)\xi_{imj}} \hat{p}_{im}(a)^{1-\kappa} \mathrm{d}m \right)^{\frac{1}{1-\kappa}} \tag{13}$$

Proof. Use the fully differentiated version in Appendix A.2 with $q_i = \bar{\Gamma}$ for all i. □

Unlike many macroeconomic models with differentiated goods, even if all consumers are given the same nominal income, they can have different real incomes, $\Omega_j(a)$. The issue is that the price index in (13) depends on the products in their choice set, which is idiosyncratic. However, given a continuum of product categories and independence of ξ from awareness evolution, infinitely lived consumers are shown to have a common bundle price P for aggregation in Section 4.

Demand

The quantity demanded *conditional on purchasing* product i is a function of ξ_i in (12). Accordingly, the distribution of ξ_i conditional on purchasing product i will not be the unconditional distribution of ξ_i in the economy, except for the subset of agents aware of only firm i. Over time, as the relative proportion of consumers with awareness of multiple firms increases, the average match intensity moves away from the mean. With the multiplicative preference parameter, this adds in a sorting effect of higher demand from a better average match value. To solve for the demand curve faced by firm i, integrate (12) over all consumers $j \in [0,1]$:

Definition 1 (Total Demand)**.** *Given the distribution over $A_j(a)$ and ξ_j, the total demand for firm i as a function of the price vector p is,*

$$y_i(a,p) \equiv \int_{[0,1]} y_{ij}(p,\xi_{ij})\mathbb{1}\{\textit{Choose } i \textit{ from } A_j \textit{ given } p \textit{ and } \xi_j\}\,\mathrm{d}j \tag{14}$$

Since demand is a function of the ξ_{ij} conditional on purchasing product i, it is necessary to ensure that the degree of differentiation is not too high relative to the structure of the idiosyncratic preferences, or (14) may not be defined. This property is similar to the comparison of within- and between-industry substitutability in Atkeson and Burstein (2008).[35]

Assumption 1 (Degree of Differentiation)**.** *Assume that $0 < \sigma < \frac{1}{\kappa-1}$ and $\xi_j \sim$ Gumbel.*

Recall that the random variable of awareness set sizes (conditional on being non-zero) is $\hat{n}$, and the time-dependent mass of agents with no empty awareness sets is $f_0(a)$—both deriving from the properties of $\mathbb{Q}$. From this, for symmetric firms, Definition 1 is:

Proposition 2 (Total Demand for N Symmetric Firms)**.** *Given Assumption 1, the independence of ξ_j and $A_j(a)$, and that every firm $i' \neq i$ chooses the price $\bar{p}$, the demand curve from Definition 1 faced by firm i choosing p is*

$$y(a,p,\bar{p}) = \frac{1-f_0(a)}{N}p^{-1-1/\sigma}\mathbb{E}_a\left[\hat{n}\left(p^{-1/\sigma}+(\hat{n}-1)\bar{p}^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}\right]\Omega \tag{15}$$

If an equilibrium exists where $p = \bar{p}$, then

$$Ny(a,p) = \underbrace{(1-f_0(a))}_{\substack{\textit{Limited}\\ \textit{Awareness}}}\underbrace{\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right]}_{\textit{From Sorting}}\underbrace{p^{-\kappa}\Omega}_{\substack{\textit{Typical}\\ \textit{CES}}} \tag{16}$$

[35] The σ parameter of the Gumbel changes the cross-product elasticity, as in typical discrete-choice models. However, unlike typical discrete-choice models, if elasticity were measured from market shares, it would show time variation due to dynamic choice set sizes. More mature industries would tend to have higher elasticities.

If the firm is a monopolist, then

$$y(a,p) = (1 - f_0(a))p^{-\kappa}\Omega \tag{17}$$

Proof. See Proposition 9 and Appendix A.4 for a derivation with both symmetric and fully asymmetric firms and awareness sets. □

If $N = 1$, then (17) is difficult to distinguish from a model of monopolistic competition with a mean-reverting "productivity" shock. In contrast, even with symmetric firms and prices, if $N > 1$, the dispersion of idiosyncratic preferences, σ, changes the total demand through consumer sorting. The intuition is that the more intense the degree of product differentiation, the larger is the mean idiosyncratic match quality conditional on purchasing from a firm. From the consumer's preferences, conditional on consumer j choosing product i, the quality per unit consumed is proportional to $e^{\sigma\xi_{ij}}$—which, in turn, depends on the match quality, ξ_{ij}.

Crucially, with symmetric prices, the total product category demand, $Ny(a,p)$, in (16) is independent of the number of firms N, except through the stochastic process for $\hat{n}$. Comparing (16) and (17), the only difference in product category demand with multiple firms vs. a monopolist is the additional $\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right]$ term, which summarizes the effective quality growth from the sorting of consumers into better matches. Note that if $\sigma = 0$ (i.e., no product differentiation), then total product category demand conditional on a price would be the same as that of a monopolist due to the lack of sorting effects (though the equilibrium price strategy would be different). Also, from Assumption 1, if $\hat{n}$ is a sub-martingale, then conditional on an income and price, demand is weakly increasing over time due entirely to customer sorting into better matches. A sub-martingale property would hold for any reasonable $\mathbb{Q}$ with a low-awareness initial condition.

3.4 Firms

The objective of a firm is to maximize the PDV of profits using the discount rate r (see Appendix A.6 for the standard definition of firm value), subject to a standard production function.

Production Technology, Profits, and Value

All firms have identical production functions with TFP z and a constant returns-to-scale Cobb-Douglas production function in labor ℓ_i and capital K_i—with the output elasticity of capital of $\alpha \in (0,1)$. Labor and capital are rented from competitive markets at real factor prices $w(t)$ and $r(t)$. The cost minimization problem to produce y goods is: $\min_{\ell,K}\{rK + w\ell\}$ subject to $y = zK^{\alpha}\ell^{1-\alpha}$. From the standard properties of Cobb-Douglas production functions and competitive markets for the factors, the optimal capital-labor ratio is $k = \frac{\alpha}{1-\alpha}\frac{w}{r}$, and marginal cost mc as a function of aggregates z, k, and w is,

$$mc \equiv \frac{1}{1-\alpha} z^{-1} k^{-\alpha} w \tag{18}$$

Firms' Problem

Firms in an industry compete through repeated Bertrand competition, choosing a price function $p_i(a+\tau)$ for $\tau \geq 0$ to maximize the firm value, given the equilibrium pricing decisions of the other firms in the industry.[36]

There are no dynamic incentives for this pricing game. For example, since $A_j(a)$ is independent of consumption choices, there is no profitable deviation for a firm to choose lower prices to attract customers early in its life cycle since consumers can costlessly switch to any product in their information set. This initially seems to contradict papers such as Burdett and Coles (1997), in which firms have an incentive to lower initial prices to build customer habits for their good. However, as will be discussed further in Section 6.1, this model provides an explanation for how new entrants to an existing industry may initially have lower prices than incumbents, but average industry markups themselves are decreasing as the product category ages (and as I found in Figure 4).

As with most dynamic games, there are a multiplicity of Nash Equilibria, but this paper focuses on repetition of a pure-strategy equilibrium for the static Nash Equilibrium in every period. By standard results of game theory, the strategy of repetition of the Nash Equilibrium of a stage game is a Nash Equilibrium of the repeated game. Given a set of prices for other firms in the industry, each firm chooses the price to maximize its period profits:

Definition 2 (Bertrand Nash Equilibrium (BNE))**.**
A BNE is a pure-strategy Nash Equilibrium for each stage game at each period a. That is, $p(a) \in \mathbb{R}^N$, $\forall\, a$ such that

$$p_i(a) = \arg\max_{\tilde{p}\geq 0} \left\{ (\tilde{p} - mc) y_i(a, \{\tilde{p}, p_{i'}(a)\}_{i' \neq i}) \right\}, \; \forall i \in \mathcal{I}, a \geq 0 \tag{19}$$

Unlike Bertrand Competition under undifferentiated products and symmetric firms, prices are not driven to marginal costs with only a duopoly. Part of the reason is the market power inherent in product differentiation, as seen in the downward-sloping demand curve of Proposition 1. This ensures that an infinitesimal change in price does not result in a discrete jump in profits. The more interesting reason in the context of this model is information

[36] Atkeson and Burstein (2007, 2008), Edmond, Midrigan, and Xu (2015) and Peters (2015) investigate markup heterogeneity across firms and industries. In Arkolakis (2010) and Eaton, Eslava, Krizan, Kugler, and Tybout (2014), consumers are heterogeneous over which firms they meet in every period, but not over their preferences for a particular good.

frictions. Even with very low product differentiation, a firm can extract monopolistic profits from consumers who are aware of only its product, which ensures that deviations from marginal cost are always profitable in the absence of full information.

For symmetric equilibria, define a measure of the age-dependent average quality of matches, $q(a)$, and markup over marginal cost, $\Upsilon(a)$:

$$q(a) \equiv \mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right] \tag{20}$$

$$\Upsilon(a) \equiv 1 + \sigma\left[1 - (1 - \sigma(\kappa-1))\frac{\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)-1}\right]}{\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right]}\right]^{-1} \in [1+\sigma, \frac{\kappa}{\kappa-1}] \tag{21}$$

Then, for symmetric firms, Definition 2 can be solved as:

Proposition 3 (Symmetric Bertrand-Nash Equilibrium)**.** *If a symmetric pure-strategy equilibrium exists for N firms according to Definition 2, then*[37]

$$Y(a) \equiv Ny(a) = (1 - f_0(a))p(a)^{-\kappa}q(a)\Omega \tag{22}$$

$$\Pi(a) \equiv N\pi(a) = (1 - f_0(a))(p(a) - mc)p(a)^{-\kappa}q(a)\Omega \tag{23}$$

$$p(a) = \Upsilon(a)mc \tag{24}$$

Proof. See Appendix A.5. □

The solutions in Proposition 3 hold for any law of motion as parameterized by $\mathbb{Q}$—which, in turn, determines the $f_0(a)$ mass of unaware consumers, and the $\hat{n}$ stochastic process for awareness set sizes.[38] Comparing to the earlier price-dispersion literature, such as Burdett and Judd (1983) and Burdett and Coles (1997), we have the possibility here for a symmetric price due to the downward sloping demand functions when $\sigma > 0$. However, if $\sigma = 0$, the dynamics would be more similar to Burdett and Judd (1983) with price dispersion at the

[37]While a symmetric pure-strategy equilibrium always exists with the parameters calibrated in this model, they would not in general. If σ is very low, then competition over customers with $\hat{n} > 1$ can intensify towards marginal cost, according to the usual Bertrand logic. However, unlike in a standard Bertrand equilibrium with differentiated goods, the competition may be so fierce that firms have a profitable deviation to charge the monopolistic price and profit from customers with $\hat{n} = 1$ in their awareness set (i.e., become a small, niche firm with high markups and low growth potential). Beyond showing that marginal cost pricing with Bertrand competition (under full awareness) is a knife-edge result, this mixed strategy is largely uninteresting. The conditions for uniqueness are complex and depend on a particular $\mathbb{Q}$.

[38] The first-order approximation to (24) is a simple function of the expected awareness set size,

$$p(a) \approx \left(1 + \sigma\left(1 - (1 - \sigma(\kappa-1))/\mathbb{E}_a[\hat{n}]\right)^{-1}\right)mc, \quad \text{for } a > 0 \tag{25}$$

Furthermore, if $\mathbb{Q}$ follows *Technical Appendix* Example 2 with $\mu = 0$, then $\bar{n}(a) = \theta a$, and the markup converges towards $1 + \sigma$ at the rate of $1/a$. This result is consistent with the general results in Gabaix, Laibson, Li, Li, Resnick, and de Vries (2016).

monopolistic competitive and marginal cost—the proportion dynamically changing as the consumer information sets expand.

Industry Equilibrium The equilibrium of a particular industry, formally defined in Appendix A.6, is fully parameterized by the awareness evolution process $\mathbb{Q}$ and the preference parameters σ and κ, with an aggregate state—exogenous to the decisions of a particular industry—of real income Ω and real marginal cost mc.[39] The evolution of awareness is deterministic if $\mathbb{Q}$ does not itself vary stochastically, while changes in aggregates Ω and mc would lead to changes in the period prices and profits—as they are static decisions in this simple repeated game—but not to markups $\Upsilon(a)$ or quality $q(a)$ as shown in Proposition 3.

4 Model Aggregation

From Proposition 3, the aggregate state necessary for firm decisions is summarized by the real income $\Omega(t)$ and the marginal costs $mc(t)$, while the state of a product category is summarized entirely by its age. The age, in turn, is used to find the age-dependent moments of awareness set sizes, $\hat{n}$, through the infinitesimal generator $\mathbb{Q}$ and (2). Furthermore, from Proposition 3, the industry profits, demand, and prices are independent of the number of firms in the industry N (except through any dependence of $\mathbb{Q}$ on N). These properties of the industry equilibrium enable an aggregation to a nearly standard neoclassical growth model with a single productivity wedge due to incomplete awareness, and clear distortions on the profit share.

For simplicity, assume a common awareness process $\mathbb{Q}$ across all product categories, and define the aggregate age- and time-dependent product category variables as real output $Y(t,a) \equiv Ny(t,a)$, real profits $\Pi(t,a) \equiv N\pi(a)$, real capital $K(t,a) \equiv Nk(t,a)$, and real valuation $V(t,a) \equiv Nv(t,a)$. In all cases, when the a is suppressed, it is the aggregate across all product categories (e.g., aggregate real output $Y(t)$ is the sum of $Y(t,a)$ across the distribution of all product categories of age a.)

While the markups $\Upsilon(a)$ in (24) are functions of the age of the industry, the marginal costs $mc(t) = \frac{1}{1-\alpha}z(t)^{-1}k(t)^{-\alpha}w(t)$ in (18) are completely standard. Consequently, the proportional prices $p(t,a) = \Upsilon(a)mc(t)$ are easily decomposed into a time-dependent aggregate component and an age-dependent industry component. Appendix B.1 derives this using standard techniques. Just keep in mind that, due to distortions caused by incomplete choice

[39] As the evolution of awareness is deterministic from the perspective of a firm, more elaborate entry and exit decisions could be modeled through standard approaches such as in Jovanovic (1982) and Hopenhayn (1992). Here, in the baseline examples in Section 3.2 with invariant productivity, all entry would optimally happen at $a = 0$ since returns from entry are decreasing as the market matures. Models of exit and entry are well studied, and nesting them would add little to my model.

sets, the output elasticity of capital will not be the same as the capital share of output.

The lack of any sensitivity to the number of firms, N, in the calibrations and solutions is especially important for aggregation, and makes the assumption that the same number of firms exist in each industry entirely harmless. As discussed in Section 3.2, the key is the effective number of competitors as summarized by moments of $\hat{n}$, rather than the actual number of competitors, N. Here, it is not necessary for the $\mathbb{Q}$ to have a stationary distribution for $\hat{n}$, or for $\mathbb{E}_a[\hat{n}]$ to converge–as long as it grows more slowly than the obsolescence rate.

4.1 Consumption Goods Aggregation

Assume that product categories can be born at different times, t, and have an age of $a \geq 0$. Define: (1) $\hat{\Phi}(t,a)$ as the mass of product categories with age less than a (i.e., an unnormalized cdf of product category ages at time t); (2) the total number of product categories at time t as $M(t) \equiv \hat{\Phi}(t,\infty)$; (3) the normalized cdf of product category ages by the $\Phi(t,a) \equiv \hat{\Phi}(t,a)/M(t)$, so that $\Phi(t,\infty) = 1$; and (4) the integral over the time t age distribution as $\mathbb{E}_t[g(a)] \equiv \int_0^\infty (g(a)\boldsymbol{\partial}_a\Phi(t,a))\,\mathrm{d}a$. For example, $\mathbb{E}_t[a]$ is the mean product category age at time t.

If the product category age distribution, $\hat{\Phi}(t,a)$, or the mass, $M(t)$ changes over time, then the price index will be time-dependent. Use the "composite good" for the demand-weighted CES preferences as $Y_j(t)$ from (6). In Appendix B.2, the accompanying price index to this composite good is shown to be identical for all consumers,

$$P(t) \equiv \left(\underbrace{M(t)}_{\substack{\text{Variety}\\\text{Effect}}} \int_0^\infty \underbrace{q(a)\hat{p}(t,a)^{1-\kappa}}_{\text{Quality adjusted price}} \underbrace{(1-f_0(a))}_{\substack{\text{Proportion}\\\text{Aware}}} \underbrace{\mathrm{d}\Phi(t,a)}_{\substack{\text{Age}\\\text{Distribution}}}\right)^{\frac{1}{1-\kappa}} \tag{26}$$

Proposition 4 shows that (26) delivers an aggregation generalizing that of monopolistic competition. Define the aggregate "measured" TFP, $Z(t)$, the factor share distortion, $B(t)$, and the cost of living quality adjustment, $Q(t)$, as,

$$Q(t) \equiv \left[\mathbb{E}_t\left[(1-f_0(a))\Upsilon(a)^{1-\kappa}q(a)\right]\right]^{\frac{1}{\kappa-1}} \tag{27}$$

$$B(t) \equiv \frac{\mathbb{E}_t\left[(1-f_0(a))\Upsilon(a)^{-\kappa}q(a)\right]}{\mathbb{E}_t\left[(1-f_0(a))\Upsilon(a)^{1-\kappa}q(a)\right]} \tag{28}$$

$$\underbrace{Z(t)}_{\substack{\text{"Measured"}\\\text{TFP}}} \equiv \underbrace{z(t)}_{\substack{\text{Physical}\\\text{TFP}}} \underbrace{M(t)^{\frac{1}{\kappa-1}}}_{\text{Varieties}} \underbrace{Q(t)}_{\substack{\text{Quality}\\\text{Distortion}}} \underbrace{B(t)^{-1}}_{\substack{\text{Factor Share}\\\text{Distortion}}} \tag{29}$$

With these definitions, the effects of limited choice sets are summarized by the $B(t)$ and $Q(t)$ distortions on otherwise standard aggregate variables:

Proposition 4 (Time Varying Price Index, TFP, and Real Wages)**.** *As functions of the aggregate state, $z(t), k(t)$, $\Phi(t,z)$, and $M(t)$, the real marginal cost and wages are*

$$mc(t) = M(t)^{\frac{1}{\kappa-1}}Q(t) \tag{30}$$
$$w(t) = (1-\alpha)Z(t)B(t)k(t)^{\alpha} \tag{31}$$

"Composite" good production aggregates to a function of TFP and is identical to the real income,

$$Y(t) = Z(t)k(t)^{\alpha} = \Omega(t) \tag{32}$$

Proof. See Appendix B.2. □

(32) shows that this aggregates to a representative consumer in the same sense as monopolistically competitive models, and that all consumers gain the same utility per unit of expenditure—confirming the guess in (26). As I assume identical incomes, (26) also shows that consumers have identical price indices and utility.

When $N > 1$, the term $Q(t)B(t)^{-1}$ is a measure of average quality adjusted for awareness and markup distortions. The wedge in productivity, factor shares, etc. enters through $Q(t)$ and $B(t)$ as calculated from the age distribution, $\Phi(t,a)$.

In (30) to (32), the $1 - f(a,0)$ adjustment embedded in $Q(t)$ is the age-dependent limited awareness product categories. Due to the continuum of product categories, this can be calculated by the probability that a consumer is aware of one or more firms in a product category through $\mathbb{Q}$. This adjustment to the marginal cost and price index calculations shows the importance of weighting new products within a bundle by the degree of awareness in the economy (e.g., calculating the CPI by the BLS). For example, if the proportion of new product categories were over-weighted, then price inflation could be overstated compared to a typical consumer's bundle. The weighting by quality $q(a) \equiv \mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right]$ within $Q(t)$ encapsulates the quality growth due to the sorting effect and should be considered in the measurement and weighting of empirical price indices.

Distortions to TFP The new aggregate TFP index in (29) takes the physical TFP, $z(t)$, and adjusts for increasing quality due to sorting, variety effects, and variable markups—all weighted by the age distribution. By comparing (32) to a standard neoclassical growth model with monopolistic competition, this can be thought of as the TFP for a representative firm producing a generic unit of consumption.

I have labeled this $Z(t)$ as "measured TFP" due to its aggregation to a standard Cobb-Douglas production function in (32). If we naively estimated the TFP from the residual of a growth regression, we would find $Z(t)$ rather than the physical productivity $z(t)$. While growth regressions often take into account the number of varieties, as in the $M(t)$, they would not take into account the $Q(t)B(t)^{-1}$ term in (29). Therefore, even if physical TFP $z(t)$ were to remain constant, variations in $Z(t)$ could occur if the age distribution of product categories changes. In particular, if there is too much product obsolescence and/or too many new product categories in the bundle, the distribution of products will have poor sorting (i.e., low average $q(a)$), incomplete market penetration for consumers (i.e., high average $f_0(a)$), and distortions in the relative markups (i.e., skewed $\Upsilon(a)$). This, in turn, can decrease TFP relative to an economy with more mature industries. Even if newer varieties had a higher $z(t)$, this would slow down the impact of new varieties on aggregate output, and provide a countervailing effect on productivity in the meantime.

In the nested model with monopolistic competition and constant markups, the cost-of-living distortion and the factor share distortion cancel out, leaving only a variety effect and physical TFP—i.e., $Z(t) \equiv z(t)M(t)^{\frac{1}{\kappa-1}}\mathbb{E}_t\left[(1-f_0(a)\right]^{\frac{1}{\kappa-1}}$. Awareness is important only insofar as it distorts the effective number of varieties available to consumers through $1-f_0(a)$. As is standard in models with monopolistic competition, constant markups do not distort aggregate allocations in production with inelastic labor supply.

4.2 Evolution of the Age Distribution

To provide a baseline model for a stationary age distribution, assume that consumer taste shocks (potentially induced by the creation of new varieties, or due to Shumpeterian forces) make product categories obsolete and kill industries at a constant rate $\delta_M > 0$.[40] On the other side, new product categories/industries enter as a result of R&D investment at a rate $\hat{x}(t) > 0$.[41] As discussed in Section 2.1, the obsolescence rate δ_M approximates the rate at which narrowly defined product categories cease to be in consumers' choice sets.

Conditional on the product category creation rate $\hat{x}(t)$—as chosen by consumers through R&D investment i_M in Section 4.3 and (10)—the KFE for the normalized age distribution is

$$\boldsymbol{\partial}_t\hat{\Phi}(t,a) = \underbrace{-\boldsymbol{\partial}_a\hat{\Phi}(t,a)}_{\text{Age Increase}} - \underbrace{\delta_M\hat{\Phi}(t,a)}_{\text{Obsolescence}} + \underbrace{\hat{x}(t)}_{\substack{\text{New Prod.}\\ \text{Categories}}} \tag{33}$$

[40] An alternative version of the model would be a (potentially endogenous) growth model with no product category depreciation, and some decreasing returns in $\boldsymbol{\partial}_t \log M(t)$. Atkeson and Burstein (2015) provide an interpretation of "social discounting of innovation" in models with innovation of varieties and productivity as analogous to physical capital depreciation if there was no investment in innovation. If the version presented here is considered a normalization to a BGP, my model could be interpreted in a similar way.

[41] This approach emphasizes the creation and destruction of new products and varieties rather than of firms. Broda and Weinstein (2010) similarly discusses changes in the varieties available to consumers. In this paper, while firms produce only one product, at the aggregate level this distinction does not matter, and the organization of products among firms is indeterminate as long as firms have no span-of-control issues.

Given the total number of product categories $M(t)$, define the proportional entry rate as $x(t) \equiv \hat{x}(t)/M(t)$ and the normalized age distribution as $\Phi(t,a) \equiv \hat{\Phi}(t,a)/M(t)$. Then, given entry rate $\hat{x}(t)$ and an initial condition $\hat{\Phi}(0,a)$, the number of product categories evolves according to

$$\boldsymbol{\partial}_t M(t) = -\delta_M M(t) + \hat{x}(t), \quad \text{s.t. } M(0) = \hat{\Phi}(0,\infty) \tag{34}$$

With constant entry and destruction rates, the proportional entry converges to the obsolescence rate, $x = \delta_M$; the total mass of product categories is $M = \hat{x}/\delta_M$; and the normalized, stationary distribution of product category ages is exponentially distributed,

$$\Phi(a) = 1 - e^{-\delta_M a} \tag{35}$$

See Appendix B.3 for a standard derivation of these results.

4.3 Consumers' Problem with Capital Investment and R&D

Through (29) and (32), I use the aggregation of consumer decisions into a "composite" good from the perspective of the consumers, $Y(t)$, and an aggregate TFP of $Z(t)$. So, even though consumers have idiosyncratic preferences, and choice sets are heterogeneous, I can solve a model with a representative agent.[42] Following standard growth theory to keep the model simple, I assume that capital investment is reversible and is normalized to have the same productivity as that of consumption goods. The marginal productivity of creating new product categories (and accompanying industries) is $z_m(t)$, and I assume that product category investment, $i_M(t) \geq 0$, is not reversible—though this constraint will not be binding in most of my analysis. Use the consumer's utility, laws of motion in (6), (9) and (10), and the aggregation in (29) and (32) with choices for consumption and investment to find:

Proposition 5 (Problem of the Representative Agent)**.** *Given initial conditions* $k(0), M(0)$,

[42] A consideration here is the assumption of conversion of consumption goods to capital or new industries. In order to simplify the interpretation of comparative statics, I assume that the markets for creating new industries and new capital have the same awareness friction as the market for consumption goods, and the same α. If I wanted to have completely frictionless markets for homogeneous capital and variety intermediates, I could simply adjust the productivity to undo the awareness wedge for that market—e.g., $\boldsymbol{\partial}_t k(t) = -\delta_k k(t) + Z(t)Q(t)^{-1}B(t)k(t)^{\alpha} i_k(t)$.

and $\Phi(0, a)$*, the representative consumer solves*

$$\max_{i_k(t), i_M(t), C(t)} \left\{ \int_0^\infty e^{-\rho t} \frac{1}{1-\gamma} C(t)^{1-\gamma} \right\} \tag{36}$$

$$s.t.\ \boldsymbol{\partial}_t k(t) = -\delta_K k(t) + i_k(t) \tag{37}$$

$$\boldsymbol{\partial}_t M(t) = -\delta_M M(t) + z_M(t) i_M(t) \tag{38}$$

$$i_M(t) \geq 0 \tag{39}$$

$$C(t) \equiv z(t) \underbrace{Q(t)B(t)^{-1}}_{\substack{\textit{Awareness} \\ \textit{Wedge}}} M(t)^{\frac{1}{\kappa-1}} k(t)^\alpha - i_k(t) - i_M(t) \tag{40}$$

where $\Phi(t, a)$ *evolves according to* (33)*, which, in turn, determines* $Q(t)B(t)^{-1}$ *through* (27) *and* (28)*.*

Proof. See Technical Appendix A. □

Proposition 5 shows that—at the aggregate level—all of the effects of awareness embedded in a neoclassical growth model manifest in the $Q(t)B(t)^{-1}$ wedge, which depends on the evolving age distribution of firms $\Phi(t, a)$ and, hence, the underlying awareness process $\mathbb{Q}$.

4.4 Equilibrium

The general solution to Proposition 5 is given in Technical Appendix Proposition 1. The analysis of this is mechanically very similar to that of the neoclassical growth model with endogenous human capital accumulation, as in Acemoglu (2009), Proposition 10.1. Here, the product categories $M(t)$ are analogous to the level of human capital. As in the neoclassical model with human capital, the $M(t)$ and $k(t)$ are maintained as a statically determined ratio to whatever extent reversibility in capital and product categories is allowed. The main difference comes out of the time variation of the awareness wedge, as captured in the $Q(t)^{-1}B(t)$ term.

The stationary equilibrium is defined as the solution to a set of equations, given the parameters $\rho, \delta_k, \delta_M, \alpha, \kappa, \sigma, z, z_M$ and $\mathbb{Q}$—where z and z_M affect only the scale:

Proposition 6 (Stationary Equilibrium)**.** *Let the stationary capital and number of product categories be* k *and* M*. Then, with a* $z(t) = 1$ *normalization, and a constant* z_M*, the stationary equilibrium is an* M *and* k *solving the system of equations,*

$$\delta_M - \delta_k = QB^{-1} k^\alpha M^{\frac{1}{\kappa-1}} \left(\frac{z_M}{\kappa - 1} M^{-1} - \alpha k^{-1} \right) \tag{41}$$

$$\rho + \delta_k = \alpha QB^{-1} M^{\frac{1}{\kappa-1}} k^{\alpha-1} \tag{42}$$

Given the k and M, the equilibrium C is

$$C = QB^{-1}M^{\frac{1}{\kappa-1}}k^{\alpha} - \delta_k k - \delta_M M/z_M \tag{43}$$

where $\Phi(a) = 1 - e^{-\delta_M a}$, and $\Upsilon(a), q(a), Q,$ and B are given by (20), (21), (27) and (28)—and parameterized by any $\mathbb{Q}$. The capital share, labor share, and profit share of output are $\alpha B, (1-\alpha)B$ and $(1-B)$, respectively.

Proof. See Technical Appendix A. □

From (27) and (28), in monopolistic competition where $N = 1$, the markup is constant, $\Upsilon(a) = 1/\varsigma$ and $q(a) = 1$, and the wedge QB^{-1} is distorted only by the slow implementation of varieties through $1 - f_0(a)$. If the variety produced by the monopolist were immediately in all consumers' choice sets, then QB^{-1} would include only the average quality, and the equilibrium would be fully efficient—as is standard in models of monopolistic competition. Furthermore, in the labor share of output wedge in (28), B is constant at $\varsigma \in (0, 1)$ for any distribution of awareness, showing that labor shares are invariant under monopolistic competition regardless of awareness—the standard result.

5 Calibrated Simulations

In order to better understand the forces of the model with reasonable parameter values, this section uses a calibrated model to simulate industries and perform comparative statics. First, Section 5.1 looks at a single industry and shows patterns of profits, markups, and Tobin's Q, consistent with the evidence presented in Figure 4 and Sections 2.2 and 2.3. Understanding that the composition of industry ages matters for the aggregates, Section 5.2 shows how secular changes could be consistent with the evidence presented in Section 2.1. Finally, Section 5.3 looks at the transition dynamics of the model to consider the role that cyclical changes in the age distribution play in affecting aggregates.

Appendix E discusses the calibration and related data in detail, and Table 4 provides a summary of the parameters.

5.1 Industry Equilibrium

Figure 7 provides a simulation of an industry with a $\mathbb{Q}$ from the calibrated awareness process of Example 1. In order to highlight the role of within-product category differentiation, the figure shows valuations for both σ_ℓ and σ_h—both within the range discussed in Appendix E.2. The markups, quality, profits, and output are shown for the σ_ℓ case and are normalized to the maximum value on the interval. The low degree of product differentiation shows up as rapidly decreasing markups and, consequently, non-monotone profit and value functions, even while industry output continues to climb.

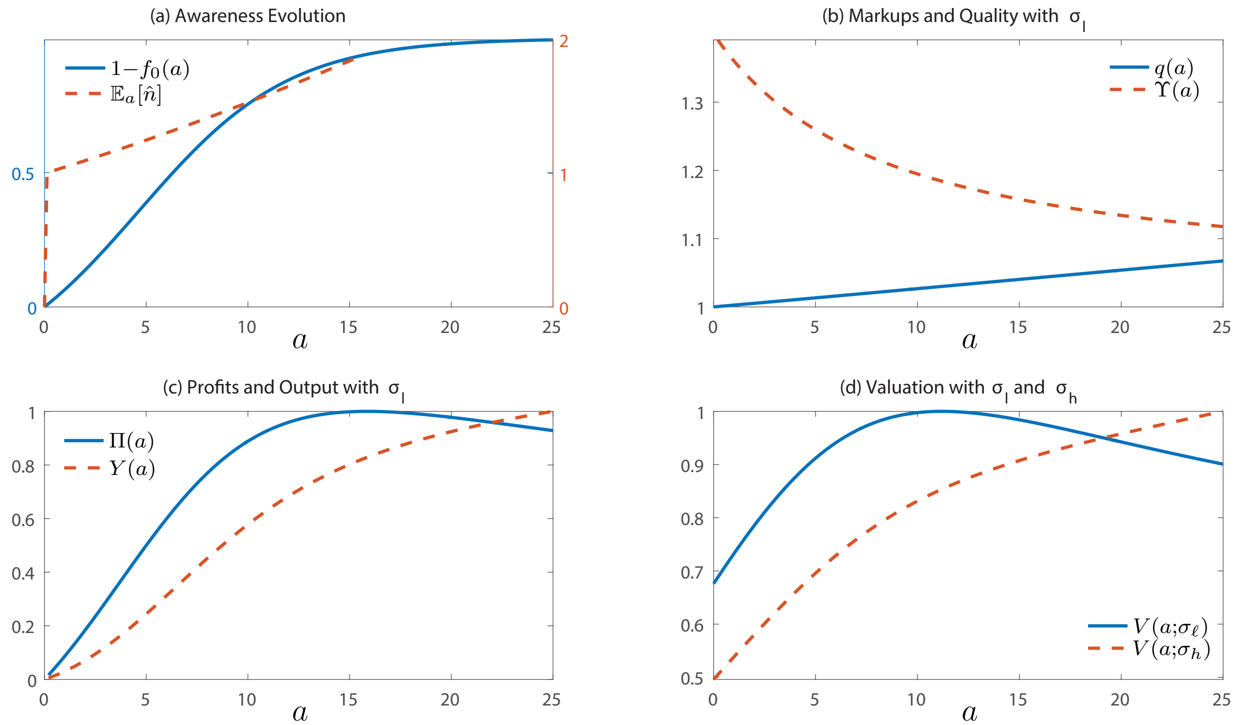

Figure 7: Example Industry and Awareness Evolution for $\sigma_\ell < \sigma_h$

Panel (a) shows an S-shaped diffusion curve through an inflection point in the mass of customers aware of at least one firm, $1 - f_0(a)$. The growth in $\mathbb{E}_a[\hat{n}]$ with these calibrated values is also evident, showing that the increase in awareness set sizes drives many model dynamics.

Panel (b) shows the markups and average quality term for the low differentiation parameter σ_ℓ. First, notice the fairly large drop-off in the markups, $\Upsilon(a)$, from the monopolistically competitive level. The asymptote of $\Upsilon(a)$ is low for the σ_ℓ case (as shown in Proposition 10) since all long-run market power must come from differentiation of the product itself (as opposed to coming from strategic considerations of information asymmetry in the transition). In the case of a high differentiation version, σ_h, markups would start at the monopolistically competitive level and fall much less, as market power is inherent in the product differentiation itself. The quality growth for σ_ℓ is modest, primarily because quality growth from consumer sorting is a function of how differentiated the products are. With this parameter, there are few quality gains to be had from sorting customers to products they are nearly indifferent between.

Continuing with the σ_ℓ example, panel (c) shows the profits and output. The output function has the familiar monotonically increasing S-shaped diffusion curve, in part from the mechanical growth of awareness in panel (a), but it continues to rise. There is also a contribution to output from the quality growth increasing intensive demand, but the quality growth in panel (b) is insufficient to have a large effect. Curiously, profits peak and then decrease in absolute terms—as is common in the data, and difficult to reconcile with models of monopolistic competition. This effect occurs because of the intensification of competition as choice sets become larger leads to large drops in markups, as in panel (b), and because there is insufficient countervailing quality growth to make up for the loss of market power.

Finally, panel (d) shows a simulation of the valuation for the two levels of product differentiation. The σ_ℓ example is non-monotone due to the non-monotone profits in panel (c), reflecting that profits and valuations peak before competition intensifies, while the σ_h example shows that profits can be monotonically increasing with sufficient quality growth from matching and if markups do not drop too rapidly.

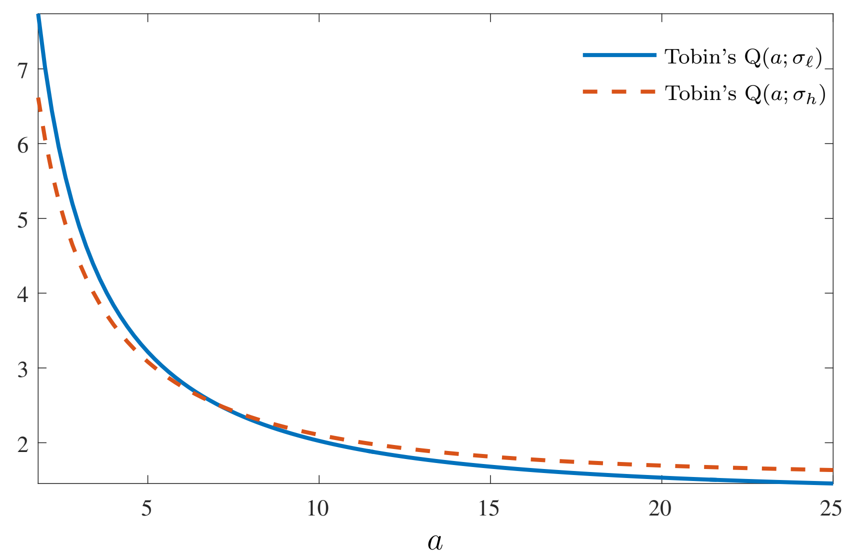

Figure 8: Evolution of Tobin's Q for $\sigma_\ell < \sigma_h$

Tobin's Q For a given firm, industry, or economy as a whole, Tobin's average Q is the market-to-book ratio and can be calculated as: the PDV of profits + replacement cost of capital, all divided by the replacement cost of capital. Due to the changes in growth options and markups, this model predicts a decreasing Tobin's Q for an industry—consistent with the evidence of age effects in Figure 4.

Tobin's Q for a product category of age a in a stationary economy is derived in Appendix B.2 as

$$\text{Tobin's Q}(a) \equiv 1 + \frac{BY}{k} \frac{\int_0^\infty e^{-r\tau} \left[(1 - f_0(\tau + a))q(\tau + a)(\Upsilon(\tau + a) - 1)\Upsilon(\tau + a)^{-\kappa}\right] \mathrm{d}\tau}{(1 - f_0(a))\Upsilon(a)^{-\kappa} q(a)} \tag{44}$$

Using (44) with calibrated parameters, Figure 8 graphs the evolution of Tobin's Q for industries of different σ to match the parameters of Figure 7.

Tobin's Q declines rapidly in Figure 8 as industries mature, especially if they have a low degree of differentiation, σ_ℓ. The decline is seen through panel (d) of Figure 7, where valuations can decline in absolute terms due to increasing competition—even while output (and, hence, book value) remains high.[43]

In Figures 9 and 10, the effects of individual changes in Tobin's Q are aggregated, and any secular change in σ or parameters for the age distribution of firms would change the composition of the aggregate Tobin's Q.

5.2 Comparative Statics and Secular Changes

To gain some intuition into the aggregate role of profits in understanding the aggregate evidence presented in Section 2.1, consider the case in which $\kappa = 2$. From (41) and (42), it can be shown that the output-to-capital ratio is undistorted by awareness, $\frac{Y}{k} = \frac{\rho+\delta_k}{\alpha}$. Furthermore, Appendix B.2 yields a simple Tobin's Q distorted by the awareness wedge on factor shares:

$$\text{Tobin's Q} = 1 + (1 - B)\frac{\rho + \delta_k}{\alpha(1 - \rho)} \tag{45}$$

The factor share distortion, B, can be calculated entirely from the age distribution of firms in (28), and is decreasing in δ_M (and increasing in θ for the example $\mathbb{Q}$ specifications). Hence, variation in the obsolescence rate of product categories leads to secular changes in Tobin's Q.

Comparing the competitive and monopolistically competitive limits, recall that in the competitive limit, the factor share distortion has the limit $B \to 1$, which nests the typical value for Tobin's Q with no market power. In contrast, in full-information monopolistic competition, the factor share distortion B remains constant at $\frac{\kappa-1}{\kappa}$ and invariant to the age distribution of product categories. Hence, within the context of this model, secular changes

[43] When comparing to the industry panel regressions in Figure 4, recall that the industry does not show up in the data until some time after it has actually been born. For example, the data in the first age decile of the panel data regressions might be closer to an age $a = 10$ in the model—and, hence, a 40% long-run drop-off in Figure 8 is closer to reality. This is discussed further in the calibration of growth rate parameters in Appendix E.1.

to Tobin's Q and the related factor shares in interpreting Figure 3 are tightly bound to changes in the factor share distortion B from (28).

Secular Changes in the Profit Share and Tobin's Q This section connects the calibrated model of aggregated industries to the empirical motivation from Section 2. Recall the evidence in Figure 3: (1) although cyclical, corporate profit shares seem to be going up; (2) as the profit share is taken away from aggregate income, this could be a major contribution to the decline in the labor share; and (3) other aggregate indicators of the role of profits are the stock market capitalization to GDP ratio, and Tobin's Q, which also seem to be trending up in this period.

Through the lens of this model, long-run changes would be driven by the factor share distortion B. The most likely candidates are a change in the average obsolescence rate of new product categories, δ_M, or in the degree of differentiation within product categories, σ. Figures 9 and 10 show comparative statics of the stationary equilibrium.[44] Both cases exhibit a similarly increasing profit share (and, consequently, a decreasing labor share) and a modest increase in Tobin's Q. However, the reasons are very different: in the case of increases in the obsolescence rate δ_M, the extra weighting of young and new products in consumer bundles is taking market power away from more mature industries. In the case of a higher product differentiation σ, the asymptotic profits are higher for the more mature product categories, with little contribution from younger ones. This can be seen in the asymptote of Figure 7.

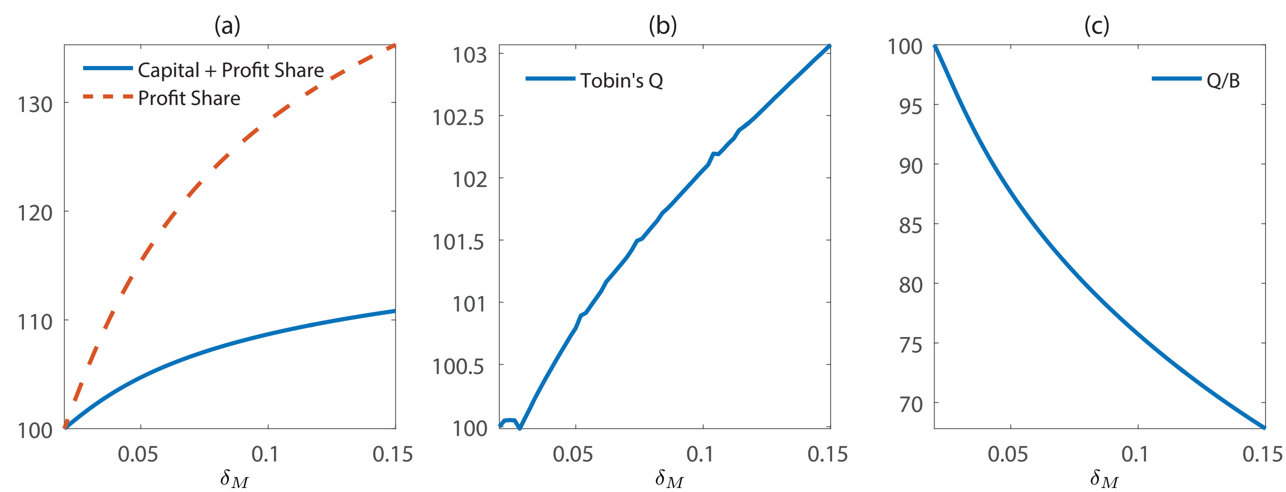

Figure 9: Comparative Statics for Product Category Obsolescence δ_M

As both δ_M and σ represent average values for a variety of product categories, the most likely explanation for secular changes is a compositional change in the consumption bundle over time. For example, products such as smartphones are more heavily differentiated than

[44] The calibration is discussed in Appendix E.

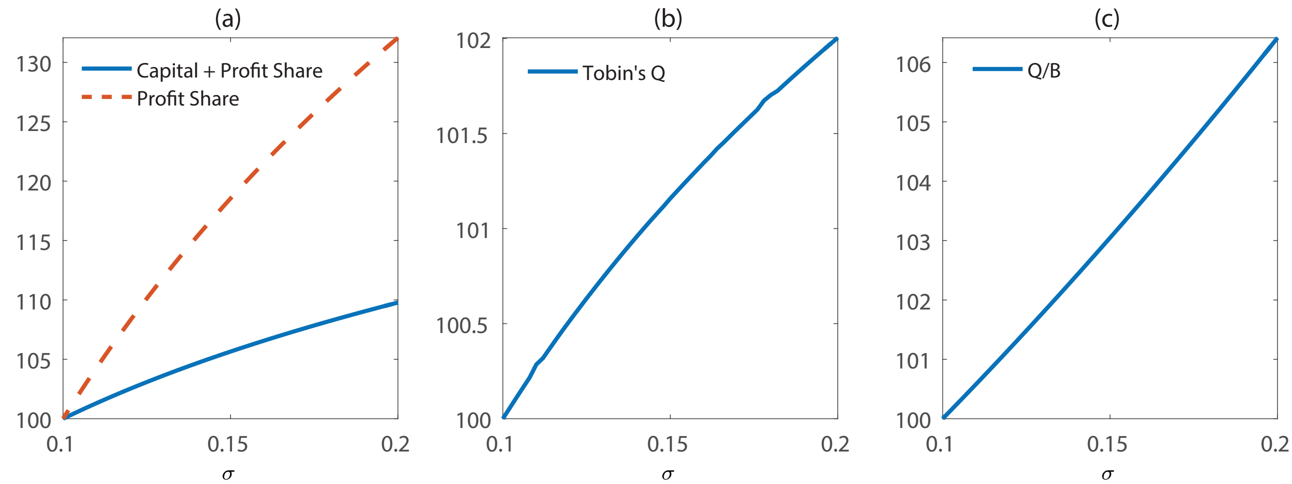

Figure 10: Comparative Statics for Product Differentiation σ

older vintages such as land-line telephones—suggesting an increase in σ for that product category. Furthermore, many services may exhibit a higher obsolescence rates (e.g., transitions in the popularity of aerobics, spinning, pilates, and yoga).

From simple aggregate consumption, investment, and valuation data, it would be difficult to decompose the contributions of σ or δ_M. The evidence in Section 2 from the proxies on obsolescence and the ratio of new product indicators to GDP suggests the importance of new products and obsolescence. Furthermore, in the transition to an economy dominated by services, increases in obsolescence and product differentiation may go hand in hand.

The productivity wedge, QB^{-1}, determines the proportionate loss in productivity due to the awareness and competition frictions—independent of any growth in intrinsic productivity $z(t)$. A declining QB^{-1} means that the actual output relative to a frictionless potential output is decreasing due to compositional changes. For this, if changes are in the obsolescence rate δ_M, then effective productivity relative to potential productivity is decreasing due to rapid changes in consumer tastes. Alternatively, if changes in the degree of differentiation σ are occurring, then effective productivity is increasing relative to potential productivity because of the greater opportunity for sorting consumers into better matches.

5.3 Product Entry and Cyclical Changes in the Profit Share

This model predicts a pattern for the early part of business cycles: if an expansion is generated by the growth of a large number of new product categories (i.e., not just TFP shocks), then the profit share will initially rise or stay stagnant and then slowly drop, eventually going below the stationary level before returning to it after a long period.

Recall from (28) that the profit share is calculated using the age distribution, $\Phi(t, a)$, but it is otherwise independent of aggregates. Therefore, the calculation of profit and labor shares in transition simply requires the solution to the PDE in (33), with an exogenously given entry rate $\hat{x}(t)$. The reason for changes in the entry rate of new product categories is irrelevant for calculating factor shares. Examples of policies that could create this sort of impulse are R&D tax subsidies and aggregate technology shocks that provide an especially fruitful cohort of new technologies and lead to a greater z_M (e.g., popularization of the internet or invention of general purpose technologies, as in Greenwood and Jovanovic (1999)).

To simulate an impulse response, from the stationary $\Phi(t, a)$, assume that a shock to the product creation rate leads to a 10% increase in varieties, after which the entry rate returns, and $\Phi(\cdot)$ eventually returns to the original stationary level of $\hat{x}(t)$ due to obsolescence.[45] The increase of 10% is not intended to be a typical entry shock but, rather, a sense of how extensive effects generated from a large entry of new products could be (e.g., products following the commercialization of the Internet in the late 1990s). Figure 11 shows the "impulse" on the entry rate on the corresponding average age of a product category relative to the stationary level (i.e., $\mathbb{E}_t [a] / \mathbb{E}_\infty [a]$). Both the stationary average age and the profit share are normalized to 100. In order to understand the role of obsolescence, the impulse is shown for two obsolescence levels in our calibrated range: $\delta_M = 0.0025$ and $\delta_M = 0.056$.

As the entry returns to the stationary obsolescence rate, δ_M, immediately after the shock, the economy will converge back to the stationary level. Here, the average product category age drops rapidly due to the magnitude of the impulse, but then can take a long time to return to the stationary level (after first going above the stationary level as the impulse ages).

During this time, the profit share in Figure 11 first rises and then slowly drops as the new products mature. Eventually, the profit share goes below its stationary value before returning. As a reminder, all of these changes are independent of aggregate TFP or other changes (except insofar as those may have generated the $\hat{x}(t)$ impulse).

The key to the speed of convergence back to the stationary age distribution is the entry and obsolescence rate δ_M after the impulse. When the obsolescence rate is high, the effects of a new product entry shock wear off faster.

Is this consistent with business cycle properties? The upward changes in profit share happen rapidly, over a few years, consistent with the first stages of a turnaround of a business cycle (and consistent with relatively stagnant wages during the early parts of an expansion). In terms of magnitude, however, there is an increase of, at most, 2.5% above the stationary profit share in this model, which is unlikely to be a significant contribution to the cyclicality of labor shares given this calibration.

[45] For $\hat{x}(t)$, I will space the additional entry over the span of the first year, calibrate the magnitude of the entry in that impulse to match the aggregate change target, and then immediately go back to the stationary new product entry rate rate, $\hat{x}/M = \delta_M$. Shorter impulses in continuous-time, such as a Dirac Delta, would be very similar, though a little harder to calculate numerically, as it would introduce an atom in $\hat{\Phi}(\cdot)$.

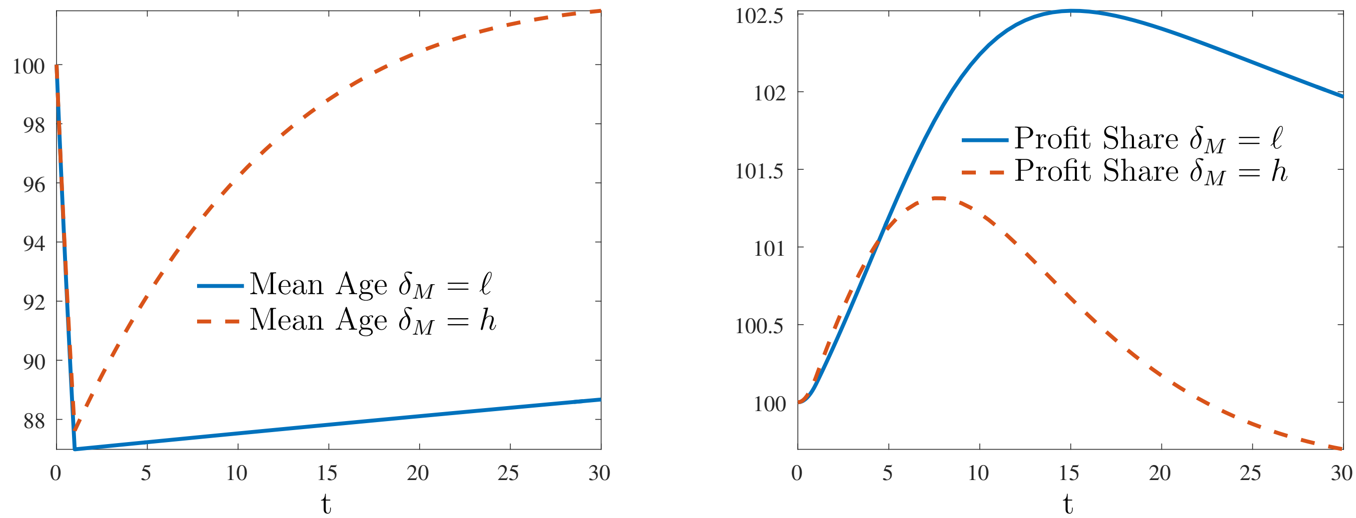

Figure 11: Impulse Response of an Entry Shock on the Profit Share

To summarize the lessons, while secular changes in the business cycle fluctuations in Section 5.2 are significant, the business cycle dynamics are relatively muted. This suggests that the mechanism at work in this model is irrelevant at business cycle frequencies due to the relatively slow changes in the age distribution $\Phi(t, \cdot)$. But a more general corollary is that the effects of new product entry are themselves smoothed out by the slow speed of growth in demand—which, in turn, urges caution with *any* business cycle model driven by new product entry with these calibrated values. New products mix into consumer bundles slowly, so we would expect to see weak time-series evidence of product creation having immediate effects on TFP or firm profits.

6 Extension: Asymmetric Quality and Entry

This section gives a few examples of asymmetry to clarify the empirical predictions for different intrinsic quality and/or entry timing. The asymmetric version of demand and prices is nested in the derivation of Appendix A, while details on the evolution of product awareness are given in Technical Appendix C.[46]

To demonstrate the forces and qualitative results, the numerical examples use a duopoly. The theory, however, fully extends to an arbitrary number of firms (or types of firms).

[46]To summarize the changes to the evolution of the distributions: with asymmetry, I need to add in a placeholder for those different types of firms into the awareness states, and expand the size of $\mathbb{Q}$. To deal with future entry, I can make the $\mathbb{Q}$ matrix time varying and/or add in placeholders for future entry types. Either way, there is no harm in having a placeholder awareness state which is of measure 0 until the entry occurs. The $f(a)$ for a given $\mathbb{Q}$ is calculated numerically, and efficiently, as a system of ODEs. Hence, even if $\mathbb{Q}$ is time varying and has a cardinality of tens of thousands, a solution is numerically tenable.

Remember that, in practice, awareness needs to be tracked only for firms with a distinct quality and entry date, which simplifies the evolution of awareness and the computational burden.

These examples are also consistent with the explanation in Syverson (2004) that the degree of productivity dispersion that can be sustained in an industry is a function of the degree of substitutability between the products.

6.1 Asymmetric Entry

Section 2.3 presented evidence that entrants have systematically different prices and productivity than incumbents, but grow very slowly—even when the physical TFP and intrinsic quality of the incumbent and entrant are identical.

Assuming that the process for awareness set sizes, $\hat{n}$, is a sub-martingale, (24) shows that prices (and markups) are weakly decreasing over time. At first, the fact that younger firms tend to have higher prices would seem to contradict the micro-evidence. For example, Foster, Haltiwanger, and Syverson (2016) finds that new entrants temporarily have only slightly higher productivity, but they tend to price significantly lower than incumbents before both prices and productivity converge. However, recall that the solution in (25) is for *both* symmetric intrinsic quality and symmetric time of entry. In this model, if a firm with identical intrinsic quality entered later, consumers would have asymmetric awareness sets.

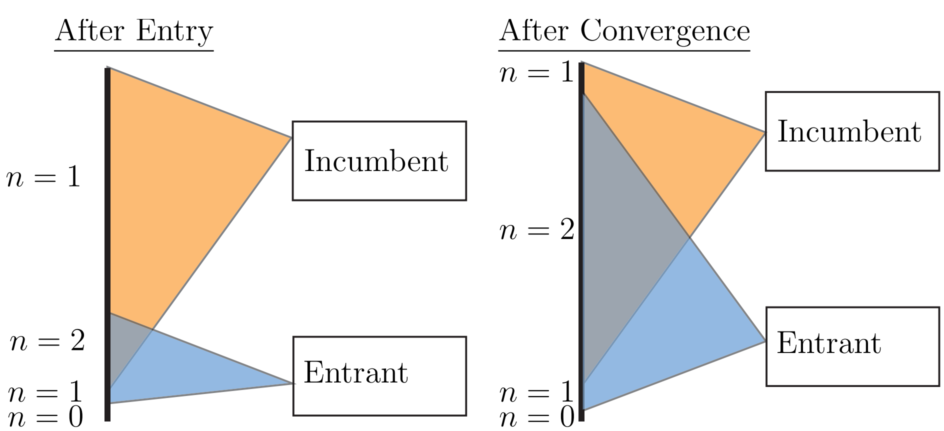

Figure 12: Asymmetric Entry Leads to Asymmetric Market Power and Sorting

This asymmetry in awareness makes the pricing decision dependent on firm age, as shown in Figure 12. For an incumbent, decreasing the price would decrease profits from existing customers who are unaware of the entrant. In contrast, the entrant has monopoly power over very few of its customers and, in the BNE, can lower prices to temporarily attract the incumbent's customers. Moreover, the average match quality of a consumer—conditional on purchasing the product—is different between the incumbent and the entrant. Incumbents sell to a number of consumers who would otherwise choose the entrant if it were in the consumer's choice set, while the entrant needs to compete for most of its consumers. This leads to an asymmetric degree of sorting, and, consequently, better average match quality for entrants, and it would show as higher revenue TFP even if absolute demand is low. Over time, as the incumbent loses customers due to more direct price competition with the incumbent, the retained customers will tend to have a higher average match quality, leading to converging revenue TFP as the incumbent's market share decreases. If the awareness process $\mathbb{Q}$ is ergodic, the prices and quality levels converge as the awareness sets become similar—as the micro-evidence suggests.

This suggests caution in interpreting increases in quality or TFP after an increase in the number of competitors as evidence of competition spurring quality innovation—especially if market shares decrease post-entry or after splitting a monopoly. Here, the increase in quality and aggregate output of the industry after entry is purely a passive process of consumer sorting. Nevertheless, even if the intrinsic properties of the products have not changed, the increase in average quality from sorting is real from a consumer's perspective.

Numerical Example To see this effect, the following experiment solves the calibrated model with 80% of the population initially aware of firm 1, a monopolist—i.e., $\int_{[0,1]} \mathbb{1}\{1 \in A_j(0)\}\mathrm{d}j = 0.8$. At that point, firm 2 enters with identical quality and no initial awareness—i.e., $\int_{[0,1]} \mathbb{1}\{2 \in A_j(0)\}\mathrm{d}j = 0$.

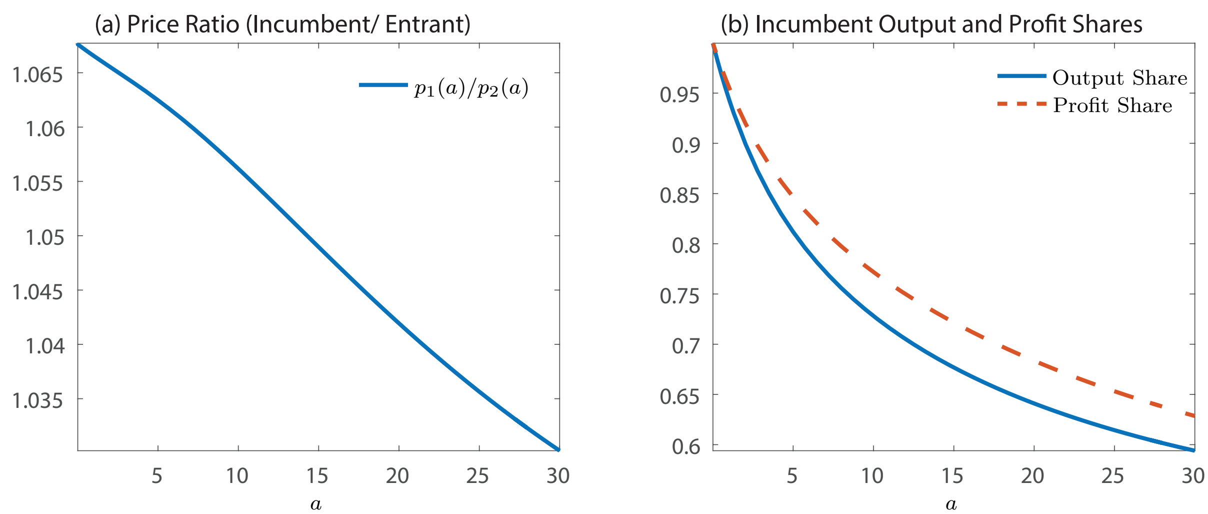

Figure 13: Entry into a Monopoly

The effect of asymmetry on pricing is evident in panel (a) of Figure 13, where the entrant has weakly lower prices throughout the industry life cycle. To understand the incentives in the BNE game: Initially, the incumbent charges the standard constant markup over marginal price inherent in monopolistic competition, and captured in the κ elasticity of substitution (and independent of the dispersion of idiosyncratic preferences within the product category,

σ). As discussed, the key is that the incumbent does not want to lose monopoly profits from the 80% of consumers who start with awareness of the incumbent, but no awareness of the entrant. While the entrant could charge the identical price as the incumbent, it has the incentive to lower its prices and capture a fraction of consumers who would otherwise choose the incumbent if prices were identical. How much the entrant is able to exploit this asymmetry in information sets is inherent in the dispersion of idiosyncratic preferences, σ. For the incumbent, lowering the price decreases monopoly profits from existing consumers without full awareness, which prevents the incumbent from retaliating by also lowering its prices.

During the transition towards symmetric information sets, note that the output and profit shares of the incumbent in panel (b) of Figure 13 are skewed. In particular, the previous monopolist is able to capture a greater proportion of industry profits than the entrant due to its ability to exploit the market power inherent in limited choice sets. Following through with this logic by considering standard model extensions: (1) this shows that if a firm is able to be a first mover in a market, there are positive profits inherent in the slow growth of firms and information frictions, even without further incentives, such as intellectual property protection. So, contrary to many endogenous growth models, policies to ensure the protection of market power (e.g., the patent system) are not necessary to ensure positive profits for the creation of a new industry; and (2) if fixed costs were added to the model through standard mechanisms, the asymmetry in the fierceness of competition for consumers could either drive entrants out of business, or simply deter entry. On the other hand, if there are complementaries in the evolution of awareness (such as high probabilities that consumers will find other providers once they have at least one product in their awareness set for the product category), there could be incentives to let other firms build initial awareness of the product category before entry.

The convergence of the prices in Figure 13 shows that as awareness sets become more similar, this effect disappears and prices converge towards the symmetric equilibrium of Proposition 3. Hence, driven only by the information asymmetry in awareness, entrants have higher quality (or, equivalently, revenue productivity) and lower prices than incumbents, but these differences disappear over time—consistent with Foster, Haltiwanger, and Syverson (2016). Beyond urging caution in interpreting evidence of entry spurring intrinsic changes in incumbents, this highlights that firms are symmetric only if they have both identical production technologies and customer awareness.

6.2 Asymmetric Quality (or Productivity)

As discussed in Section 2.3, many industries have a different skewness of market share versus profit share. For example, Apple took 91% of smartphone profits in 2015, while having only

17.2% of the market share—all in a competitive and fairly mature industry. This model shows that small differences in the intrinsic quality of products can lead to larger differences in profit-share than market-share, and that the differences in quality (and the different skews) would seem more pronounced as the industry evolved. Furthermore, Syverson (2004) shows enormous and sustained variations in profitability and productivity can be sustained within an industry, and this example shows how this is possible in my model.

With asymmetry in underlying quality, the key force in this model is that with a high dispersion of preferences within the product category, σ, sorting of consumers to preferred products can have a strong effect on profit shares. However, sorting takes time to develop, as it requires choice sets with more than a single firm. Consequently, as an industry matures and consumers become aware of all of the competitors, the profit share becomes much more skewed.

Numerical Example In order to understand the dynamics of market and profit shares, the following experiment of the calibrated model takes a duopoly with asymmetric firm quality. Both firms enter at age $a = 0$, but one firm has a 10% higher average quality than the other (i.e., $q_h/q_\ell = 1.1$ in the notation of Appendix A). As always, in this model, quality and productivity differences would be isomorphic given only revenue and profit data.

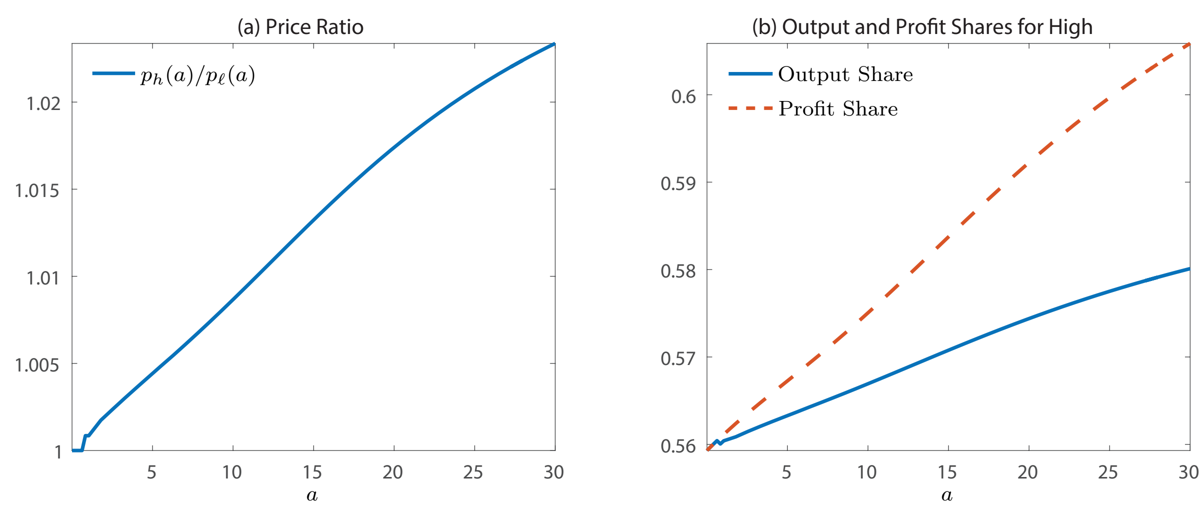

Figure 14: Evolution of High vs. Low Quality

In panel (a) of Figure 14, the higher-quality firm has slightly higher prices per unit of the good (reflecting the higher average-quality consumer's gain), but the price difference is asymptotically very small. The prices start out as identical because both firms are able to charge the monopolistically competitive price. It is only as the industry matures that the extent of the higher quality becomes evident, when consumers have several goods in their choice sets.

In contrast, panel (b) of Figure 14 shows that the profit and output-share become much more skewed. The output and profit shares are identical at the beginning since most consumers have only a single good in their choice set, but starts slightly higher than 50/50 since consumers have higher intensive demand for the higher quality product. This diverges as consumers sort into their preferred product, and the higher quality good can be priced accordingly to maximize profits given their position. The asymmetry between the profit ratio and customer ratio explains how a few firms in an industry can capture the majority of the industry's profits, but not necessarily capture the same proportion of customers or output.[47]

7 Extension: Simple Controlled Awareness Evolution

The intuition for the role of awareness in Sections 3 and 4 holds for a general class of stochastic processes. While exogenous processes highlight the unique implications and forces of the model, they are potentially unsatisfactory for policy and comparative statics. In this section, I take the equilibrium of Sections 3 and 4 but endogenize the awareness growth—and, consequently, the stochastic process $\mathbb{Q}$. After aggregating in the same manner as in Section 4, I repeat comparative statics and compare.

7.1 Sales and Marketing

Instead of an exogenous $\mathbb{Q}$, assume that each firm can control the evolution of product awareness through investment in sales, marketing, and advertising. As before, I maintain the assumption that the evolution of awareness is independent of prices.[48]

For simplicity, I assume that the firm makes its investment decision at entry by purchasing consumption goods to produce a permanent "storefront," which provides advertising throughout the lifetime of the product. The investment is irreversible, so while firms could conceivably scrap and rebuild a new storefront, they would never choose to rebuild in a stationary environment. A more complicated model with control at every time period is

[47] To gain a profit share split as skewed as the Apple example, we would need to combine asymmetric quality with the first-mover entry in Section 6.1 (which also leads to a more skew), and probably add in fixed operating costs to both amplify the profit shares dispersion and show why many phone producers have negative profits.

[48] For example, no experimentation is required to overcome asymmetric information or switching costs. As discussed, this contribution to price dynamics has already been studied extensively in both the macro and micro literature; see, for example, Nakamura and Steinsson (2011) and Bergemann and Välimäki (2006). Moreover, I argued in Section 6.1 that we need to make a distinction between lower pricing of new entrants to established industries vs. entry of new products in an entirely new product category.

possible, but this one-time decision simplifies the analysis of stationary equilibria.[49] I will solve a model assuming symmetric equilibria, in which all firms choose the same storefront size, and with a sufficiently large number of firms per industry, N, to eliminate complicated strategic considerations.

Let the parameter determining the growth rate of awareness be θ (e.g., Example 1). Otherwise, the evolution of the awareness sets is still left generic. If all firms symmetrically choose the same θ, then the generator of awareness set sizes conditioned on the θ choice is denoted by the parameterized evolution of awareness $\mathbb{Q}(\theta)$.

The convex cost of building the storefront with a particular rate of awareness growth θ is $\frac{\theta^\eta}{\eta N \nu}$ consumption goods, with $\eta > 1$. This function is decreasing in the number of initial entrants, N, to reflect that more firms mean a smaller number of consumers to access per firm.[50] Scaling of the cost by the number of firms ensures the independence of the equilibrium choices from a particular N, so that the total number of consumption goods to create the storefronts for the new industry is $\frac{\theta^\eta}{\eta \nu}$. As before, the consumer chooses to invest in a new product category, which generates entry of firms in the new industry producing the product category.

To solve for the storefront size θ for new product categories in the stationary equilibria, I need to consider off-equilibrium actions. A storefront size choice for firm i of $\theta_i \neq \theta$ at entry leads to: (1) changes in the growth of awareness sets; and (2) asymmetry in the proportion of awareness sets that include firm i. Given a large N, I assume that the evolution of the count distribution for consumers, $\mathbb{Q}(\theta)$, is unaffected by the choice of a particular θ_i for an arbitrarily small firm—which helps remove complicated strategic considerations.

While a large number of firms ensures that the aggregate evolution of awareness count is unaffected by individual deviations from the symmetric strategy, as shown in Appendix C, deviations do distort the proportion of awareness sets containing that particular firm. I assume that the evolution of awareness with asymmetric choices of θ is modeled as an urn with different "weighting" of the draws. If all N firms chose the same θ, then the probability that an individual firm would be added to the consumer's choice set (conditional on an arrival) is simply $1/N$, as reflected in Proposition 3. However, off-equilibrium, if a firm chooses a larger awareness growth rate θ relative to the other firms, it will skew the probability that it is added to awareness sets relative to competitors.

To model this: whereas in the symmetric version of Proposition 2, the likelihood of a

[49] With aggregate shocks, such as in Gourio and Rudanko (2014a,b) and Drozd and Nosal (2012), this assumption would not be innocuous, as cyclical investment in advertising is central to their mechanisms. To understand what is lost by the entry decision, Lee and Wilde (1980) describes the differences between period-by-period and committed upfront decisions in the R&D investment game of Loury (1979).

[50] This normalization is also necessary since the effects of θ are *not independent* of other firms, as they interact to provide an aggregate arrival rate of meetings within the product category, as in Example 1 and *Technical Appendix* Example 2.

firm being in an awareness set of size n is distributed as a Hypergeometric, with asymmetric weighting, the likelihood of a firm being in an awareness set of size n is distributed as a Fisher's Non-central Hypergeometric. For a large N with all other firms choosing the same storefront size θ, the probability that firm i is in a particular awareness set of size n is $\frac{\theta_i}{\theta}\frac{n}{N}$ (see Appendix C for the derivation). Writing the off-equilibrium profits, values, and pricing decisions, we find that

$$\pi(a, p, \theta_i | \theta, \bar{p}) = \frac{\theta_i}{\theta}\pi(a, p, \theta | \theta, \bar{p}) \tag{46}$$

Furthermore, the price chosen by firm i is identical to the other firms', and given by the $\Upsilon(a)$ from (21), conditioned on the symmetric θ—i.e., $p(a|\theta) = \Upsilon(a|\theta)mc$. Finally, from these, $v(\theta_i|\theta) = \frac{\theta_i}{\theta}v(\theta|\theta)$. Define the symmetric value $v(\theta|\theta)$ as $v(\theta)$. Given the θ choice for all of the $N-1$ firms, each entering firm then chooses its idiosyncratic θ_i to solve the following:

$$\max_{\theta_i \geq 0}\left[\frac{\theta_i}{\theta}v(\theta) - \frac{\theta_i^\eta}{\eta\nu N}\right] \tag{47}$$

where, in a symmetric equilibrium, $\theta_i = \theta$. The first-order necessary condition evaluated at the symmetric equilibrium is $\theta^\eta = \nu v(\theta)$, where $v(\theta)$ is the value of a new industry conditional on the symmetric choice of θ.

When aggregating, as the cost of creating a new industry is now endogenous, the consumer's productivity of creating a new product category, $z_M(t)$ in (38), is augmented with a function of the chosen θ. As one unit of final goods produces storefronts for $\eta\nu/\theta^\eta$ product categories, the productivity of creating the new industry with N entrants aggregates to

$$\hat{z}_M(\theta) \equiv z_M + \eta\nu\theta^{-\eta} \tag{48}$$

The stationary equilibrium is similar to that of Proposition 6, except that an additional equilibrium condition on θ is required; expectations are calculated conditional on this particular θ; and the cost of creating new industries is $\hat{z}_M(\theta)$.

Proposition 7 (Aggregation with Controlled Awareness)**.** *A symmetric, stationary equilib-*

rium is a θ, k, and M solving the system of implicit equations,

$$\delta_M - \delta_k = Q(\theta)B(\theta)^{-1}k^{\alpha}M^{\frac{1}{\kappa-1}}\left(\frac{\hat{z}_M(\theta)}{\kappa-1}M^{-1} - \alpha k^{-1}\right) \tag{49}$$

$$\rho + \delta_k = \alpha Q(\theta)B(\theta)^{-1}M^{\frac{1}{\kappa-1}}k^{\alpha-1} \tag{50}$$

$$\theta^{\eta} = \nu Q(\theta)^{2-\kappa}M^{\frac{2-\kappa}{\kappa-1}}B(\theta)^{-1}k^{\alpha}\int_0^{\infty} e^{-(\rho+\delta_M)a}\left[(1-f_0(a|\theta))q(a|\theta)(\Upsilon(a|\theta)-1)\Upsilon(a|\theta)^{-\kappa}\right]\mathrm{d}a \tag{51}$$

where $\hat{z}_M(\theta)$ is given by (48) *and $\Phi(a) = 1 - e^{-\delta_M a}$. Given the θ, k and M,*

$$C = Q(\theta)B(\theta)^{-1}M^{\frac{1}{\kappa-1}}k^{\alpha} - \delta_k k - \delta_M M/\hat{z}_M(\theta) \tag{52}$$

Proof. See Appendix C. Take the first-order condition of (47) with respect to θ, and evaluate at the $\theta = \bar{\theta}$ to get $\theta^{\eta} = \nu N v(\theta)$. Then, substitute for (29) and (B.32) to get (51). □

Given an equilibrium θ, k, and M, aggregate expenditures on sales and marketing as a fraction of GDP is $\frac{\text{S\&M}}{\text{GDP}} \equiv \frac{\delta_M}{\hat{z}_M}\frac{M}{Y}$.

7.2 Analysis and Comparative Statics

Repeating the experiment of Figure 9, the average obsolescence rate, δ_M, is allowed to change with endogenous θ. Figure 15 shows that the results are similar to those of the previous experiment, and endogeneity does not unravel changes in the δ_M as an explanation for the motivating empirics. Obsolescence, and the related preference for newly created product categories, is very powerful.

As an alternative, consider Figure 16, in which advertising or S&M investment becomes cheaper, resulting in a change in productivity of creating storefronts, ν—for example, due to the invention of the internet or television. At first, one might assume that this would simply speed up the expansion of awareness and, consequently, undermine market power, but here the profit share and Tobin's Q remain largely constant. For the most part, the main change is a modest drop in the proportion of S&M/GDP and a faster expansion of awareness (but not enough to undermine much market power in calculating B).

To summarize, technological innovations in spreading awareness (such as the invention of the internet) result in primarily a larger number of varieties M, which may be part of the increase in trademarks per GDP in Figure 1 of Technical Appendix D. There is some inconclusive evidence in Figure 2 of Technical Appendix D.2 of enormous changes in SG&A to revenue or advertising to GDP ratios to support changes in the productivity ν due to the invention of the internet. For many products, internet advertising is simply a cheaper and more targeted replacement for older forms of advertising—but it may not be increasing the

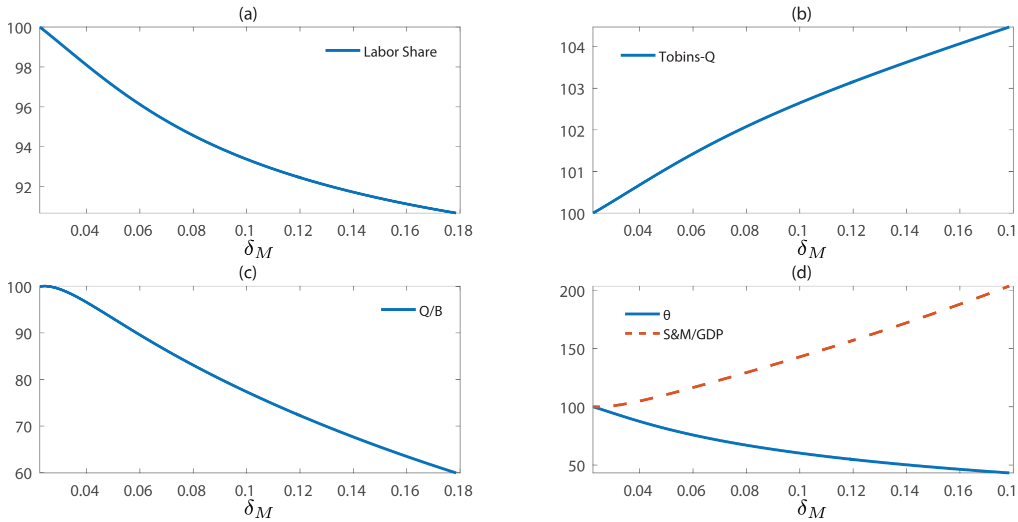

Figure 15: Comparative Statics for Product Category Obsolescence δ_M, with Endogenous θ

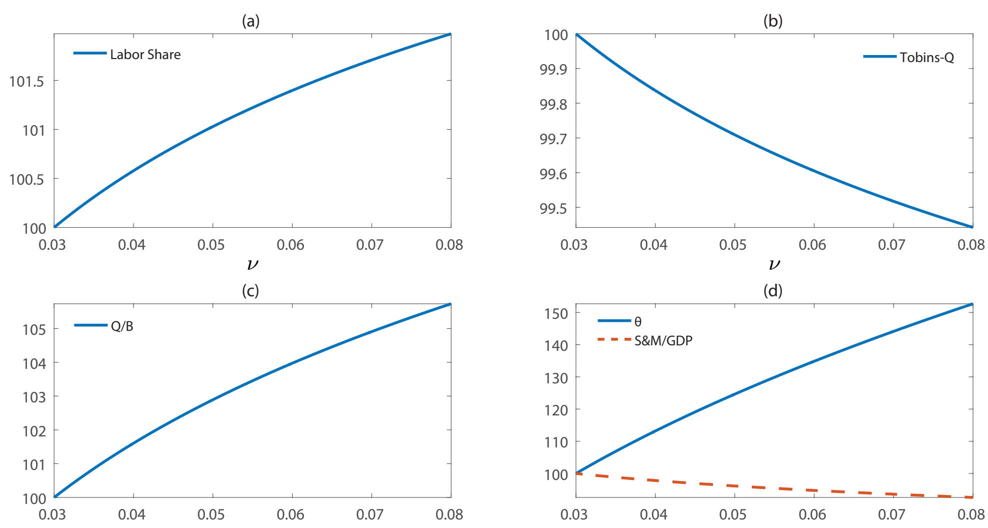

Figure 16: Comparative Statics for S&M productivity ν, with Endogenous θ

speed at which choice sets grow for a given product category. Regardless, Figure 16 shows that the expansion of more-productive S&M technologies has limited impact on profit shares and Tobin's Q and is unable to explain Figure 3, while changes in the obsolescence rate suggested by Figure 2 and Figure 1 of Technical Appendix D have a large effect in the right direction. See Technical Appendix D.2 for more empirics and analysis.

8 Conclusion

The primary theoretical contribution of this paper is a novel theory of demand and information heterogeneity, isolated from quality and productivity changes, and kept stylized to aid in interpreting aggregates. This information friction is modeled as a stochastically evolving set of firms that each consumer is aware of, where consumers can purchase only products from within this information set. At the aggregate level, this information friction manifests as changes in factor shares and the PDV of future growth opportunities in Tobin's Q. While the model is kept stylized in most of the paper to provide analytical results, it is extensible along a number of dimensions (at the cost of requiring numerical solutions).

The expansion of awareness causes two countervailing forces on firm profitability. First, an increase in the size of the average information set increases the intensity of competition among firms, thus decreasing market power. Second, as consumers gain access to more goods, they choose the product that best matches their needs. Since consumers tend to demand more of goods that they strongly prefer, the sorting of consumers into their preferred products increases the average quantity demanded for all goods in that industry.

Using a calibrated version of the model, I show that the information frictions are significant, and that secular changes in the average degree of product differentiation or average product obsolescence rates might cause large changes in aggregate factor shares. In contrast, changes in the speed of information diffusion (e.g., a new advertising technology) have a more modest effect on factor shares—although they significantly increase the equilibrium number of products in the economy. At the business cycle frequency, I show with these calibrated parameters that the effect of an impulse in the creation of new products is modest and slow. A broader lesson from the impulse response exercise is that, in the presence of these sorts of information frictions, any model focusing on new product creation is unlikely to have implications for the business cycle frequency.

Finally, I contribute new empirical results at the industry and aggregate levels. First, using USPTO data, I show strong evidence that the obsolescence and creation rates for products are much higher than previous estimates that use one-product-per-firm and firm entry/exit rates. There is also weaker, but still significant, evidence that secular changes in product obsolescence have occurred since the 1980s. Finally, using an industry panel from the Census and Compustat, I show strong evidence that markups and Tobin's Q tend to decline as industries age—even after controlling for measures of market concentration and the number of firms.

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Appendix A Proofs with Differentiated Firms

This section nests the cases of symmetric and asymmetric firms: Sections 3 and 6. Here, the additional quality term q_{im} is isomorphic(for all the usual reasons) to adding an idiosyncratic productivity for each firm (i, m).

A.1 State Space of Consumers and Information Sets

This section provides more details on tracking awareness with differentiated firms. For the most generality, I use the joint distribution of all consumer states (i.e., awareness sets A_j and idiosyncratic preferences ξ_j) to construct the marginal pmf $\hat{f}(a, A)$ over all the possible awareness set types. This is required because, in general, the count pmf, $f_n(a)$, is no longer sufficient for firm and consumer decisions with full differentiation.

Distribution of Consumer States Recall that consumer j is heterogeneous over both choice sets $A_{jm}(a)$ and permanent idiosyncratic match preferences ξ_{mj} for all industries. To forecast profits and make optimal pricing decisions, firms need to form expectations over the evolution of this state space. I will maintain assumptions to ensure that these states are independent across industries.

Define the joint distribution of consumer states for a particular industry as $\hat{\Psi}(a, A_j(a), \xi_j)$, with density pdf $\hat{\psi}(a, \cdot)$. This must sum to 1 for all consumers at each industry age a.

To aid in computation, factor this into two marginals: (1) the *marginal* probability mass function (pmf) of the awareness states A across consumers as a discrete distribution over the power-set of $\mathcal{I}$, $\hat{f}(a, A) : \mathbf{2}^{\mathcal{I}} \to \mathbb{R}$; and (2) the marginal distribution of idiosyncratic preferences, $G(\xi) : \mathbb{R}^N \to \mathbb{R}$, with density $g(\xi)$. While we can leave these marginals fairly general, to simplify integrals, I will maintain the following assumption throughout this paper:[51]

Assumption 2 (Independence of Preferences and Awareness)**.** *Assume conditions such that: (1) awareness evolves independent of preferences—i.e., $\hat{\psi}(a, A, \xi) = \hat{f}(a, A)g(\xi)$— and the distribution of preferences is independent across industries: $\xi_{mj} \perp \xi_{m'j}$ for all $m \neq m'$; and (2) g is continuous.*

The canonical case of the idiosyncratic distribution of preferences is the independent product: $\xi \sim G \equiv G^u(\xi_1) \times \dots G^u(\xi_N)$, where G^u is the univariate Gumbel distribution. In that case, the idiosyncratic preferences are independent across product categories, products, and consumers.

[51] An example of a model extension that breaks this assumption is if the evolution of $A_{jm}(a)$ is a function of the particular ξ_i for $i \in A_{jm}(a)$. An example of this is when a consumer controls the evolution of choice sets through shopping intensity lowers her intensity when she finds a high match value. These sorts of additions would not destroy the mechanism but would require more-complicated integrals for (A.3) and (A.23).

Information Sets and Timing To summarize the information structure and timing with this state-space: (1) consumers have incomplete information about firms, as captured in $A_j(a)$ choice sets, but complete information on their idiosyncratic preference ξ_i for $i \in A_j$; (2) firms have complete information on the joint distribution $\hat{\Psi}(\cdot)$ but cannot price discriminate based on awareness or preferences; (3) firms have complete information on the pricing and production decisions of other firms in the industry; (4) simultaneously, firms post prices, $p_i(a)$ (i.e., repeated Bertrand competition with no price discrimination), while consumers choose quantity demanded, $y_{ij}(a)$, to clear markets for each good; and (5) $A_j(a)$ stochastically evolves for each consumer as the industry ages (through any process, as long as Assumption 2 is maintained).

A.2 Consumers' Static Problem

In order to save on notation, the consumer's static problem written here uses y_{imj} directly instead of c_{imj}. As in models of monopolistic competition, this distinction is irrelevant if the aggregation technology (and/or preferences under the laws of large numbers of consumers, as proven in Appendix B.2) for creating consumer goods is identical to that for creating investment goods. To have a separate technology and productivity for the creation of capital and R&D goods, I would need to specify a different version of (A.1) for each—or simply undo the awareness-specific elements, as discussed in Section 4.3.

Suppress the m, t, and a indices where convenient. Assume that firms are differentiated, and may have a persistent quality difference, $q_{im} > 0$. Firm differentiation leads to an additional quality term in the objective compared to the static decisions of (6),

$$\max_{y_{imj} \geq 0} \left(\int_0^M \left(\sum_{i \in A_{mj}} q_{im} e^{\sigma \xi_{imj}} y_{imj} \right)^{\varsigma} \mathrm{d}m \right)^{1/\varsigma} \quad \text{s.t.} \quad \int_0^M \left[\sum_{i \in A_{mj}} \hat{p}_{im} y_{imj} \right] \mathrm{d}m \leq P_j \Omega_j \tag{A.1}$$

Define the set of industries with awareness of at least one firm as $\mathcal{M}_j \equiv \{m \in [0, M] \text{ s.t. } |\mathcal{I}_{jm}| > 0\}$.

Definition 3 (Total Demand)**.** *Given $\hat{\Psi}(\cdot)$, the total demand for firm i is defined as,*

$$y_i(a, p) \equiv \int y_{ij}(p, \xi_{ij}) \mathbb{1}\{\textit{Choose } i \textit{ from } A_j \textit{ given } p \textit{ and } \xi_j\} \mathrm{d}\hat{\Psi}(a, A_j, \xi_j) \tag{A.2}$$

Proposition 8 (Intensive Demand with Differentiated Firms)**.** *Given real prices p and real income Ω, for each industry m:*

1. *Of those with non-empty awareness sets, almost every consumer purchases from a*

single firm per industry. A consumer purchases product i and no others if and only if

$$\log\left(\frac{p_{i'}}{q_{i'}}\right) - \log\left(\frac{p_i}{q_i}\right) > \sigma\left(\xi_{i'j} - \xi_{ij}\right), \quad \forall\, i' \in A_j \setminus \{i\} \tag{A.3}$$

2. *The intensive demand for product i is*

$$y_{ij}(a, \xi_{ij}) = q_i^{\kappa-1} e^{\sigma(\kappa-1)\xi_{ij}} p_i^{-\kappa} \Omega_j, \quad y_{i'j} = 0, \forall\, i' \in A_j \setminus \{i\} \tag{A.4}$$

3. *The price index is a function of the preferences, ξ_{mj}, firm average quality q_m, and nominal prices, $\hat{p}_m$,*

$$P_j \equiv \left(\int_{\mathcal{M}_j} \left(e^{\sigma\xi_{imj}} q_{im}\right)^{\kappa-1} \hat{p}_{im}^{1-\kappa} \mathrm{d}m\right)^{\frac{1}{1-\kappa}} \tag{A.5}$$

Proof of Proposition 8. First, define the Lagrangian for the consumers' optimization problem in (A.1) with $\lambda_j > 0$ as the Lagrange multiplier on the budget constraint, and $\mu_{imj} \geq 0$ as the Lagrange multipliers ensuring weak positivity on every choice of y_{imj}.

$$\mathcal{L} = \left(\int_0^M \left(\sum_{i' \in A_m} q_{i'm} e^{\sigma\xi_{i'mj}} y_{i'mj}\right)^{\varsigma} \mathrm{d}m\right)^{1/\varsigma} - \lambda_j \left(\int_0^M \sum_{i' \in A_m} \hat{p}_{i'm} y_{i'mj} \mathrm{d}m - P\Omega\right) + \int_0^M \sum_{i' \in A_m} \mu_{i'mj} y_{i'mj} \mathrm{d}m \tag{A.6}$$

The first-order necessary conditions with respect to y_{imj} are

$$S_j \left(\sum_{i' \in A_m} e^{\sigma\xi_{i'mj}} q_{i'm} y_{i'mj}\right)^{\varsigma-1} q_{im} e^{\sigma\xi_{imj}} = \lambda_j \hat{p}_{im} - \mu_{imj} \tag{A.7}$$

$$\lambda_j > 0, \quad \mu_{imj} \geq 0, \quad \mu_{imj} y_{imj} = 0, \quad \forall\, i, m \tag{A.8}$$

with the following definition:

$$S_j \equiv \left(\int_0^M \left(\sum_{i' \in A_m} q_{i'm} e^{\sigma\xi_{i'mj}} y_{i'mj}\right)^{\varsigma} \mathrm{d}m\right)^{1/\varsigma - 1} \tag{A.9}$$

Intensive Demand: Maintain Assumptions 1 and 2 throughout. Assume, to be verified, that if $|A_m| > 0$, the consumer will almost certainly consume a single product per industry.

To solve for λ_j and the price index, I will follow standard CES algebra under the assumption of consuming, at most, one product per industry. The i index is dropped since there is only one good per industry, and the j index is dropped for simplicity—though given the assumptions for the aggregate consumer, I show in Appendix B.2 that S_j is identical for all j, and, hence, all consumers have the same price index. From (A.7), for industries with non-empty awareness sets,

$$S_j \left(e^{\sigma\xi_m} q_m\right)^{\varsigma} y_m^{\varsigma-1} = \lambda_j \hat{p}_m \tag{A.10}$$

Take the ratio of two industries m' and m with positive demand

$$\frac{y_{m'}^{\varsigma-1}}{y_m^{\varsigma-1}} = \left(\frac{e^{\sigma\xi_{m'}} q_{m'}}{e^{\sigma\xi_m} q_m}\right)^{-\varsigma} \frac{\hat{p}_{m'}}{\hat{p}_m} \tag{A.11}$$

Rearrange,

$$y_{m'} = y_m \hat{p}_m^{\kappa} \left(e^{\sigma\xi_m} q_m\right)^{1-\kappa} \hat{p}_{m'}^{-\kappa} \left(e^{\sigma\xi_{m'}} q_{m'}\right)^{\kappa-1} \tag{A.12}$$

Multiply both sides by $\hat{p}_{m'}$ and integrate over all industries with positive consumption, $m' \in \mathcal{M}_j$,

$$\int_{\mathcal{M}_j} \hat{p}_{m'} y_{m'} \mathrm{d}m' = y_m \hat{p}_m^{\kappa} \left(e^{\sigma\xi_m} q_m\right)^{1-\kappa} \int_{\mathcal{M}_j} \left(e^{\sigma\xi_{m'}} q_{m'}\right)^{\kappa-1} \hat{p}_{m'}^{1-\kappa} \mathrm{d}m' \tag{A.13}$$

Recognize that industry m is infinitesimal, so the integrals are identical with or without industry m. Hence, the consumer cannot affect the price index either through a change in the intensive demand or by switching between different i in $\mathcal{I}_m$. Define the price index as

$$P_j \equiv \left(\int_{\mathcal{M}_j} \left(e^{\sigma\xi_m} q_m\right)^{\kappa-1} \hat{p}_m^{1-\kappa} \mathrm{d}m\right)^{\frac{1}{1-\kappa}} \tag{A.14}$$

Reorganize (A.13) using the price index, noting that the left-hand side of the equality is the budget

$$y_m = \left(e^{\sigma\xi_m} q_m\right)^{\kappa-1} \left(\frac{\hat{p}_m}{P}\right)^{-\kappa} \Omega \tag{A.15}$$

This function is the intensive demand, as in (12) and (A.4). Note that since $\kappa > 1$, $\frac{\partial y_i}{\partial \hat{p}_i} < 0$, $\frac{\partial y_i}{\partial q_i} > 0$, $\frac{\partial y_i}{\partial \xi_i} > 0$. Also, if the real income and nominal price, $\hat{p}$, are is kept constant, then $\frac{\partial y_i}{\partial P} > 0$, reflecting substitution away from other goods to this industry.

Extensive Demand: The proof strategy is to assume that a single product is consumed, to use the non-negativity of the Lagrange multipliers for the other products to determine a set of inequalities necessary for this choice to hold, and then to show that multiple products will be chosen only under measure 0 events. The inequality constraints in (A.7) for all products $i, i' \in \mathcal{I}_m$ give

$$S_j \left(e^{\sigma\xi_i} q_i y_i\right)^{\varsigma-1} q_{i'} e^{\sigma\xi_{i'}} \leq \lambda_j \hat{p}_{i'} \tag{A.16}$$

Rearrange (A.10)

$$\left(y_i q_i e^{\sigma\xi_i}\right)^{\varsigma-1} = \frac{\lambda_j}{S_j} \frac{\hat{p}_i}{q_i e^{\sigma\xi_i}} \tag{A.17}$$

Combine with (A.16)

$$\tfrac{\lambda_j}{S_j} \tfrac{\hat{p}_i}{q_i} e^{-\sigma\xi_{ij}} q_{i'} e^{\sigma\xi_{i'j}} \leq \tfrac{\lambda_j}{S_j} \hat{p}_{i'} \tag{A.18}$$

Take logs and rearrange

$$\log\left(\tfrac{\hat{p}_{i'}}{q_{i'}}\right) - \log(\tfrac{\hat{p}_i}{q_i}) \geq \sigma(\xi_{i'j} - \xi_{ij}) \tag{A.19}$$

Using $p \equiv \hat{p}/P$, this expression gives (A.3). Finally, show that only measure 0 consumers choose multiple products. Without loss of generality, assume that there are only two products in the industry and that $y_1 > 0$ and $y_2 > 0$. The first-order conditions are then

$$S_j \left(e^{\sigma\xi_{ij}} q_i y_{ij} + e^{\sigma\xi_{i'j}} q_{i'} y_{i'j}\right)^{\varsigma-1} q_i e^{\sigma\xi_{ij}} = \lambda_j \hat{p}_i \tag{A.20}$$

$$S_j \left(e^{\sigma\xi_{ij}} q_i y_{ij} + e^{\sigma\xi_{i'j}} q_{i'} y_{i'j}\right)^{\varsigma-1} q_{i'} e^{\sigma\xi_{i'j}} = \lambda_j \hat{p}_{i'} \tag{A.21}$$

Take the ratio and the log to find an equation in ξ space,

$$\xi_{i'j} - \xi_{ij} = \sigma^{-1}\left(\log\left(\tfrac{\hat{p}_{i'}}{q_{i'}}\right) - \log\left(\tfrac{\hat{p}_i}{q_i}\right)\right) \tag{A.22}$$

For a given set of prices and distribution of ξ, there are an infinite number of agents with the particular combination of these $(\xi_{i'j}, \xi_{ij})$. However, the solution is an affine subset of the ξ_1, ξ_2 space. Given the independence of the ξ preferences from Assumption 2, the measure of this affine subset is 0. The conclusion is that the set of agents who purchase multiple products is measure 0 if prices are positive, and (A.19) can be written as a strict inequality for almost every consumer. □

A.3 Total Demand

In this section, I assume conditions such that consumers have identical real incomes, Ω, and I derive the demand curve faced by a firm. Due to the intensive margin, the market shares for firms are less useful than in a discrete-choice model (see Technical Appendix C.4 for a derivation). I assume that the consumers have identical nominal incomes, and that they have the same price index in (A.5) due to a law of large numbers—as proven in Appendix B.

Define the set of possible awareness states that contain product i as $A \,|\, i \equiv \left\{A \,|\, A \in \mathbf{2}^{\mathcal{I}} \text{ s.t. } i \in A\right\}$.

Proposition 9 (Total Demand for Gumbel Preferences). *Given Assumptions 1 and 2,*

$$y(a,p) = \bar{\Gamma}^{1-\kappa}\Omega\, q_i^{\kappa-1} p_i^{-1/\sigma-1} \sum_{A\,|\,i} \left(\hat{f}(a,A) \left[\sum_{i' \in A} \left(\frac{q_i}{q_{i'}} p_{i'} \right)^{-1/\sigma} \right]^{\sigma(\kappa-1)-1} \right) \tag{A.23}$$

If the firm is a monopolist, then

$$y_i(a,p_i) = (1 - \hat{f}(a,\emptyset))\bar{\Gamma}^{1-\kappa} q_i^{\kappa-1} p_i^{-\kappa}\Omega \tag{A.24}$$

Proof. Since intensive demand is a function of ξ_j, to find the total demand for product i, the firm will sum up demand with ξ_{ij} conditional on product i being chosen. From (A.2), define the total demand for product i given a price vector p as

$$y_i(a,p) = \sum_{A\,|\,i} \hat{f}(a,A) \int y_i(\xi_{ij}) \mathbb{1}\{\log\left(\tfrac{p_{i'}}{q_{i'}}\right) - \log\left(\tfrac{p_i}{q_i}\right) > \sigma(\xi_{i'j} - \xi_{ij}) | \forall\, i' \in A \setminus i\} \mathrm{d}G(\xi_j) \tag{A.25}$$

Simplify using Assumption 2 and (A.4):

$$y_i(a,p) = p_i^{-\kappa} q_i^{\kappa-1}\Omega \sum_{A\,|\,i} \hat{f}(a,A) \int e^{\sigma(\kappa-1)\xi_{ij}} \mathbb{1}\{\log\left(\tfrac{p_{i'}}{q_{i'}}\right) - \log\left(\tfrac{p_i}{q_i}\right) > \sigma(\xi_{i'j} - \xi_{ij}) | \forall\, i' \in A \setminus i\} \mathrm{d}G(\xi_j) \tag{A.26}$$

This equation sums the demand across the distribution of A. For a particular A, find the total demand from agents conditional on having awareness set A

$$y_i(p,A) \equiv p_i^{-\kappa} q_i^{\kappa-1}\Omega \int e^{\sigma(\kappa-1)\xi_{ij}} \mathbb{1}\{\log\left(\tfrac{p_{i'}}{q_{i'}}\right)/\sigma - \log\left(\tfrac{p_i}{q_i}\right)/\sigma + \xi_{ij} > \xi_{i'j} | \forall\, i' \in A \setminus i\} \mathrm{d}G(\xi_j) \tag{A.27}$$

Define the marginal distribution of ξ_j for all products other than product i as $G_{-ij}(\xi_{-ij})$. For arbitrary $g(\xi)$, this expression could be calculated numerically. For iid Gumbel distributions with pdf $g(\xi_i)$, the integral is solved in two parts: first, use Fubini's Theorem and Assumption 2 to solve for the inner non-ξ_i variables, defined as ξ_{-ij}; and then integrate with respect to the the ξ_i variable. This is the standard technique in the derivation of the Logit probabilities.

$$\frac{y_i(p,A)}{p_i^{-\kappa} q_i^{\kappa-1}\Omega} = \int_{-\infty}^{\infty} e^{\sigma(\kappa-1)\xi_{ij}} \left[\int \mathbb{1}\{\log\left(\tfrac{p_{i'}}{q_{i'}}\right)/\sigma - \log\left(\tfrac{p_i}{q_i}\right)/\sigma + \xi_{ij} > \xi_{i'j} | \forall\, i' \in A \setminus i\} \mathrm{d}G_{-i}(\xi_{-ij}) \right] \mathrm{d}G_i(\xi_{ij}) \tag{A.28}$$

The inner integral is the cdf of the joint distribution of ξ_{-ij}. Use the cdf of the Gumbel along each dimension other than i; then, substitute in the pdf of the Gumbel, and recognize that $\log\left(\frac{p_i}{q_i}\right)/\sigma - \log\left(\frac{p_i}{q_i}\right)/\sigma = 0$ allows combining the exponent as a sum for all i', including i

$$= \int_{-\infty}^{\infty} e^{\sigma(\kappa-1)\xi_{ij}} \prod_{i'\in A\setminus i} e^{-e^{-\left(\log(\frac{p_{i'}}{q_{i'}})/\sigma - \log(\frac{p_i}{q_i})/\sigma + \xi_{ij}\right)}} \mathrm{d}G_i(\xi_{ij}) \tag{A.29}$$

$$= \int_{-\infty}^{\infty} e^{\sigma(\kappa-1)\xi_{ij}} \exp\left(-\sum_{i'\in A} e^{-\left(\log\left(\frac{p_{i'}}{q_{i'}}\right)/\sigma - \log\left(\frac{p_i}{q_i}\right)/\sigma + \xi_{ij}\right)}\right) e^{-\xi_{ij}} \mathrm{d}\xi_{ij} \tag{A.30}$$

Simplify by factoring the exponential

$$= \int_{-\infty}^{\infty} \exp\left(-(1-\sigma(\kappa-1))\xi_{ij}\right) \exp\left(-\exp\left(-\xi_{ij}\right)\left(\frac{p_i}{q_i}\right)^{1/\sigma} \sum_{i\in A}\left(\frac{p_{i'}}{q_{i'}}\right)^{-1/\sigma}\right) \mathrm{d}\xi_{ij} \tag{A.31}$$

If $B > 0$ and $A > 0$, then $\int_{-\infty}^{\infty} e^{-Ax} e^{-Be^{-x}} \mathrm{d}x = B^{-A}\Gamma(A)$, where $\Gamma(\cdot)$ is the Gamma function.

$$= \Gamma(1-\sigma(\kappa-1)) \left(\frac{p_i}{q_i}\right)^{\frac{\sigma(\kappa-1)-1}{\sigma}} \left(\sum_{i\in A}\left(\frac{p_{i'}}{q_{i'}}\right)^{-1/\sigma}\right)^{\sigma(\kappa-1)-1} \tag{A.32}$$

Assumption 1 ensures that the variance of the idiosyncratic preferences is not so large that the total demand explodes as the demand from agents with large ξ_{ij} is summed. To find the total demand for product i given price vectors p, integrate over the distribution of A states in the economy. From (A.32) and (A.26), the total demand is the sum of all awareness states that contain product i in (A.26)

$$y_i(p) = \frac{\Gamma(1-\sigma(\kappa-1))\Omega}{q_i} \left(\frac{p_i}{q_i}\right)^{-1-1/\sigma} \sum_{A\,|\,i}\left[\hat{f}(a,A)\left(\sum_{i\in A}\left(\frac{p_{i'}}{q_{i'}}\right)^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}\right] \tag{A.33}$$

Reorganize and use the definition of $\bar{\Gamma}$ to get (A.23)

$$y_i(p) = \bar{\Gamma}^{1-\kappa}\Omega\, q_i^{\kappa-1} p_i^{-1/\sigma-1} \sum_{A\,|\,i}\left(\hat{f}(a,A)\left[\sum_{i'\in A}\left(\frac{q_i}{q_{i'}}p_{i'}\right)^{-1/\sigma}\right]^{\sigma(\kappa-1)-1}\right) \tag{A.34}$$

$\square$

A.4 Total Demand for Symmetric Firms

Proof of Proposition 2. Using (3) gives sums in terms of moments of the $\hat{n}$ random variable,

$$\sum_{n=1}^{N} f_n(a)g(n) = (1 - f_0(a))\mathbb{E}_a\left[g(\hat{n})\right] \tag{A.35}$$

First, note that in a symmetric equilibrium, finding the probability that a particular firm is in an awareness set of size n with total N firms is distributed Hypergeometric (i.e., an urn problem without replacement). From this, as there is only one possible successful state, the pmf of the Hypergeometric evaluated at the successful state is $\frac{\binom{N-1}{n-1}}{\binom{N}{n}} = \frac{n}{N}$.

Given N firms and symmetric evolution of awareness, the mass of consumers aware of firm i who have a total awareness set of size n is, then, $\frac{n}{N}f_n(a)$. Simplify (A.23) with $q_i = \bar{\Gamma}$, the price of the firm p, and the symmetric price of all other firms $\bar{p}$,

$$y(a, p, \bar{p}) = \frac{p^{-1-1/\sigma}}{N}\left[\sum_{n=1}^{N} f_n(a)n\left(p^{-1/\sigma} + (n-1)\bar{p}^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}\right]\Omega \tag{A.36}$$

Simplify with (A.35) to get (15). Substitution of $\bar{p} = p$ gives (16) □

A.5 Prices for Symmetric Firms

Proof of Proposition 3. Assume a symmetric price $\bar{p}$, and use (15) with (19) to form the optimization problem

$$\bar{p} = \arg\max_{p\geq 0}\left\{(p - mc)\frac{1 - f_0(a)}{N}p^{-1-1/\sigma}\mathbb{E}_a\left[\hat{n}\left(p^{-1/\sigma} + (\hat{n}-1)\bar{p}^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}\right]\Omega\right\} \tag{A.37}$$

Define $g(p, \bar{p}) \equiv \mathbb{E}_a\left[\hat{n}\left(p^{-1/\sigma} + (\hat{n}-1)\bar{p}^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}\right]$. Assume existence and take the first-order condition with respect to p

$$0 = (-p + mc + \sigma\, mc)g(p, \bar{p}) + p(p - mc)\sigma\frac{\partial g(p, \bar{p})}{\partial p} \tag{A.38}$$

Evaluate at the symmetric equilibrium $\bar{p} = p$ and simplify,

$$0 = -p + mc + \sigma\, mc + (p - mc)(1 - \sigma(\kappa - 1))\frac{\mathbb{E}_a\left[\hat{n}^{\sigma(k-1)-1}\right]}{\mathbb{E}_a\left[\hat{n}^{\sigma(k-1)}\right]} \tag{A.39}$$

Finally, solve for the price

$$p(a) = \left(1 + \sigma\left[1 - (1 - \sigma(\kappa - 1))\frac{\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)-1}\right]}{\mathbb{E}_a\left[\hat{n}^{\sigma(\kappa-1)}\right]}\right]^{-1}\right) mc \tag{A.40}$$

□

Given the symmetric pricing equilibria, find asymptotic prices by taking limits as $a \to 0, a \to \infty$, etc.

Proposition 10 (Asymptotic Properties of Prices)**.** *Define $p(a, N)$ as the equilibrium prices conditional on an industry with N firms. For symmetric firms, if a pure-strategy equilibrium exists and the stochastic process has $\lim_{a\to\infty} \mathbb{E}_a[\hat{n}] = N$, then: (1) $p(\infty, N) \equiv \lim\limits_{a\to\infty} p(a) = \frac{(N-1+\sigma(N\kappa-1))}{N-1+\sigma(\kappa-1)} mc < p_1(N)$; (2) $p(a, 1) = p(0, N) = \frac{mc}{\varsigma}$; and (3) $p(\infty, \infty) \equiv \lim\limits_{N\to\infty} p_\infty = (1 + \sigma)mc$*

Proof. The $\lim_{a\to0}$ price uses the first-order expansion of the counting process for any $\mathbb{Q}$. Regardless of the particular Markov chain, after an infinitesimal amount of time, the support of n will be 0 or 1.

$$p = \arg\max_{\tilde{p}\geq0} \left\{(\tilde{p} - mc)\tilde{p}^{-1-1/\sigma}\left(\tilde{p}^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}\right\} = \arg\max_{\tilde{p}\geq0} \left\{(\tilde{p} - mc)p^{-\kappa}\right\} = \frac{mc}{\varsigma} \tag{A.41}$$

For any $\mathbb{Q}$ with a stationary distribution of $f(\infty) = \begin{bmatrix}0 & \dots & 0 & 1\end{bmatrix}$, use $\lim_{a\to\infty} \frac{\mathbb{E}_a\left[\hat{n}^{\sigma(k-1)-1}\right]}{\mathbb{E}_a\left[\hat{n}^{\sigma(k-1)}\right]} = \frac{1}{N}$ and substitute into (24). □

A.6 Firm Value and Industry Equilibrium

Define the prices of all of the other firms in the industry at age a as $p_{-i}(a) \equiv \{p_{i'}(a)|i' \in \mathcal{I} \neq i\}$. Using the total demand derived in (15), firm i's value at age a is the present discounted value (PDV) of profits. The following is the dynamic value of the firm given the sequence of prices:

Definition 4 (Firm Value)**.** *Given a discount rate $r > 0$ and prices $p(a)$, the firm's valuation is the present discounted value of profits*

$$v(a, p_i, p_{-i}) \equiv \int_0^\infty e^{-r(a+\tau)}\left[\underbrace{(p_i(a + \tau) - mc(a + \tau))y_i(p_i(a + \tau), p_{-i}(a + \tau))}_{\equiv\pi_i(a+\tau),\ \textit{Profits}}\right] \mathrm{d}\tau \tag{A.42}$$

Given the value, I can define a standard pure-strategy symmetric equilibrium for a given industry:

Definition 5 (Symmetric Industry Equilibrium).
A (post-entry) symmetric industry equilibrium with history-independent prices is a set of: (1) demand functions $y(a, p, A, \xi) \to \mathbb{R}$*; (2) firm pricing functions* $p(a)$*; and (3) evolving distributions of the consumer awareness count* $f(a)$*, such that: (a) given* $p(a)$*,* $y(\cdot)$ *is optimal for the consumer according to* (12)*; (b) given* $y(\cdot)$ *and the aggregation in* (15)*,* $p(a)$ *are a symmetric pure strategy BNE of the game as in* (19)*; and (c)* $f(a)$ *evolves according to the law of motion discussed in* (2)*.*

Appendix B Aggregation

This section provides proofs for the aggregation to a neoclassical growth economy with an awareness-dependent wedge. It relies on the derivations for markups, output, etc. in Appendix A.

B.1 Static Industry Conditions

Proof. The following starts with a standard derivation of the marginal cost of a Cobb-Douglas production function, and then applies the heterogeneous markups and sorting quality. The cost minimization problem is

$$\min_{\ell, K} \{rK + w\ell\} \text{ s.t. } y = zK^\alpha \ell^{1-\alpha} \tag{B.1}$$

The first-order conditions for the cost minimization in (B.1) are

$$r = \lambda z \alpha \frac{y}{K} \tag{B.2}$$

$$w = \lambda z (1 - \alpha) \frac{y}{L} \tag{B.3}$$

Combine the FONCs and define k to find the optimal capital-labor ratio for all firms

$$k \equiv \frac{K}{\ell} = \frac{\alpha}{1 - \alpha} \frac{w}{r} \tag{B.4}$$

From the production technology,

$$\ell = k^{-\alpha} \frac{y}{z} \tag{B.5}$$

Substitute (B.4) and (B.5) into the total cost and simplify

$$rK + w\ell = \frac{1}{1-\alpha} w\ell = \frac{k^{-\alpha} wy}{(1-\alpha)z} \tag{B.6}$$

Taking $\boldsymbol{\partial}_y$ gives the constant marginal cost,

$$mc = \tfrac{1}{1-\alpha} z^{-1} k^{-\alpha} w \tag{B.7}$$

Substitute (20), (21) and (24), into (22) and (23), and factor into age and time dependent components,

$$Y(t,a) = (1 - f_0(a))\Upsilon(a)^{-\kappa} q(a)\, mc(t)^{-\kappa}\Omega(t) \tag{B.8}$$

$$\Pi(t,a) = (1 - f_0(a))(\Upsilon(a) - 1)\Upsilon(a)^{-\kappa} q(a)\, mc(t)^{1-\kappa}\Omega(t) \tag{B.9}$$

Use (B.5), (B.7) and (B.8) to find labor demand by industry age

$$L(t,a) = (1 - f_0(a))\Upsilon(a)^{-\kappa} q(a)\ (1-\alpha) w(t)^{-1} mc(t)^{1-\kappa}\Omega(t) \tag{B.10}$$

The capital used by the industry of age a comes from (B.4) and (B.10), which is also the book value of tangible assets

$$K(t,a) = k(t)L(t,a) \tag{B.11}$$

At this point, I have collapsed all industry-specific functions into the proportion of consumers unaware of any firm $1 - f_0(a)$, the markup $\Upsilon(a)$ and quality $q(a)$. The aggregate contributions to prices, profits, and output are the components of marginal cost (i.e., $k(t)$, real wages $w(t)$, aggregate productivity $z(t)$) and real income $\Omega(t)$). □

B.2 Price Index and Aggregation

Proof of Proposition 4. First, derive the price index, (13), in terms of the age distribution of industries. Given Assumption 2 and that all consumers were alive at the birth of every industry, the price index will be identical for all consumers.[52] Take (13), temporarily drop

[52] These assumptions can be relaxed if additional consumer state variables in the $\hat{\Psi}(\cdot)$ distribution. For example, if agents are born and die at different times, the price index would be consumer age-dependent, as well. However, consumers born in the same year would have identical pricing indices. Finally, as consumers would be entering and exiting, the evolution of $f(a)$ in (1) would need to be modified since it is the distribution conditional on survival.

the t index where appropriate for clarity, and denote the age of industry m as $a(m)$,

$$P_j(\xi) = \bar{\Gamma}^{-1} \left(\int_{\mathcal{M}_j} e^{\sigma(\kappa-1)\xi_{mj}} \hat{p}(a(m))^{1-\kappa} \mathrm{d}m \right)^{\frac{1}{1-\kappa}} \tag{B.12}$$

While the price in equilibrium is only a function of age, I need to take into account the idiosyncratic $\mathcal{M}_j$ and matches based on ξ. As derived from $\mathbb{Q}$, the proportion of firms of age a that a consumer is aware of (i.e., $n > 0$) is $1 - f_0(a)$. Hence, given the unnormalized cdf of industry age, $M(t)\Phi(t, a)$, I can replace $\mathcal{M}_j$ with an integral over the age distribution weighted by the proportion that they have an awareness of.

While the price is directly a function of age, the idiosyncratic ξ_{mj} match value can be shown to be a function of age in expectation. Independent of age, recall that in the symmetric equilibrium, the consumer chooses the product with the highest match value given an awareness set of size n. If the ξ_{imj} are independent, then the distribution of the maximum of n draws from the $g(\xi)$ distribution is the first order-statistic, $g_{(n)}(\xi)$. Industry age enters the matches through a distribution of n for the continuum of industries of a particular age. Convert to the $\hat{n}$ random variable and use (A.35),

$$P = \bar{\Gamma}^{-1} \left(\int (1 - f_0(a))\hat{p}(a)^{1-\kappa} \mathbb{E}_a \left[\int_{-\infty}^{\infty} e^{\sigma(\kappa-1)\xi} g_{(\hat{n})}(\xi) \mathrm{d}\xi \right] M \mathrm{d}\Phi(t, a) \right)^{\frac{1}{1-\kappa}} \tag{B.13}$$

For the Gumbel distribution, the order-statistic for the maximum of $\hat{n}$ draws is also Gumbel due to max-stability,

$$g_{(\hat{n})}(\xi) = \hat{n} e^{-\xi} e^{-\hat{n}e^{-\xi}} \tag{B.14}$$

From this, the following integral is calculated using $\int_{-\infty}^{\infty} e^{-Ax} e^{-Be^{-x}} \mathrm{d}x = B^{-A}\Gamma(A)$:

$$\int_{-\infty}^{\infty} e^{\sigma(\kappa-1)\xi} g_{(\hat{n})}(\xi) \mathrm{d}\xi = \Gamma(1 - \sigma(\kappa - 1))\hat{n}^{\sigma(\kappa-1)} \tag{B.15}$$

Substitute into (B.13) and use definitions for $\bar{\Gamma}$ and (20) to find the price index in (26)

$$P(t) = \left(\int (1 - f_0(a))\hat{p}(t, a)^{1-\kappa} q(a) M \mathrm{d}\Phi(t, a) \right)^{\frac{1}{1-\kappa}} \tag{B.16}$$

Divide both sides by $P(t)$, substitute from (24) and (27), use $\hat{p} \equiv pP$, and reorganize for $mc(t)$,

$$mc(t) = M(t)^{\frac{1}{\kappa-1}} Q(t) \tag{B.17}$$

Use (30) to solve for $w(t)$ and then use (29)

$$w(t) = (1-\alpha)k(t)^{\alpha}z(t)M(t)^{\frac{1}{\kappa-1}}Q(t) = (1-\alpha)Z(t)k(t)^{\alpha}B(t) \quad \text{(B.18)}$$

Composite Good: From (6), define the composite good,

$$Y_j \equiv \left(\int_0^M \left(\sum_{i \in A_{mj}} \bar{\Gamma} e^{\sigma \xi_{imj}} y_{imj} \right)^{\varsigma} \mathrm{d}m \right)^{1/\varsigma} \quad \text{(B.19)}$$

Use the same approach to grouping as in (B.12)

$$Y_j(\xi)^{\varsigma} = \bar{\Gamma}^{\varsigma} \int_{\mathcal{M}_j} \left(e^{\sigma \xi_m} y(a, \xi_m) \right)^{\varsigma} \mathrm{d}m \quad \text{(B.20)}$$

From (12)

$$= \bar{\Gamma}^{\varsigma} \int_{\mathcal{M}_j} \left(e^{\sigma \xi_m} \bar{\Gamma}^{\kappa-1} e^{\sigma(\kappa-1)\xi_m} p(a)^{-\kappa} \Omega \right)^{\varsigma} \mathrm{d}m = \Omega^{\varsigma} \bar{\Gamma}^{\kappa-1} \int_{\mathcal{M}_j} e^{\sigma(\kappa-1)\xi_m} p(a)^{1-\kappa} \mathrm{d}m \quad \text{(B.21)}$$

Use $\hat{p} \equiv p/P$ and reorganize

$$= \Omega^{\varsigma} P^{\kappa-1} \frac{\int_{\mathcal{M}_j} e^{\sigma(\kappa-1)\xi_m} \hat{p}(a)^{1-\kappa} \mathrm{d}m}{\bar{\Gamma}^{1-\kappa}} \quad \text{(B.22)}$$

Combine (B.12) and (B.22) and simplify to find that $Y(t) = \Omega(t)$. Hence, the composite good $Y(t)$ acts as an aggregate good and is equal to real income. This is a standard result from CES preferences and monopolistic competition, and it generalizes here. With this, I can write the consumer's dynamic and labor supply problems as those of a representative agent and representative firm (conditional on an agent distribution), with TFP given by (29).

Income and Aggregation Production Take (B.10) and aggregate to find labor demand

$$L(t) = \int_0^{\infty} L(t, a) M(t) \mathrm{d}\Phi(t, a) \quad \text{(B.23)}$$

$$= (1-\alpha) M(t) w(t)^{-1} mc(t)^{1-\kappa} Y(t) \int_0^{\infty} (1 - f_0(a)) \Upsilon(a)^{-\kappa} q(a) \mathrm{d}\Phi(t, a) \quad \text{(B.24)}$$

Use (B.17) and reorganize

$$w(t)L(t) = (1-\alpha) Y(t) Q(t)^{1-\kappa} \int_0^{\infty} (1 - f_0(a)) \Upsilon(a)^{-\kappa} q(a) \mathrm{d}\Phi(t, a) \quad \text{(B.25)}$$

With (27) and (28),

$$w(t)L(t) = (1-\alpha)B(t)Y(t) \tag{B.26}$$

Substitute from (29) and (B.18) into (B.26), and reorganize to get physical output as a function of aggregates and labor supply

$$Y(t) = Z(t)L(t)k(t)^{\alpha} \tag{B.27}$$

From (B.26), note that the labor share of output, $\frac{P(t)w(t)L(t)}{P(t)Y(t)}$, is $(1-\alpha)B(t)$, with $\alpha B(t)$ going to capital, and $1 - B(t)$ going to profits. In the case of monopolistic competition, for any age and quality distribution, $B(t) = (\kappa - 1)/\kappa$ so that the profit share is constant at $1/\kappa$.

Profits and Aggregate Value Substitute (27) and (B.17) into (B.9)

$$\Pi(t,a) = (1 - f_0(a))(\Upsilon(a) - 1)\Upsilon(a)^{-\kappa} q(a) M(t)^{-1} Q(t)^{1-\kappa} Y(t) \tag{B.28}$$

Aggregate profits and use (27)

$$\Pi(t) = \int \Pi(t,a)M(t)\mathrm{d}\Phi(t,a) = (1 - B(t))Y(t) \tag{B.29}$$

Aggregate PDV of profits in a stationary economy (without installed capital),

$$V = \frac{1-B}{1-r}Y \tag{B.30}$$

The aggregate Tobin's Q is the market to book value (i.e., (PDV profits + book value)/book value, where the book value is k due to the price normalization

$$\text{Tobin's Q} = 1 + \frac{1-B}{1-r}\frac{Y}{k} \tag{B.31}$$

Industry Profits and Allocations Calculate the valuation of an entire industry *at entry* in a stationary economy, from (A.42), (B.17) and (B.27) to (B.28),

$$V(t,a) = ZQ^{1-\kappa}M^{-1}k^{\alpha} \int_0^{\infty} e^{-r\tau} \left[(1 - f_0(\tau + a))q(\tau + a)(\Upsilon(\tau + a) - 1)\Upsilon(\tau + a)^{-\kappa}\right] \mathrm{d}\tau \tag{B.32}$$

To find the book value, take (B.10) to (B.11), (B.17), (B.18) and (B.27),

$$K(t,a) = (1 - f_0(a))\Upsilon(a)^{-\kappa} q(a)\, B^{-1} M^{-1} Q^{1-\kappa} k \tag{B.33}$$

From (B.32) and (B.33), Tobin's Q (i.e., (PDV of Profits + replacement cost of capital)/(replacement cost of capital) of an industry of age a is

$$\text{Tobin's Q}(a) \equiv 1 + \frac{BY}{k} \frac{\int_0^\infty e^{-r\tau} \left[(1 - f_0(\tau + a))q(\tau + a)(\Upsilon(\tau + a) - 1)\Upsilon(\tau + a)^{-\kappa}\right] d\tau}{(1 - f_0(a))\Upsilon(a)^{-\kappa} q(a)} \tag{B.34}$$

□

B.3 Age Distribution

Proof of (33) *to* (35). First, note that if a flow of $\hat{x}(t)$ industries are born and a proportional flow of δ_M are removed due to obsolescence, then the law of motion for the total mass of industries is

$$M'(t) = -\delta_M M(t) + \hat{x}(t) \tag{B.35}$$

Rearrange and use $\hat{x}(t) = x(t)M(t)$,

$$\frac{M'(t)}{M(t)} + \delta_M = x(t) \tag{B.36}$$

Take derivatives of $\Phi(t, a) \equiv \hat{\Phi}(t, a)/M(t)$,

$$\boldsymbol{\partial}_a \hat{\Phi}(t, a) = M(t)\boldsymbol{\partial}_a \Phi(t, a) \tag{B.37}$$

$$\boldsymbol{\partial}_t \hat{\Phi}(t, a) = M(t)\boldsymbol{\partial}_t \Phi(t, a) + M'(t)\Phi(t, a) \tag{B.38}$$

Substitute these expressions, $\hat{\Phi}(t, a) = M(t)\Phi(t, a)$, and $\hat{x}(t) = x(t)M(t)$ into (33),

$$\boldsymbol{\partial}_t \Phi(t, a) = -\boldsymbol{\partial}_a \Phi(t, a) - \left(\delta_M + \frac{M'(t)}{M(t)}\right) \hat{\Phi}(t, a) + x(t) \tag{B.39}$$

Use (B.36) and reorganize

$$\boldsymbol{\partial}_t \Phi(t, a) = -\boldsymbol{\partial}_a \Phi(t, a) + (1 - \Phi(t, a))\, x(t) \tag{B.40}$$

Also, given an $M(t)$ function, substitute from (B.36) to get

$$\boldsymbol{\partial}_t \Phi(t, a) = -\boldsymbol{\partial}_a \Phi(t, a) + (1 - \Phi(t, a)) \left(\frac{\boldsymbol{\partial}_t M(t)}{M(t)} + \delta_M\right) \tag{B.41}$$

To get the stationary distribution, note from (B.36) that a $x(t) = \delta_M$ is necessary. Substitute this into (B.40) to get the ODE,

$$0 = \boldsymbol{\partial}_a \Phi(a) + \delta_M(1 - \Phi(a)) \tag{B.42}$$

Solve the ODE subject to the initial condition $\Phi(0) = 0$ and $\Phi(a) = 1$ for $a > 0$,

$$\Phi(a) = 1 - e^{-\delta_M a} \tag{B.43}$$

□

Appendix C Controlled Awareness

While the evolution of the economy given a fixed awareness process $\mathbb{Q}$ is covered in previous sections, with endogeneity, the off-equilibrium actions need to be considered. As quality heterogeneity is left out, this means that I need only to consider the value of a single agent deviating from a symmetric Nash equilibrium.

Proof for Section 7.1. First, adapting Appendix A.4, note that with a single asymmetric firm, and $N-1$ symmetric firms, the probability of that firm being in an awareness set of size n is no longer distributed (central) Hypergeometric. Instead, with the model of investment in awareness distorting the relative probabilities, I model the probability following Fisher's non-central Hypergeometric distribution (i.e., an urn problem with no replacement and biased "weights").

Let the particular firm's choice be θ and the symmetric choice of the other firms be $\bar{\theta}$. Then, the relative weight is $\theta/\bar{\theta}$. From the probability mass function for Fisher's non-central Hypergeometric distribution, the probability of a successful draw of firm i with an awareness set of size n and N total firms is,

$$\frac{\binom{N-1}{n-1}\theta/\bar{\theta}}{\binom{N-1}{N} + \binom{N-1}{n-1}\theta/\bar{\theta}} = \frac{n}{N}\frac{\theta/\bar{\theta}}{1 + (\theta/\bar{\theta} - 1)n/N} \approx \frac{\theta}{\bar{\theta}}\frac{n}{N}, \text{ for a large } N \text{ limit} \tag{C.1}$$

In the equilibrium with symmetric weights in Proposition 2, this is identical to the n/N derived in Appendix A.4. Furthermore, with a large N limit, deviations of θ_i from θ for a single firm have a negligible effect on the distribution of awareness set sizes for each consumer. Let $f_n(a|\theta)$ be the pmf of awareness set sizes given an equilibrium θ. With (A.34), (A.37) and (C.1),

$$\pi(a,p,\theta|\bar{\theta},\bar{p}) = (p-mc)\frac{1-f_0(a|\bar{\theta})}{N}p^{-1-1/\sigma}\frac{\theta}{\bar{\theta}}\mathbb{E}_a\left[\hat{n}\left(p^{-1/\sigma}+(\hat{n}-1)\bar{p}^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}|\bar{\theta}\right]\Omega \tag{C.2}$$

Given a particular $\bar{\theta}$, the large N assumption leads to off-equilibrium changes in a particular θ having a small impact on the expectation. From this, note that the first-order condition for p in (46) is identical to (A.39), so the equilibrium choice of p is unaffected by off-equilibrium changes in θ.

As this simple version of controlled θ assumes that entering firms pay once for θ, the firm will choose θ to maximize its value, taking the symmetric $\bar{\theta}$ equilibrium as given. Define the symmetric value when $\theta = \bar{\theta}$ as $v(\bar{\theta})$, and using (46) and (A.42) gives the following post-entry value:

$$v(\theta|\bar{\theta}) = \int_0^\infty e^{-ra}\left[(p-mc)\frac{1-f_0(a|\bar{\theta})}{N}p^{-1-1/\sigma}\frac{\theta}{\bar{\theta}}\mathbb{E}_a\left[\hat{n}\left(p^{-1/\sigma}+(\hat{n}-1)\bar{p}^{-1/\sigma}\right)^{\sigma(\kappa-1)-1}|\bar{\theta}\right]\Omega\right]\mathrm{d}s \tag{C.3}$$

$$= \frac{\theta}{\bar{\theta}}v(\bar{\theta}) \tag{C.4}$$

□

Appendix D Industry Panel

This section continues the evidence of Section 2.2, and provides robustness checks on the evidence in Figure 4. Even more robustness checks on controls and on the age definition are done in Technical Appendices E.1 and E.2.

D.1 Additional Data

Figure 17 displays the histogram of birth and peak years of the 189 industries in our sample. The figure shows a wide variety of birth years, based on the definition of birth being when peak employment in the industry hits 5% of its maximum level. This also shows a wide variety of industry ages when attaining peak employment—which I use as a proxy for the life cycle length for the industry. The median is a little less than 20 years (past the 5% birth threshold) to hit the peak.

To see how the industries grow during the life cycle prior to hitting the peak, Figure 18 shows a box and whiskers plot of the employment and revenue relative to the maximums. By definition, the employment peak happens at 100% of the age relative to the peak employment

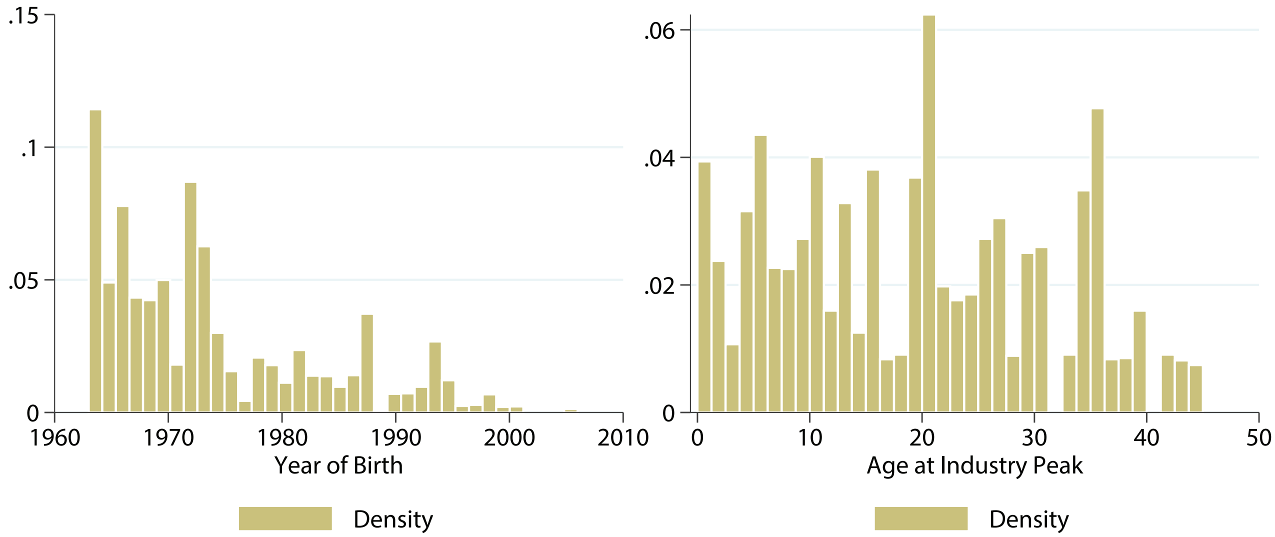

Figure 17: Histogram of Birth Year and Peak Employment Year

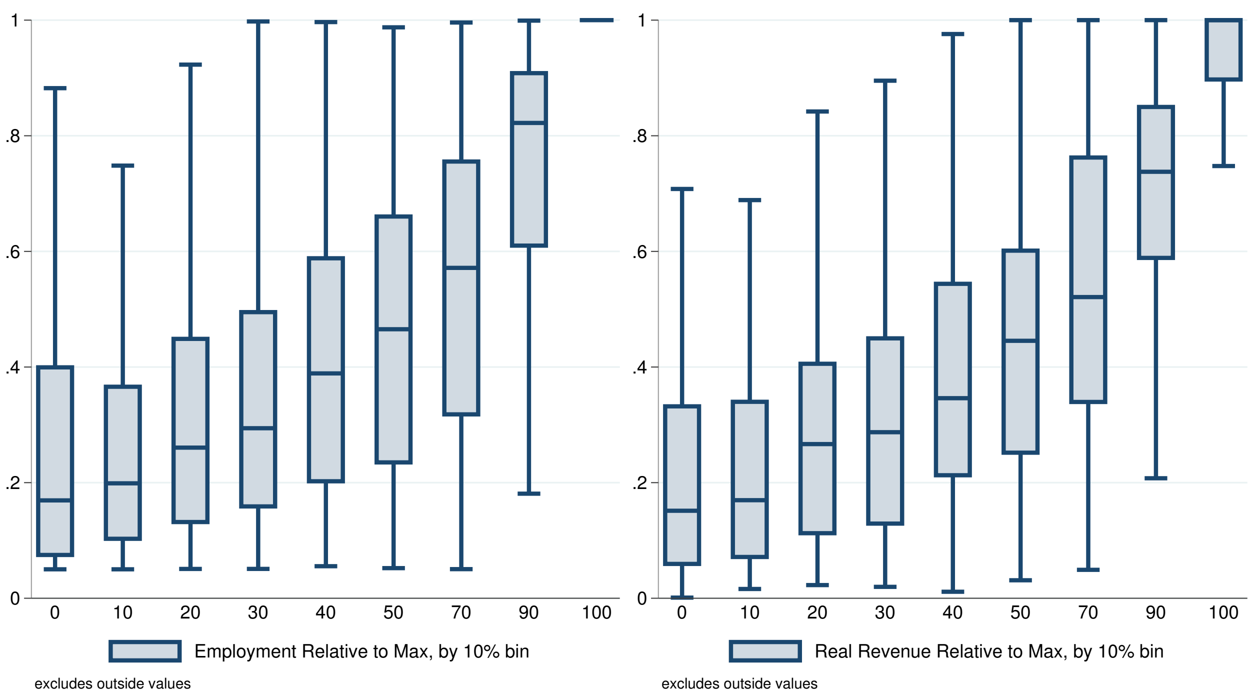

Figure 18: Employment and Real Revenue Relative to Peak Employment Year

age (i.e., normalize the age of each industry by its industries period length in Figure 17).

Figure 19: More Effects of Age Relative to Peak Employment (Controls for # Firms & Concentration)

For a robustness check on the markup measures of Figure 4 (i.e., CRSP operating profit margin and the MID price cost margin), see Figure 19. Included are the inverse share of value-added to wages, calculated from the MID and used as a proxy for markups in the same sense as in Hall (1988). Also included is the price cost margin calculated from Compustat, rather than the MID based price cost margin of Figure 4. In all of these cases, the evidence suggests that the general pattern of decreasing margins up to the peak employment level is robust. Finally, Figure 19 provides evidence on the price index of the product (calculated from the MID), and employment growth rates.

D.2 Direct Age Effects

Recall that the marginal effects of age in Figures 4 and 19 were generated by normalizing the age of each industry relative to the peak employment level (with birth defined being at 5% of the maximum employment level) and binning based on deciles of the relative age. This approach—taken from the business cycle literature—is intended to deal with the issue of widely varying life cycle length, as documented in Figure 17.

As a robustness check, Figures 20 and 21 provide a similar marginal effect to Figures 4 and 19, but using the age (in years) directly. Hence, while this still uses the birth definition as 5% of maximum employment and controls for the number of firms and industry concentration, it no longer uses any normalizations. The general patterns are the same as in the normalized version with bins.

Figure 20: Effects of Age (Controls for # Firms & Concentration & Year Fixed Effects))

D.3 Year Effects

To show that the patterns of Figure 3 are connected to changes in profitability, and not simply due to industry composition effects, Figure 22 uses a panel of industries and markups with an industry fixed-effect (and no age effects). Of course, the goal of the paper is to

Figure 21: More Effects of Age (Controls for # Firms & Concentration & Year Fixed Effects))

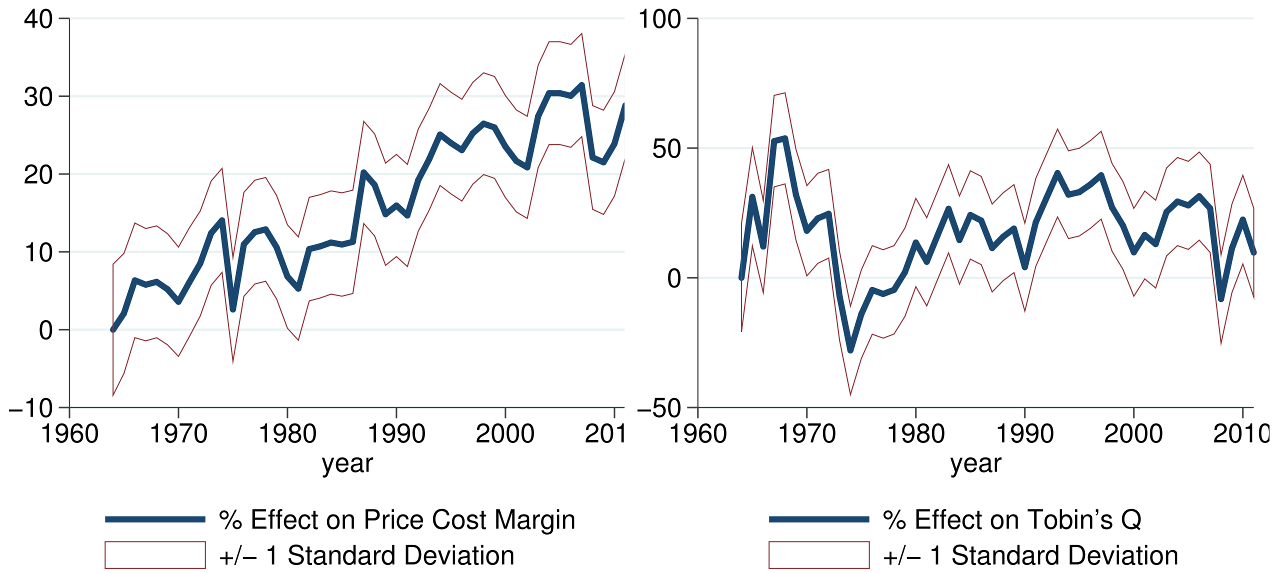

Figure 22: Year Effects on Markups and Tobin's Q (Controls for # Firms & Concentration & Industry Fixed Effects)

instead show that the year effects are in fact composed of life cycle effects, with panel results shown in Figure 4.

The data is described in detail in Section 2.2 and uses the NBER-CES Manufacturing Industry Database (MID), the Census Concentration Ratios, and Compustat. In calculating the marginal effects of the year, the panel regression controls for the industry with the fixed effect, and the industries concentration index and number of competing firms from the Census Concentration data. The pattern shows that the price cost margin is increasing fairly steadily over the sample. Tobin's Q is generally increasing after the 1970s, but is susceptible to aggregate shocks to valuations. Since this pattern exists even after controlling for industry fixed effects and changes in the competitiveness of the industry, I can then explore whether the life cycle and age of the industry are key factors, as I do in the empirics of Section 2.2.

Some additional results of the regression is given in Figure 23. This shows that the results of increasing markups over time are generally robust to the calculation method. The exception is the operating profit margin, which is very noisy and statistically insignificant in the range of the sample.

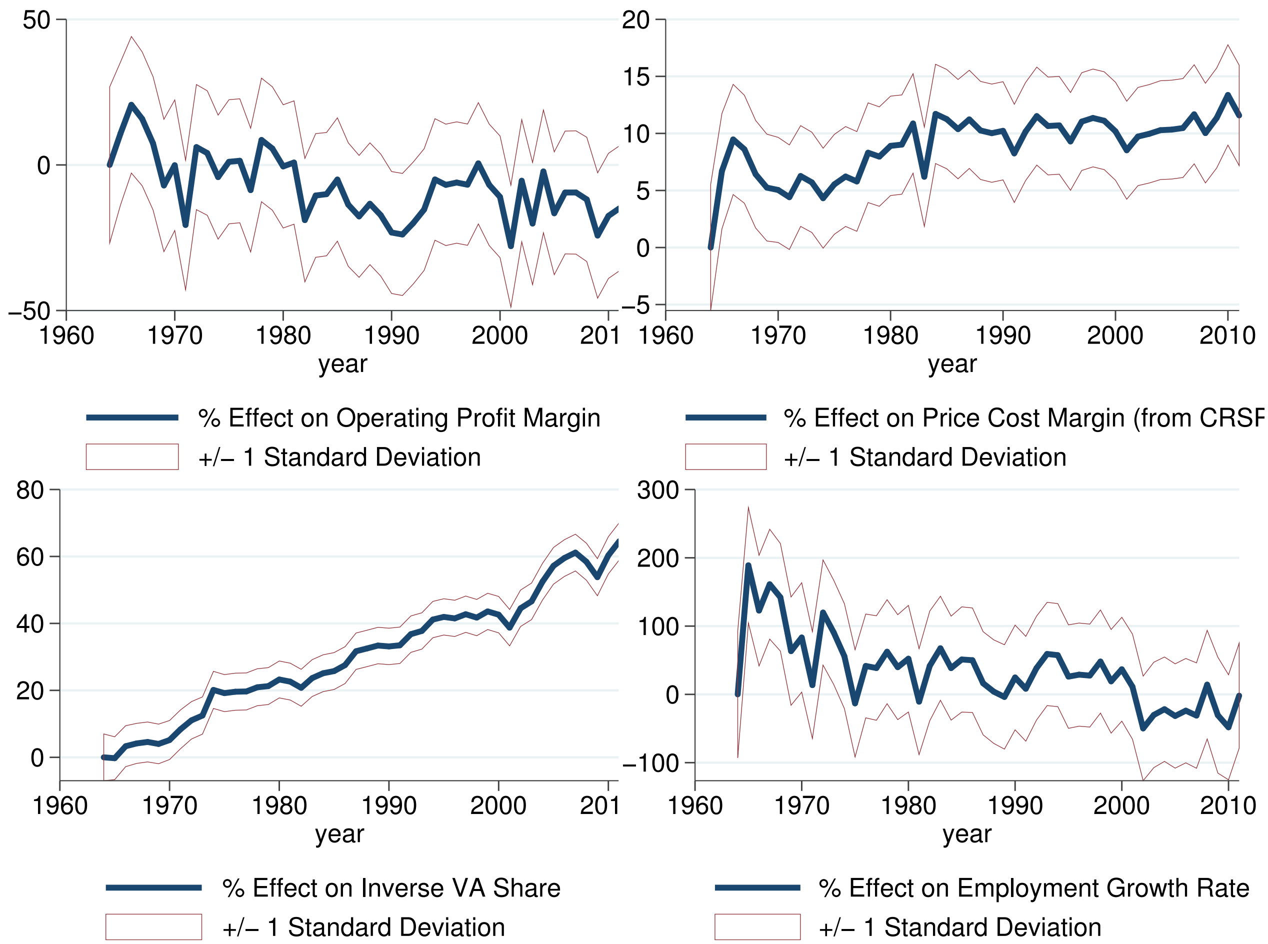

Figure 23: More Year Effects (Controls for # Firms & Concentration & Industry Fixed Effects)

Appendix E Calibration

This section describes the calibration for the key model parameters as summarized in Table 4.

E.1 Growth Rate Parameters

If the parameters for the awareness process, $\mathbb{Q}$, are θ and θ_d from Example 1, then the market saturation is

$$1 - f_0(a) = 1 - \frac{\theta + \theta_d}{\theta_d + \theta \exp\left((\theta_d + \theta)a\right)} \tag{E.1}$$

And the percent change of this is

$$\boldsymbol{\partial}_a \log(1 - f_0(a)) = (\theta + \theta_d)\left(\frac{1}{\exp\left((\theta_d + \theta)a\right) - 1} + \frac{\theta_d}{\theta \exp\left((\theta_d + \theta)a\right) + \theta_d}\right) \tag{E.2}$$

From (22), decompose the growth rate of output into the growth rate of market saturation, markups, and quality.

$$\boldsymbol{\partial}_a \log Y(a) = (\theta + \theta_d)\left(\frac{1}{\exp\left((\theta_d + \theta)a\right) - 1} + \frac{\theta_d}{\theta \exp\left((\theta_d + \theta)a\right) + \theta_d}\right) - \kappa \boldsymbol{\partial}_a \log \Upsilon(a) + \boldsymbol{\partial}_a \log q(a) \tag{E.3}$$

Recall that an age $a = 0$ in the data is based on some threshold, since we cannot see when the industry actually began. For example, in Figure 4, industry birth is defined as when the employment reaches 5% of its maximum level, and in the robustness check in Technical Appendix E.2, industry birth is defined as when the enterprise value of the industry reaches 5% of its maximum level.

Given this lag from birth, to bring estimate (E.3) from the data, I need to adjust the time frame by translating the age by a fixed number (i.e., the actual age of the industry when the data enters the sample). This translation itself is estimated, as well.

Calibrating with Age-Dependent Growth Rates with Nonlinear Least Squares From the age-dependent means in the panel data on firm growth rates, we can find a proxy for the growth rate of output, $\boldsymbol{\partial}_a \log Y(a)$, markup growth rates, $\boldsymbol{\partial}_a \log \Upsilon(a)$, and industry age (up to the undetermined translation of actual birth).

Given this data and our calibrated κ, ignoring quality growth, we can use nonlinear least squares to estimate (E.3) for the θ, θ_d and age-translation.[53] From this estimation, I find that

[53] Of course, $\Upsilon(a)$ and $q(a)$ are functions of the θ and θ_d parameters, as well. However, since these should enter independently into the function, I will not use the structure of $\Upsilon(a)$ and $q(a)$ in the nonlinear least squares.

$\theta = 0.019, \theta_d = 0.11$, and the age translation is 11.1. Since $\boldsymbol{\partial}_a \log q(a)$ would generally be increasing, these estimates end up a lower bound on the θ and θ_d parameters. For example, if the quality growth rate is 2% per annum, then these parameters are $\theta = 0.041, \theta_d = 0.14$, and 9.52.[54]

Robustness Check: Calibrate with Relative Growth Rates Alternatively, to calibrate the growth rate parameters, use the average industry peak age of 19 years from Figure 17, and then rescale the average proportion of maximum by decile in Figure 19. While the market saturation in the model does not exactly map to the growth rates, we can use it as a guideline on the 2-parameter awareness process. By picking three age bins, and comparing them to the data in Figure 19, I solve solve for the θ, θ_d, and age-translation as a system of three equations in (E.1). With the 10-20%, 20-30%, and 50-60% bins (with corresponding 26%, 31% and 46% saturation, respectively), the solution is $\theta = 0.02, \theta_d = 0.06$, and an age-translation is 9.1.

Time Scale Keeping the ratio of θ/θ_d constant, will maintain the shape of the awareness evolution, while any multiple of both will simply change the timescale. As the calibrated θ, and θ_d are based on age-dependent scales, I can adjust the ratio to hit age-independent ratios, such as those in Figure 18.

For example, to roughly hit the ratio of 50% of output at 50% of peak, the timescale can be adjusted by 1.5 so that $\theta = 0.06, \theta_d = 0.21$. As discussed, while the timescale changes the scale of the x-axis in graphs of the industry simulations, it is otherwise irrelevant. For the aggregation, the results on relative ratios, markups, etc. are not especially sensitive to changes in the values.[55]

E.2 Markups

Table 3 provides a summary of the average highest and the average lowest markups in the MID industry panel. Using the asymptotic theory of markups from Proposition 10, these numbers provide bounds on σ and κ. In particular, the minimum markup is bounded by $1+\sigma$, and the maximum markup is bounded by the monopolistically competitive $\kappa/(\kappa-1)-1$.

[54] The degree of sensitivity in the estimation of the parameters is due primarily to the low number of parameters in the $\mathbb{Q}$ process. However, the quantitative results of the paper are robust to fairly large changes in the value of θ and θ_d, so the difficulty in calibration is less troubling than it appears.

[55] These parameters can more strongly affect the scale of the economy, but I leave the scale uncalibrated to focus on factor shares and scale-independent equilibrium objects.

	mean	sd	min	max
Minimum Price Cost Markup	0.21	0.09	-0.19	0.50
Maximum Price Cost Markup	0.39	0.11	0.18	1.30
Observations	4406			

Table 3: Summary Statistics for Min and Max Markup by Industry

E.3 Product Obsolescence

In the simple aggregate model with a constant hazard, the survivor function for product categories is,

$$\text{Survival}(a) = e^{-\delta_M a} \tag{E.4}$$

The obsolescence rate δ_M is difficult to measure directly since it is related to product category—*not* firm—exit rates. I consider several alternatives to discipline the parameter:

- Reinterpreting Example 2 in Atkeson and Burstein (2015) as a measure of obsolescence, provides a δ_M of 0.0225.
- Broda and Weinstein (2010) finds that households spend 20% of their money on goods that will disappear in the next 4 years. Using the 80% survival after 4 years with (E.4) leads to $\delta_M = 0.056$.
- Finally, using trademark data directly from Table 2, if the survival rate of trademarks is 16% after 10 years, then $\delta_M = 0.18$.

As the baseline value for calibrations, I will use the 0.056 estimate since it is the only direct measure of products (calculated from Nielson scanner data) and is in the middle of the range.

E.4 Calibration Summary

The parameters for the experiments are summarized in Table 4.

[56] Recall that due to the distortions in markups and profits, this can no longer be calibrated directly from the labor share proportion.

Variable	Value	Description
σ	≤ 0.21	See Appendix E.2. Minimum industry markup bound from Proposition 10. Calculated as the *average minimum markup* from NBER-CES Manufacturing Industry Database as summarized in Table 3. Baseline is $\sigma = 0.15$.
$\kappa/(\kappa-1)-1$	≥ 0.39	See Appendix E.2. Maximum industry bound from Proposition 10. Calculated as the *average maximum markup* from NBER-CES Manufacturing Industry Database, and summarized in Table 3. Baseline is $\kappa = 3.5$
θ	> 0.019	See Appendix E.1. From Nonlinear Least Squares, industry panel growth rates, and theoretical bounds. See Appendix E.1. Uses $\theta = 0.06$ as the baseline.
θ_d	> 0.11	See Appendix E.1. From Nonlinear Least Squares, industry panel growth rates, and theoretical bounds. See Appendix E.1. Uses $\theta_d = 0.21$ as the baseline.
δ_M	$[0.0225, 0.18]$	See Appendix E.3. From Broda and Weinstein (2010), trademark obsolescence rates, or Atkeson and Burstein (2015). See Appendix E.3. Uses $\delta_M = 0.056$ from Broda and Weinstein (2010) as the baseline.
N	Irrelevant	With the θ and θ_d above, the N is essentially irrelevant (as long as it is above 5-10). Growth in $\hat{n}$ is estimated to be too slow to converge close to N prior to obsolescence.
δ_k	0.07	Typical capital depreciation rate
α	0.28	Set from the 1980 corporate labor share in the data, with the factor share distortion, $B(t)$, derived in Section 4.[56]
ρ	0.03	A typical interest rate target
γ	$[1, 5]$	Typical range of elasticity of intertemporal substitution
z, z_m, ν	N/A	Level effects, not calibrated

Table 4: Parameter Calibration

Appendix F Summary of Sources

This section provides a quick summary of the data used in constructing the figures and in the calibration. See Technical Appendix F for more details on the construction of the data.

Source	Description	Used For
Compustat/WRDS	Collapsed from firm data to a panel of 1375 industries (manufacturing and other) from 1950 to 2015. Includes financial data on firm valuations, employment, revenue, profits, operating margins, etc. for large public firms in the US across a variety of industries	Figures 3, 4 and 17 to 23 and Appendix E.1 and Technical Appendix Figure 2
NBER-CES Manufacturing Industry Database (MID)	Panel of 473 manufacturing industries from 1958 to 2011. Includes industry employment, price indices, revenue, TFP, etc.	Figures 4 and 17 to 23, Appendix E.1, and Table 3
Census Concentration Ratios	Panel of 482 manufacturing industries from 1935 to 2012. Includes various calculations of industry concentration, such as Herfindahl indices, the proportion of revenue in the top 8 firms, the # of firms, etc.	Figures 4 and 17 to 23
FRED	Yearly data from 1929 to 2015. Includes national accounts data, such as the profit shares, corporate profit share, GDP, price deflators, Tobin's Q, etc.	Figures 1 and 3 and Table 1 and Technical Appendix Figures 1 and 2
USPTO Trademark Case Files	Data on 6.7 million trademarks filed between the early 1800s and 2012. In practice, only post-1980 data is useful for calculating abandonment rates, of which there is data on 5.4 million trademarks. The data includes all events described in Technical Appendix F.1	Figure 2 and Appendix E.3 and Technical Appendix Figure 1
USPTO Patent Examination Public Pair	Detailed data on 9.2 million patent applications from the early 1900s to 2014. Useful data includes details on abandonment and disposal types, which enable more of a detailed analysis of abandonment rates and reasons for abandonment.	Figure 2 and Table 2 and Technical Appendix Figure 1
USPTO Historical Patent Data Files	Better historical data on 11 million patent applications from the mid-1800s to 2014 than the USPTO Patent Examination Public Pair, but fewer details. In practice, can only use post-1980 data for calculating abandonment rates, of which there are approximation 6.8 million applications in the data sample.	Figures 1 and 2 and Tables 1 and 2 and Technical Appendix Figure 1
World Advertising Research Center (WARC)	Total US spending on advertising by media type from 1980 to 2015.	Technical Appendix Figure 2

Table 5: Summary of Data Sources